YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

Gravitational Wave Physics and Astronomy
Genesis

Constant-roll inflation

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Martin, HM, Suyama, PRD 87, 023514 (2013), [arXiv:1211.0083]
HM, Starobinsky, Yokoyama, JCAP 1509, 018 (2015), [arXiv:1411.5021]
HM, Starobinsky, EPL 117, 39001 (2017), [arXiv:1702.05847]
HM, Starobinsky, Eur.Phys.J. C77, 538 (2017), [arXiv:1704.08188]

2017.12.11-15 CosPA2017, YITP

Canonical single field Inflation

$$3H^2 = \frac{\dot{\phi}^2}{2} + V \qquad (M_{Pl} = 1)$$

$$-2\dot{H} = \dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

Slow-roll : $\ddot{\phi}/H\dot{\phi} \simeq 0$

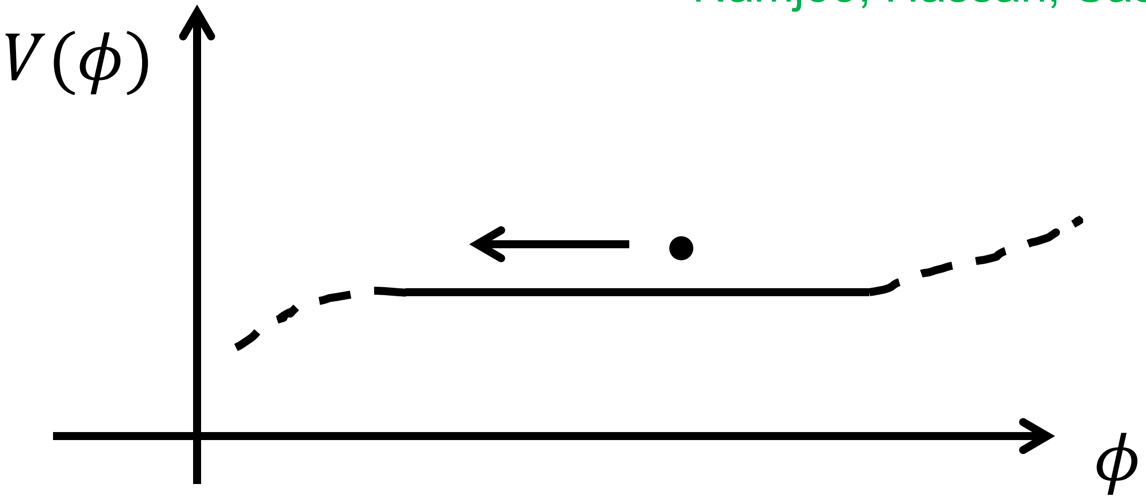
Ultra slow-roll : constant potential $\ddot{\phi} = -3H\dot{\phi}$

Constant-roll : generalization $\ddot{\phi} = \beta H\dot{\phi}$ (β: constant)

Ultra slow-roll inflation

Kinney, gr-qc/0503017

Namjoo, Hassan, Sasaki 1210.3692



Constant potential $V \simeq V_0$

$$\ddot{\phi} = -3H\dot{\phi} \quad \Rightarrow \quad \dot{\phi} \propto a^{-3}$$

$$3H^2 = \frac{\dot{\phi}^2}{2} + V \simeq V_0$$

Slow-roll parameter

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-6}$$

Ultra slow-roll inflation

Ultra slow roll $\ddot{\phi} = -3H\dot{\phi} \Rightarrow \epsilon_H \propto a^{-6}$

Curvature perturbation on superhorizon scales

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode

Slow-roll $\epsilon_H \simeq$ const $\ll 1$: Decaying mode ✓

Ultra slow-roll $\epsilon_H \propto a^{-6}$: Growing mode ✗

While ϵ_H is small, $d\ln\epsilon_H/dN = -6$ violates SR.

One could have nearly scale-inv. spectrum with growing mode and then terminate USR, after which, however, growing mode becomes decaying mode and cannot explain observables, while a smooth transition from USR to SR may allow to take over amp. of growing mode to const. mode.

