Cosmological dynamics and perturbations in Light mass Galileon models

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13^{th} December, 2017, CosPA 2017, YITP, Kyoto University, Japan

Plan

- Introduction to Light Mass Galileon
- Background Cosmology
- First and Second Order Perturbations

Cosmic History

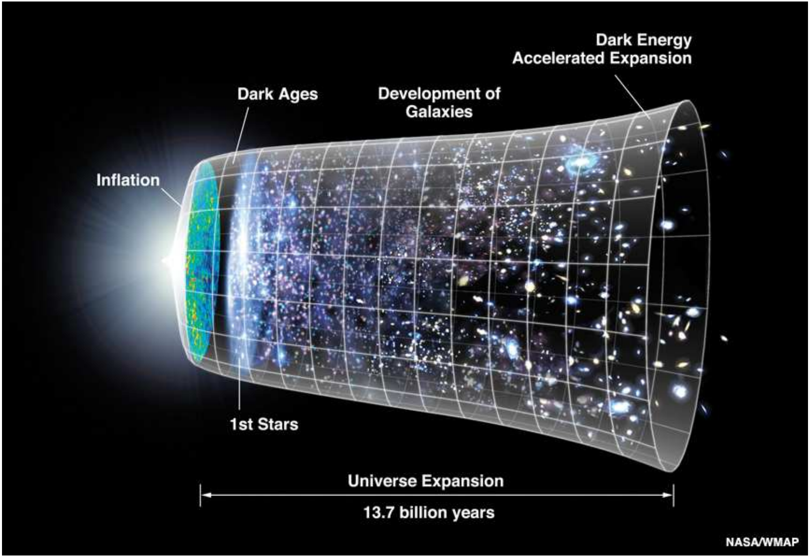

Figure: Cosmic history. Picture is taken from wfirst.gsfc.nasa.gov.

- Cosmic acceleration $\Longrightarrow$ Equation of state $\Longrightarrow$ $w =$ Pressure/Density $< -\frac{1}{3}$.
- Observationally $\Longrightarrow$ $w_{\rm inf} \approx -1$ and also currently $w_{\rm DE,0} \approx -1$.

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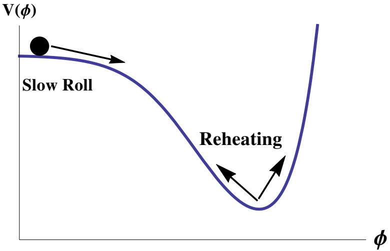
Inflation:

$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi + V(\phi)$. ϕ is a scalar field.



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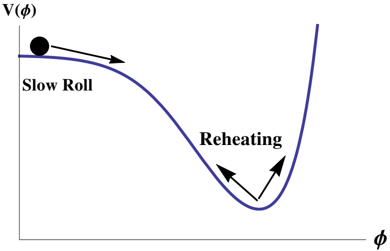

Dark Energy:

Simplest candidate can be Λ.

Fine tuning problem

$$\Longrightarrow \boxed{\frac{\rho_{\Lambda,\rm obs}}{\rho_{\Lambda,\rm theo}} = 10^{-120}}$$

Cosmic Coincidence

$$\Longrightarrow \boxed{\rho_\Lambda \approx \rho_{\rm m0}}\ .$$

Alternatives $\Longrightarrow$ Make DE dynamical $\Longrightarrow$ Modification of gravity.

Most general scalar-tensor Lagrangian

$$\mathcal{L} = \sum_{i=1}^{5} c_i \mathcal{L}_i$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X)\,, \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}(\phi, X)\Big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 \\
&\qquad + 2(\nabla_\mu\nabla_\nu\phi)^3\Big],
\end{aligned}$$

- with K and G_i $(i = 3, 4, 5)$ are arbitrary functions of ϕ and $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and $G_{iX} = \partial G_i/\partial X$.
- Second order equation of motion.

Light Mass Galileon

- $K(\phi, X) = X - V(\phi)$, $\quad G_3(\phi, X) = 2X \quad$ and $\quad G_4 = G_5 = 0$.
- $$\mathcal{L}_2 = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)\,,$$
  $$\mathcal{L}_3 = -(\partial_\mu \phi)^2 \Box \phi\,,$$

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-

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$$\mathcal{L}_3 = -(\partial_\mu \phi)^2 \Box \phi\,,$$

- if $V(\phi) \sim \phi$ then ϕ-field preserves galilean shift symmetry in the flat space time

$$\phi \to \phi + b_\mu x^\mu + c$$
$$\partial_\mu \phi \to \partial_\mu \phi + b_\mu$$

$\Longrightarrow$ Galileon field.

- There are five Lagrangians which preserve the shift symmetry and give second order equation of motion.

Galileon Lagrangian

$$\mathcal{L} = \sum_{i=1}^{5} c_i \mathcal{L}_i$$

$$\begin{aligned}
\mathcal{L}_1 &= M^3 \phi\,, \\
\mathcal{L}_2 &= \frac{1}{2}(\partial_\mu \phi)^2\,, \\
\mathcal{L}_3 &= \frac{1}{M^3}(\partial_\mu \phi)^2 \Box\phi\,, \\
\mathcal{L}_4 &= \frac{1}{M^6}\phi_{;\mu}\phi^{;\mu}\left[2(\Box\phi)^2 - 2\phi_{;\mu\nu}\phi^{;\mu\nu} - \frac{1}{2}R\phi_{;\mu}\phi^{;\mu}\right], \\
\mathcal{L}_5 &= \frac{1}{M^9}\phi_{;\mu}\phi^{;\mu}\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{\mu\nu}\phi^{;\mu\rho}\phi^{;\nu}{}_{;\rho} - 6\phi_{;\mu}\phi^{;\mu\nu}\phi^{;\rho}G_{\nu\rho}\right].
\end{aligned}$$

- Second order equation of motion.
- Can explain late time acceleration of the Universe.
- Local physics is restored through the Vainshtein mechanism.

The Model

A. Ali, R. Gannouji, MWH and M. Sami, Phys. Lett. B **718**, 5 (2012)
MWH, Phys. Rev. D **96**, no. 2, 023506 (2017)

Action

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2\Big(1 + \frac{\alpha}{M^3}\Box\phi\Big) - V(\phi)\Big]$$
$$+\mathcal{S}_m\Big[\psi_m; e^{2\beta\phi/M_{pl}}g_{\mu\nu}\Big]$$

- EH action is modified with Galileon Lagrangian $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(3)}$ and with a potential $\Longrightarrow$ Only $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(3)}$ can't give late time accelaration.
- Potential is added phenomenologically to get accelaration.
- Potential $\Longrightarrow$ breaks shift symmetry even in the flat background.
- Non-linear self interaction term of the galileon field plays the main role to preserve the local physics through Vinshtein mechanism.

$V = V_0 \left(\cosh\left(\alpha\phi/M_{\rm Pl}\right) - 1\right)^p$

V. Sahni and L. M. Wang, PRD **62**, 103517 (2000).

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$$\begin{aligned} V(\phi) &= \frac{V_0}{2}\mathrm{e}^{\alpha p\phi/M_{\rm Pl}}, \qquad \text{for} \quad \frac{\alpha\phi}{M_{\rm Pl}} \gg 1\,, \quad \phi > 0 \\ V(\phi) &= \frac{V_0}{2}\left(\frac{\alpha\phi}{M_{\rm Pl}}\right)^{2p}, \qquad \text{for} \quad \frac{\alpha\phi}{M_{\rm Pl}} \ll 1 \end{aligned}$$

M. S. Turner, PRD **28**, 1243 (1983).

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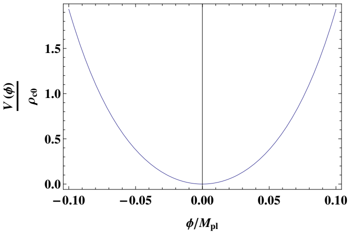
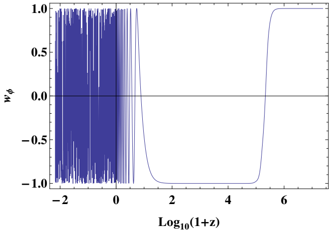
$$< w_\phi >= \left\langle \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)} \right\rangle = \frac{p-1}{p+1\,.}$$

$V = V_0 \left(\cosh\left(\alpha\phi/M_{\rm Pl}\right) - 1\right)^p$

$\alpha = 20$ and $p = 1$.