Generalization

Martin, HM, Suyama, 1211.0083

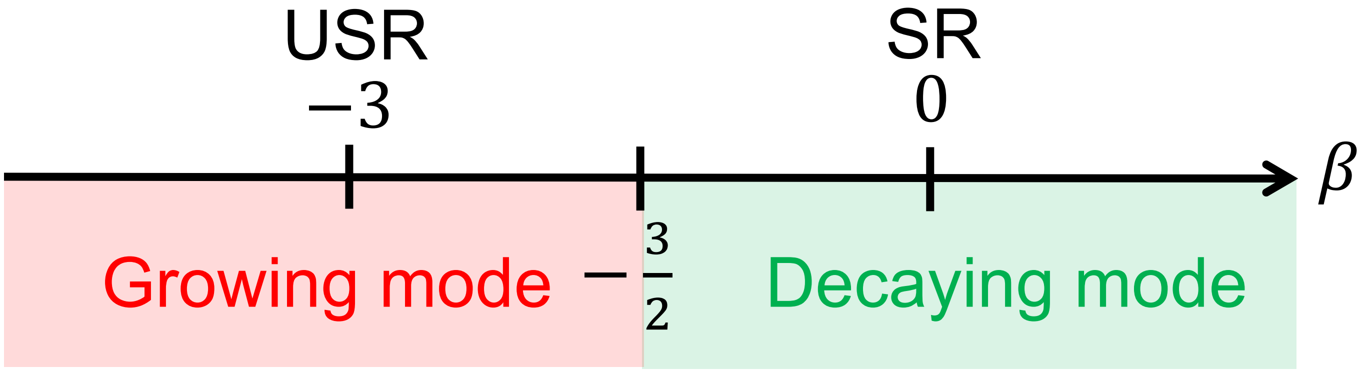
Constant roll $\ddot{\phi} = \beta H \dot{\phi} \;\Rightarrow \epsilon_H \propto a^{2\beta}$

Curvature perturbation on superhorizon scales

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode

$$\frac{d\ln\epsilon_H}{dN} = 2\beta \gtrless -3$$



HM, Starobinsky, Yokoyama, 1411.5021

Constant-roll potential

With constant-roll condition $\ddot{\phi} = \beta H \dot{\phi}$,

the evolution eq. $-2\dot{H} = \dot{\phi}^2$ or $\dot{\phi} = -2\frac{dH}{d\phi}$ implies

$$\frac{d^2H}{d\phi^2} = -\frac{\beta}{2}H$$

Analytic solution:

$H(\phi) =$ linear comb. of $e^{\pm\sqrt{-\beta/2}\phi}$

$\Rightarrow V(\phi) = 3H^2 - 2\left(\frac{dH}{d\phi}\right)^2 =$ linear comb. of $e^{\pm\sqrt{-2\beta}\phi}$

For each solution of $V(\phi)$,

we can derive analytic solution for $\phi(t), H(t), a(t)$.

Constant-roll potential

Abbott, Wise, 1984

Lucchin, Matarese, 1985

Analytic solutions include:

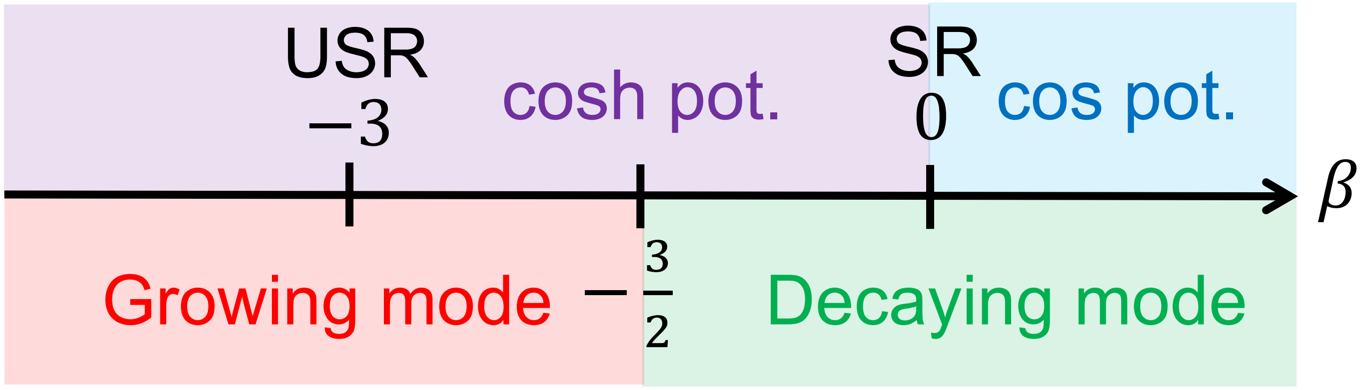
a) $V \propto e^{\sqrt{-2\beta}\phi}$ with $\beta < 0$: Power-law inflation