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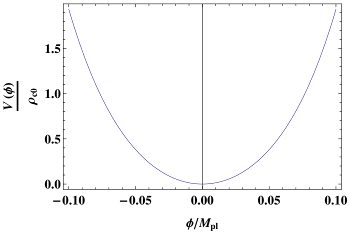
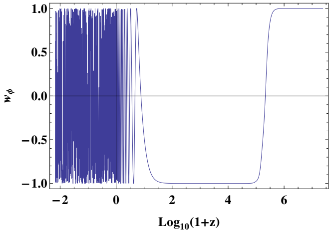
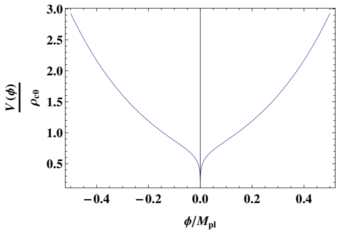
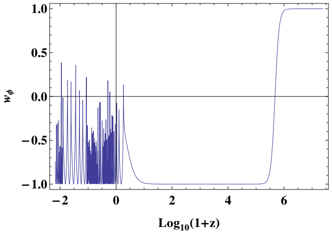




$\alpha = 30$ and $p = 0.1$.





T. Barreiro, E. J. Copeland and N. J. Nunes, PRD **61**, 127301 (2000)

$$V = V_0 \left(e^{-\mu_1 \phi / M_{Pl}} + e^{-\mu_2 \phi / M_{Pl}} \right) .$$

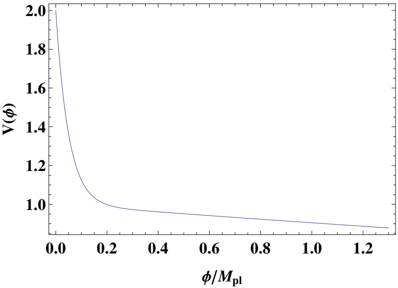


Figure: $\mu_1 = 20$ and $\mu_2 = -0.1$.

$V = V_0 \left(\mathrm{e}^{-\mu_1 \phi / \mathrm{M_{Pl}}} + \mathrm{e}^{-\mu_2 \phi / \mathrm{M_{Pl}}} \right)$

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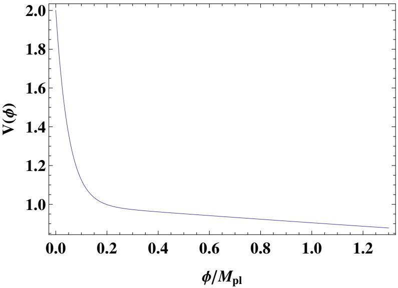

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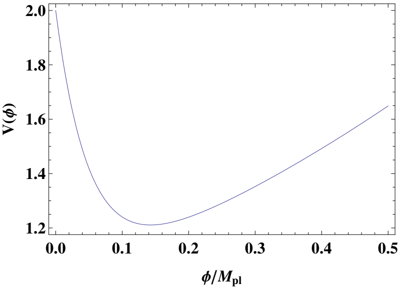

Figure: $\mu_1 = 20$ and $\mu_2 = 1$.

Cosmological Evolution

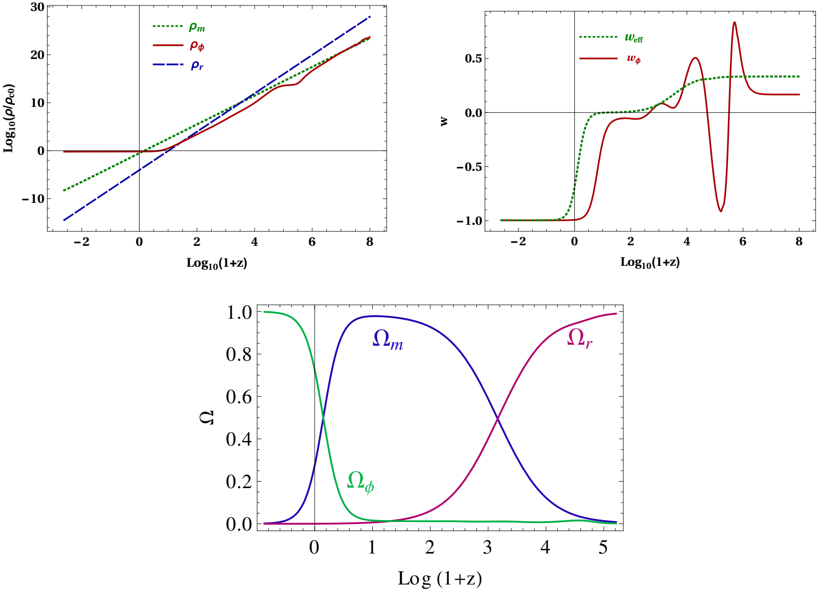

Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\beta = 0.01$.

We consider the following metric in the Newtonian gauge

$$\mathrm{ds}^2 = \mathrm{a}(\tau)^2 \Big[-(1+2\Phi)\mathrm{d}\tau^2 + (1-2\Psi)\mathrm{d}\tilde{\mathrm{x}}^2 \Big] ,$$

where τ is the conformal time, Φ and Ψ are the scalar perturbations of the metric.

We expand the perturbations in series,

$$\begin{aligned} \Phi &= \Phi_1 + \frac{1}{2!}\Phi_2 + \frac{1}{3!}\Phi_3 + \cdots , \\ \Psi &= \Psi_1 + \frac{1}{2!}\Psi_2 + \frac{1}{3!}\Psi_3 + \cdots , \end{aligned}$$

Linear Perturbation

A. Ali, R. Gannouji, MWH and M. Sami, Phys. Lett. B **718**, 5 (2012)
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Equation for the linear density contrast in subhorizon ($k^2 \gg \mathcal{H}^2$) and quasistatic ($|\ddot{\phi}| \lesssim \mathcal{H}|\dot{\phi}| \ll k^2|\phi|$) approximations,

$$\ddot{\delta}_1 + (\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\dot{\phi})\dot{\delta}_1 - 4\pi G_{\rm eff} a^2 \bar{\rho}_{\rm m}\delta_1 = 0$$

$G_{\rm eff}$

$$G_{\rm eff} = G\left(1 + \frac{\left(\frac{\alpha}{M^3 a^2 M_{\rm Pl}}\dot{\phi}^2 + 2\beta\right)^2}{2 - \frac{4\alpha}{M^3 a^2}\left(\ddot{\phi} + \mathcal{H}\dot{\phi}\right) - \frac{\alpha^2}{M^6 a^4 M_{\rm Pl}^2}\dot{\phi}^4}\right)$$

Evolution of $G_{\rm eff}$

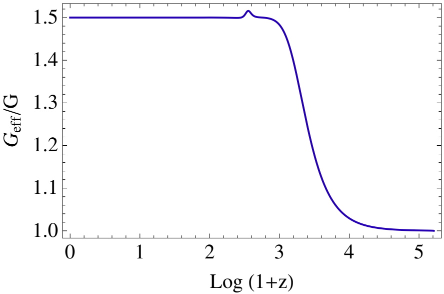

Figure: Evolution of $G_{\rm eff}/G$ with $\mu_1 = 20$, $\mu_2 = 0.5$ and $\beta = 0.5$.

MWH, Phys. Rev. D **96**, no. 2, 023506 (2017)

Using the transformation

$$\tilde{a} = a\ \mathrm{e}^{\beta\phi/\mathrm{M_{Pl}}}$$

Evolution equation of the density contrast can be written as

$$\ddot{\delta}_1 + \tilde{\mathcal{H}}\dot{\delta}_1 - 4\pi\tilde{G}_{\mathrm{eff}}\tilde{a}^2\bar{\rho}_{\mathrm{m}}\delta_1 = 0$$

where

$$\tilde{\mathcal{H}} = \frac{1}{\tilde{a}}\frac{\mathrm{d}\tilde{a}}{\mathrm{d}\tau} = \mathcal{H} + \frac{\beta}{M_{\mathrm{Pl}}}\dot{\phi} = \mathcal{H}\frac{\mathrm{d}\ln\tilde{a}}{\mathrm{d}\ln a}$$

$$\tilde{G}_{\mathrm{eff}} = G_{\mathrm{eff}}\ \mathrm{e}^{-(2\beta/\mathrm{M_{Pl}})\phi}$$