❌ but $r = 8(1 - n_s) \approx 0.28$ is too large.

Barrow, 1994

b) $V \propto \cosh(\sqrt{-2\beta}\phi) + \text{const}$ with $\beta < 0$: Known

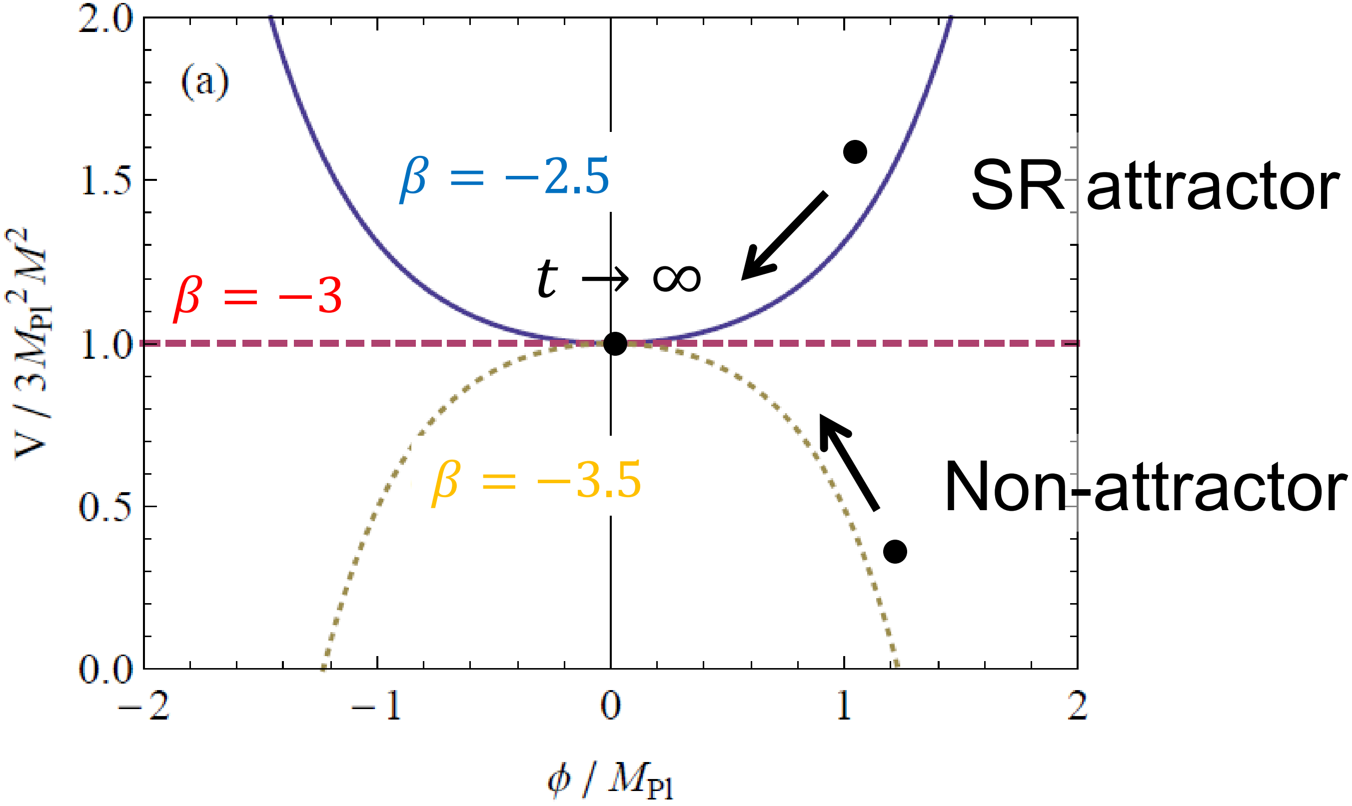
c) $V \propto \cos(\sqrt{2\beta}\phi) + \text{const}$ with $\beta > 0$: New potential



cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = 0$.



cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = 0$.

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = \sqrt{\frac{2}{-\beta}}\ln\left[\coth\left(\frac{-\beta}{2}Mt\right)\right] \to 0 \qquad (t \to \infty)$$

$$\frac{H(t)}{M} = \coth(-\beta M t) \to 1$$

$$a(t) \propto \sinh^{-1/\beta}(-\beta M t) \to e^{Mt}$$

$$\epsilon_1 \equiv -\dot{H}/H^2$$
$$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$$

Slow-roll parameters

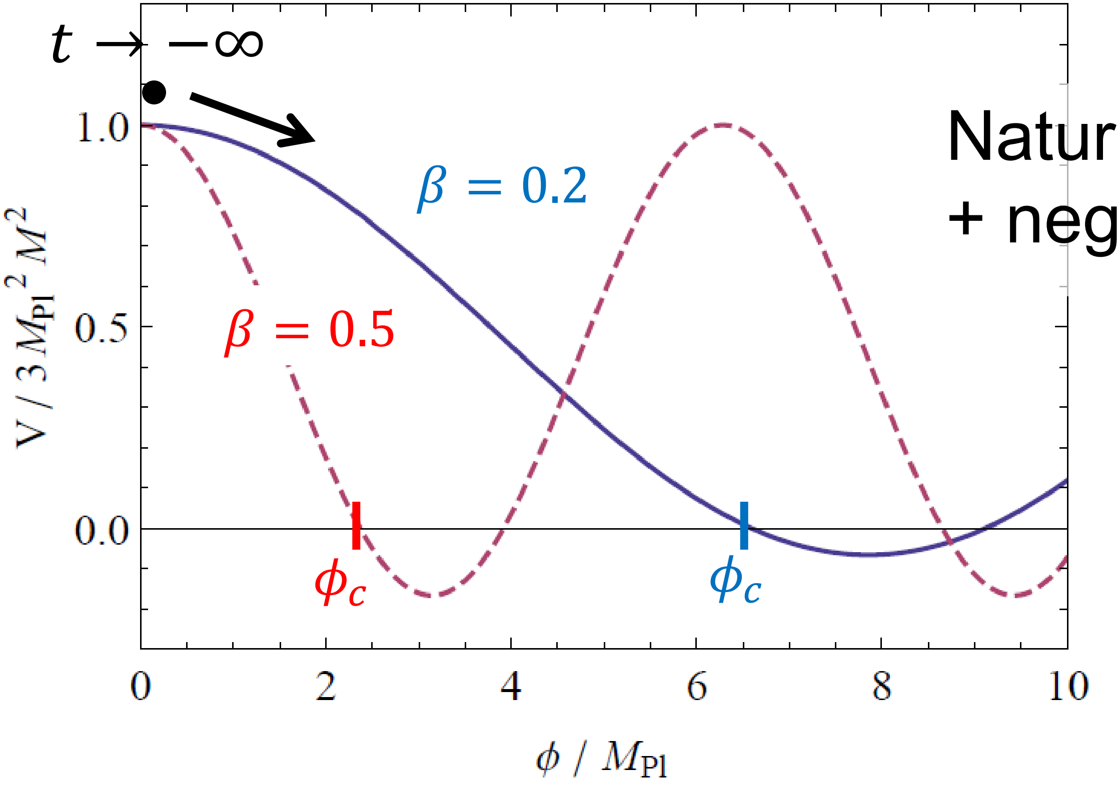
$$2\epsilon_1 = \epsilon_{2n+1} = -\beta/\cosh^2(-\beta M t) \to 0$$

$$\epsilon_{2n} = 2\beta\tanh^2(-\beta M t) \to 2\beta$$

New cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$.