Growing and decaying modes: Integral solutions

Growing Mode

$$D_+(\tilde{a}) \quad = \quad \tilde{a}_{\rm m}^{7/4} e^{-3\beta\phi_{\rm m}/4M_{\rm Pl}} \left(\frac{\mathcal{A}(\phi_{\rm m})}{\mathcal{A}(\phi_0)}\right)^{1/2} \frac{\gamma(\tilde{a})}{\tilde{a}} \sqrt{\frac{\tilde{\mathcal{H}}_0}{\tilde{\mathcal{H}}}}$$

Decaying Mode

$$D_-(\tilde{a}) \quad = \quad \tilde{a}_{\rm m}^{-3/4} e^{7\beta\phi_{\rm m}/4M_{\rm Pl}} \left(\frac{\mathcal{A}(\phi_{\rm m})}{\mathcal{A}(\phi_0)}\right)^{1/2} \frac{\gamma(\tilde{a})}{\tilde{a}} \sqrt{\frac{\tilde{\mathcal{H}}_0}{\tilde{\mathcal{H}}}} \times \left[1 - \frac{5}{2}\frac{1}{\tilde{a}_{\rm m}\mathcal{A}(\phi_{\rm m})}\int_{\tilde{a}_{\rm m}}^{\tilde{a}} \frac{d\tilde{a}'}{\gamma^2(\tilde{a}')}\right]$$

Growing and decaying modes: Integral solutions

Where,

$$\mathcal{A}(\phi) = \frac{\mathrm{d}\ln\tilde{\mathrm{a}}}{\mathrm{d}\ln\mathrm{a}} = 1 + \frac{\beta}{M_{\mathrm{Pl}}}\frac{\mathrm{d}\phi}{\mathrm{d}\ln\mathrm{a}}$$

And

$$\gamma^2(\tilde{a}) = \mathrm{e}^{-\int_{\tilde{\mathrm{a}}_\mathrm{m}}^{\tilde{\mathrm{a}}}\mathrm{d}\tilde{\mathrm{a}}'\mathrm{g}(\tilde{\mathrm{a}}')}$$

Where $g(\tilde{a}')$ satisfies

$$\frac{\mathrm{dg}(\tilde{\mathrm{a}})}{\mathrm{d}\tilde{\mathrm{a}}} - \frac{1}{2}g^2(\tilde{a}) + 2I(\tilde{a}) = 0$$

With

$$I(\tilde{a}) = A(\tilde{a}) + \frac{1}{\tilde{a}\tilde{\mathcal{H}}}\frac{\mathrm{d}\tilde{\mathcal{H}}}{\mathrm{d}\tilde{\mathrm{a}}} - \frac{1}{4\tilde{\mathcal{H}}^2}\left(\frac{\mathrm{d}\tilde{\mathcal{H}}}{\mathrm{d}\tilde{\mathrm{a}}}\right)^2 + \frac{1}{2\tilde{\mathcal{H}}}\frac{\mathrm{d}^2\tilde{\mathcal{H}}}{\mathrm{d}\tilde{\mathrm{a}}^2}$$

$$A(\tilde{a}) = 4\pi\tilde{G}_{\mathrm{eff}}\frac{\bar{\rho}_\mathrm{m}}{\tilde{\mathcal{H}}^2}$$

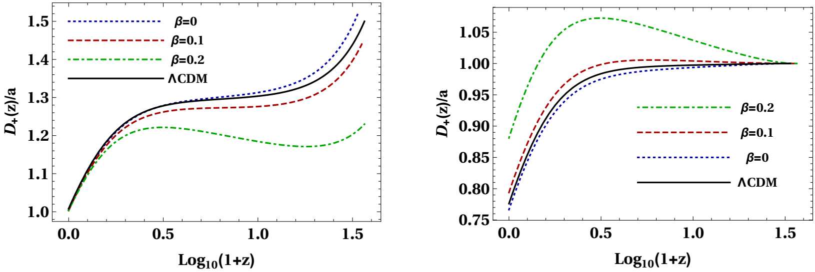

Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\beta = 0,\ 0.1,\ 0.2.$

Density Contrast: Comparison

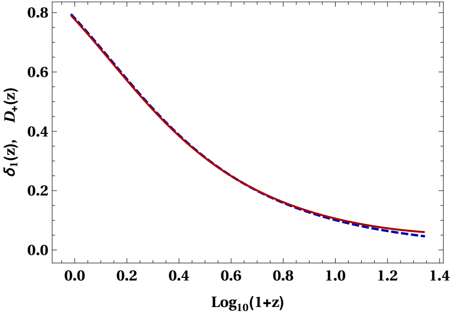

Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\beta = 0.1$.

Power spectrum and $f\sigma_8$

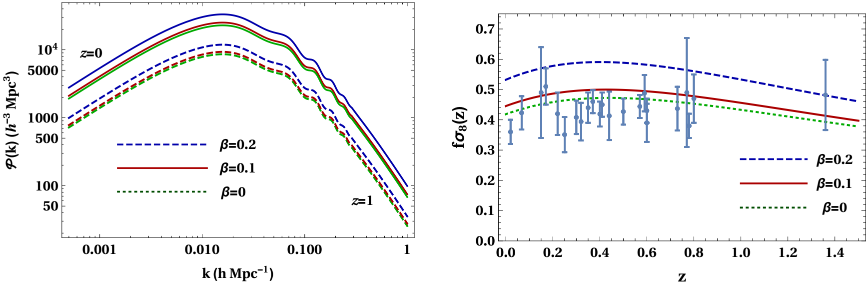

Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $\Omega_{\rm b} = 0.04$, $\Omega_{\rm m} = 0.3$ and $n_{\rm s} = 0.968$.

Second Order Perturbation

Equation for the second order density contrast,

$$\ddot{\delta}_2 + \left(\mathcal{H} + \frac{\beta}{M_{\rm Pl}}\dot{\phi}\right)\dot{\delta}_2 - 4\pi G_{\rm eff} a^2 \bar{\rho}_{\rm m}\delta_2 = S_\delta$$

Fourier transform of S_δ can be written as

$$S_\delta(a,\vec{k}) = \int {\rm d}^3{\rm k}_1 {\rm d}^3{\rm k}_2 \delta^{(3)}(\vec{k}-\vec{k}_1-\vec{k}_2)\mathcal{K}(a,\vec{k}_1,\vec{k}_2)\delta_1(a,\vec{k}_1)\delta_1(a,\vec{k}_2)$$

Second Order Perturbation

Second Order Density Contrast

$$\delta_2(\tilde{a},\vec{k}) = D_+(\tilde{a})\delta_2(\vec{k}) - D_+(\tilde{a})\int_{\tilde{a}_m}^{\tilde{a}} \frac{D_-(\tilde{a}')\hat{S}_\delta(\tilde{a}',\vec{k})}{\tilde{a}'^2\tilde{\mathcal{H}}^2(\tilde{a}')W_r(\tilde{a}')}\mathrm{d}\tilde{a}' + D_-(\tilde{a})\int_{\tilde{a}_m}^{\tilde{a}} \frac{D_+(\tilde{a}')\hat{S}_\delta(\tilde{a}',\vec{k})}{\tilde{a}'^2\tilde{\mathcal{H}}^2(\tilde{a}')W_r(\tilde{a}')}\mathrm{d}\tilde{a}'$$

$$W_r(\tilde{a}) = D_+(\tilde{a})\frac{\mathrm{d}D_-(\tilde{a})}{\mathrm{d}\tilde{a}} - D_-(\tilde{a})\frac{\mathrm{d}D_+(\tilde{a})}{\mathrm{d}\tilde{a}}$$