Natural inflation
+ negative CC

New cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$.

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = 2\sqrt{\frac{2}{\beta}}\arctan(e^{\beta M t}) \to 0 \qquad (t \to -\infty)$$

$$\frac{H(t)}{M} = -\tanh(\beta M t) \to 1$$

$$a(t) \propto \cosh^{-1/\beta}(\beta M t) \to e^{Mt}$$

$\epsilon_1 \equiv -\dot{H}/H^2$

$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$

Slow-roll parameters

$$2\epsilon_1 = \epsilon_{2n+1} = 2\beta/\sinh^2(\beta M t) \to 0$$

$$\epsilon_{2n} = 2\beta\tanh^2(\beta M t) \to 2\beta$$

Same asymptotic values

Curvature perturbation

Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

where $v_k = \sqrt{2} z \zeta_k$ with $z = a\sqrt{\epsilon_1}$.

$\epsilon_1 \equiv -\dot{H}/H^2$

$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$

Without approximation,

$$\frac{z''}{z} = a^2 H^2 \left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$$

For both cosh potential and cos potential,

$$\frac{z''}{z} \to \frac{(\beta+2)(\beta+1)}{\tau^2} = \frac{\nu^2 - 1/4}{\tau^2}$$

where

$$\nu \equiv \sqrt{(\beta+2)(\beta+1) + 1/4} = |\beta + 3/2|$$

Curvature perturbation

Since spectral index is given by

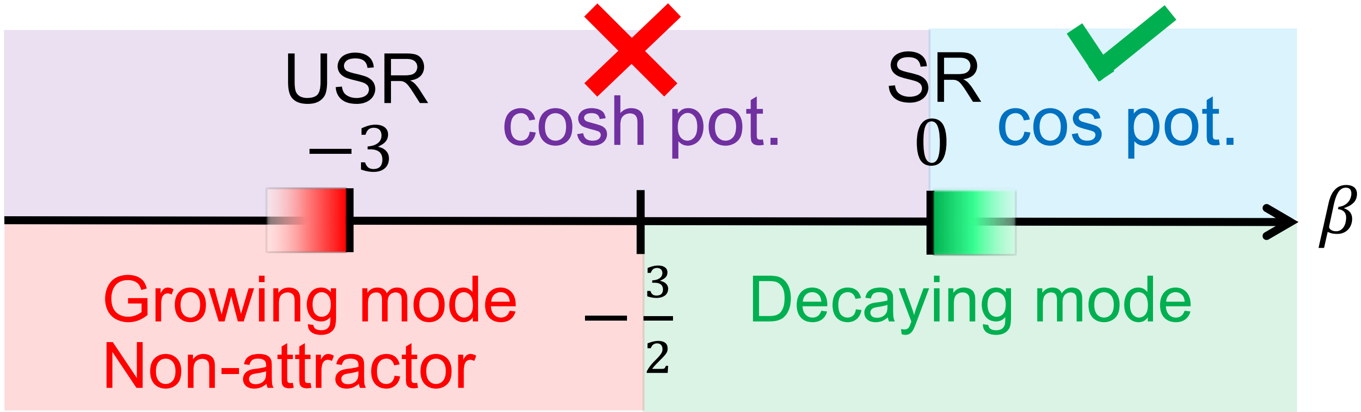
$$n_s - 1 = 3 - 2\nu = 3 - |2\beta + 3|$$

we obtain

$$\beta = \frac{n_s - 7}{2} \quad \text{or} \quad \frac{1 - n_s}{2}$$

For $n_s = 0.96$,

$$\beta = -3.02 \;\; \text{or} \;\; 0.02$$



✗

✓

USR

-3

cosh pot.

SR

0

cos pot.

β

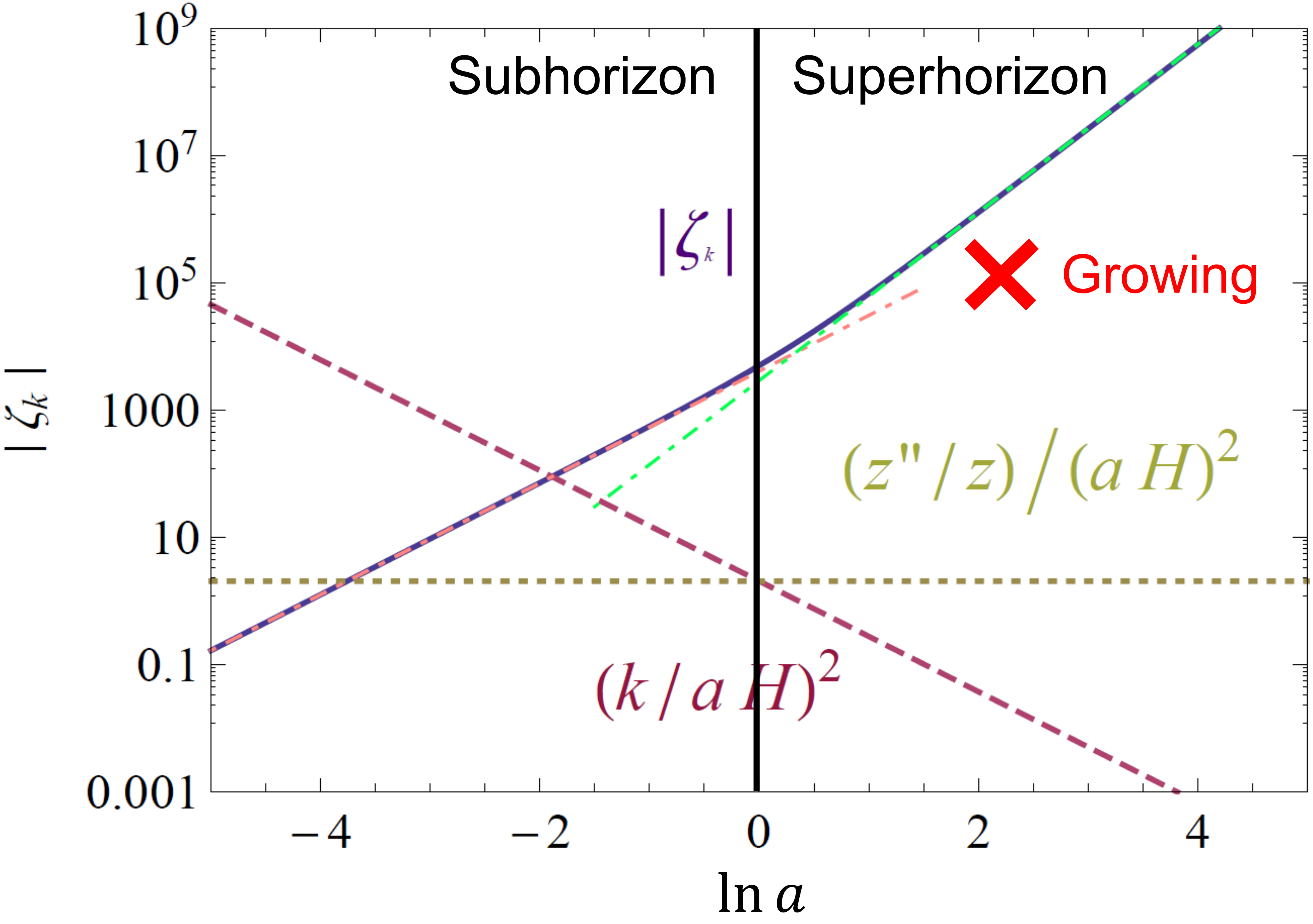
Growing mode