Second Order Perturbation

$$\delta(\tilde{a},\vec{k}) = \delta_1(\tilde{a},\vec{k}) + \frac{1}{2}\delta_2(\tilde{a},\vec{k}) = D_+(\tilde{a})\delta_1(\vec{k}) + \int \mathrm{d}^3\mathrm{k}_1\mathrm{d}^3\mathrm{k}_2\delta^{(3)}(\tilde{\mathrm{k}} - \tilde{\mathrm{k}}_1 - \tilde{\mathrm{k}}_2)$$
$$\times\mathcal{F}_2(\tilde{a},\vec{k}_1,\vec{k}_2)\delta_1(\tilde{a},\vec{k}_1)\delta_1(\tilde{a},\vec{k}_2)$$

Second Order Kernel

$$\mathcal{F}_2(\tilde{a},\vec{k}_1,\vec{k}_2) = \int_{\tilde{a}_\mathrm{m}}^{\tilde{a}} \mathrm{d}\tilde{a}' \, \frac{\mathcal{D}(\tilde{a},\tilde{a}')\mathcal{K}(\tilde{a}',\tilde{\mathrm{k}}_1,\tilde{\mathrm{k}}_2)}{2\tilde{a}'^2\tilde{\mathcal{H}}^2(\tilde{a}')\mathrm{W}_\mathrm{r}(\tilde{a}')}$$

$$\mathcal{D}(\tilde{a},\tilde{a}') = \frac{D_+^2(\tilde{a}')}{D_+^2(\tilde{a})}\left(D_-(\tilde{a})D_+(\tilde{a}') - D_+(\tilde{a})D_-(\tilde{a}')\right)$$

Bispectrum

$$\langle \delta(\tau, \vec{k})\delta(\tau, \vec{k}')\delta(\tau, \vec{k}'')\rangle = \delta^{(3)}(\vec{k} + \vec{k}' + \vec{k}'')\mathcal{B}(\tau, k, k')$$

$$\mathcal{B}(\tau, k, k') = 2\mathcal{F}_2(\vec{k}, \vec{k}')\mathcal{P}(k)\mathcal{P}(k') + \mathrm{cyc}$$

$$\mathcal{Q} = \frac{\mathcal{B}(\tau, k, k')}{\mathcal{P}(\tau, k)\mathcal{P}(\tau, k') + \mathcal{P}(\tau, k')\mathcal{P}(\tau, k'') + \ldots}$$

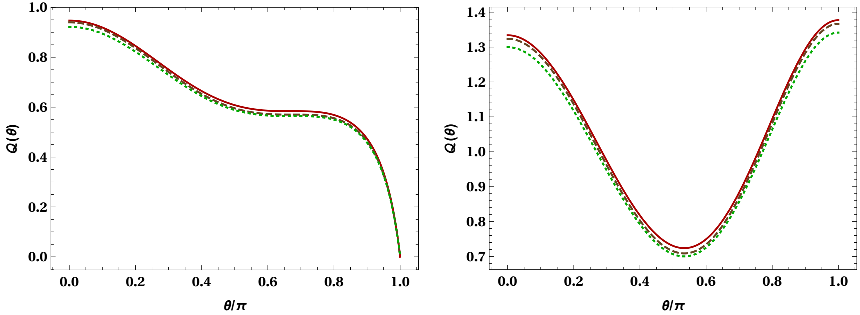

Figure: $\mu_1 = 20$, $\mu_2 = 0.1$ and $k = k' = 0.01 \ \mathrm{hMpc}^{-1}$ and $5k = k' = 0.05 \ \mathrm{hMpc}^{-1}$ for $\beta = 0, \ 0.5$ and ΛCDM.

Summary

- We have discussed the effect of the conformal coupling at the perturbation level in a tracker scalar field model with a cubic Galileon correction term.
- Integral solution of the growing and decaying modes are calculated in the subhorizon approximation.
- Effect of the conformal coupling constant on matter power spectrum and bispectrum has been observed.
- The power spectrum changes for different conformal constant but there is no significant change in the reduced bispectrum.
- Comparison with $f\sigma_8$ data shows that higher values of the conformal constant can be ruled out.

THANK YOU