Non-attractor

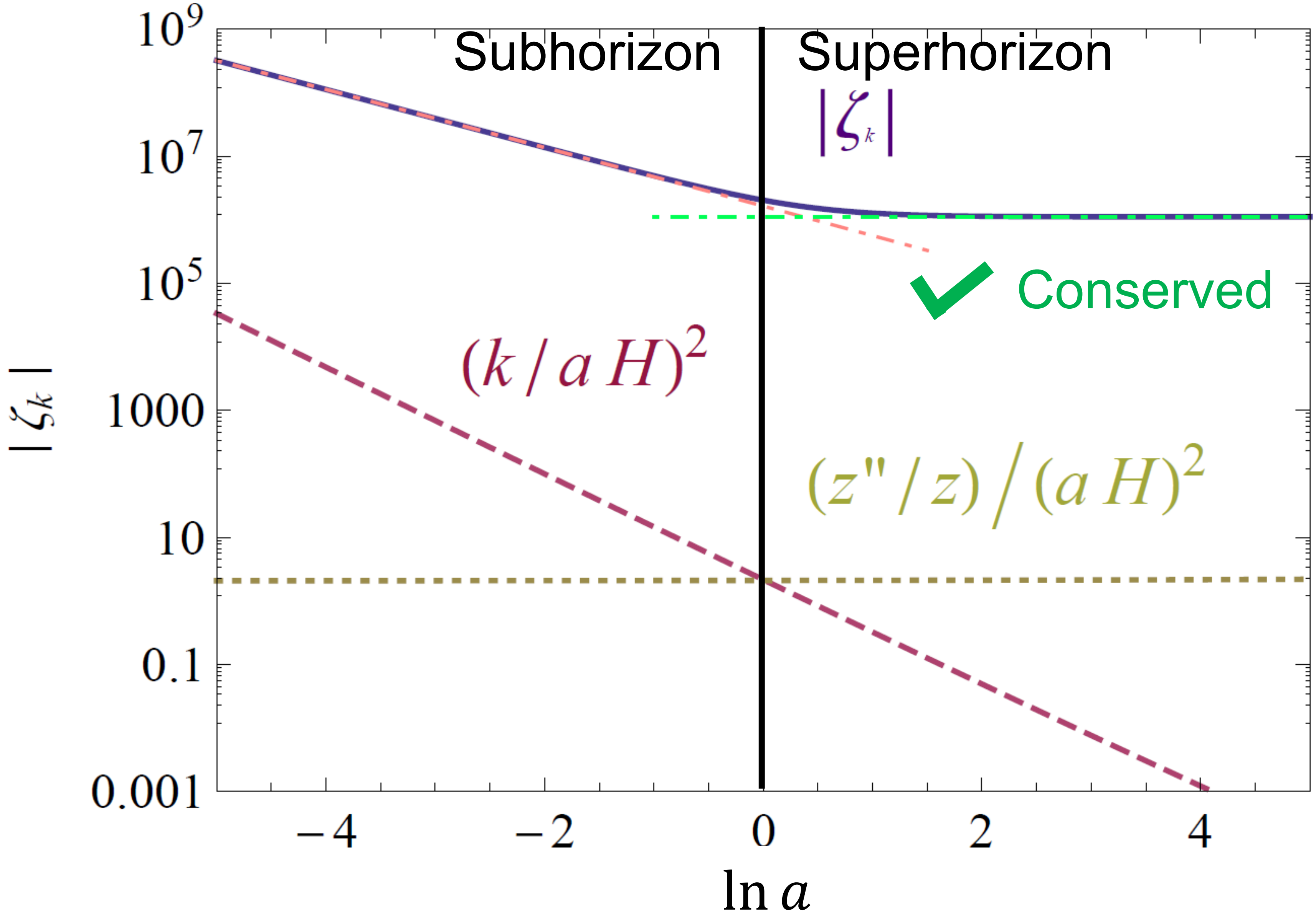
$-\frac{3}{2}$

Decaying mode

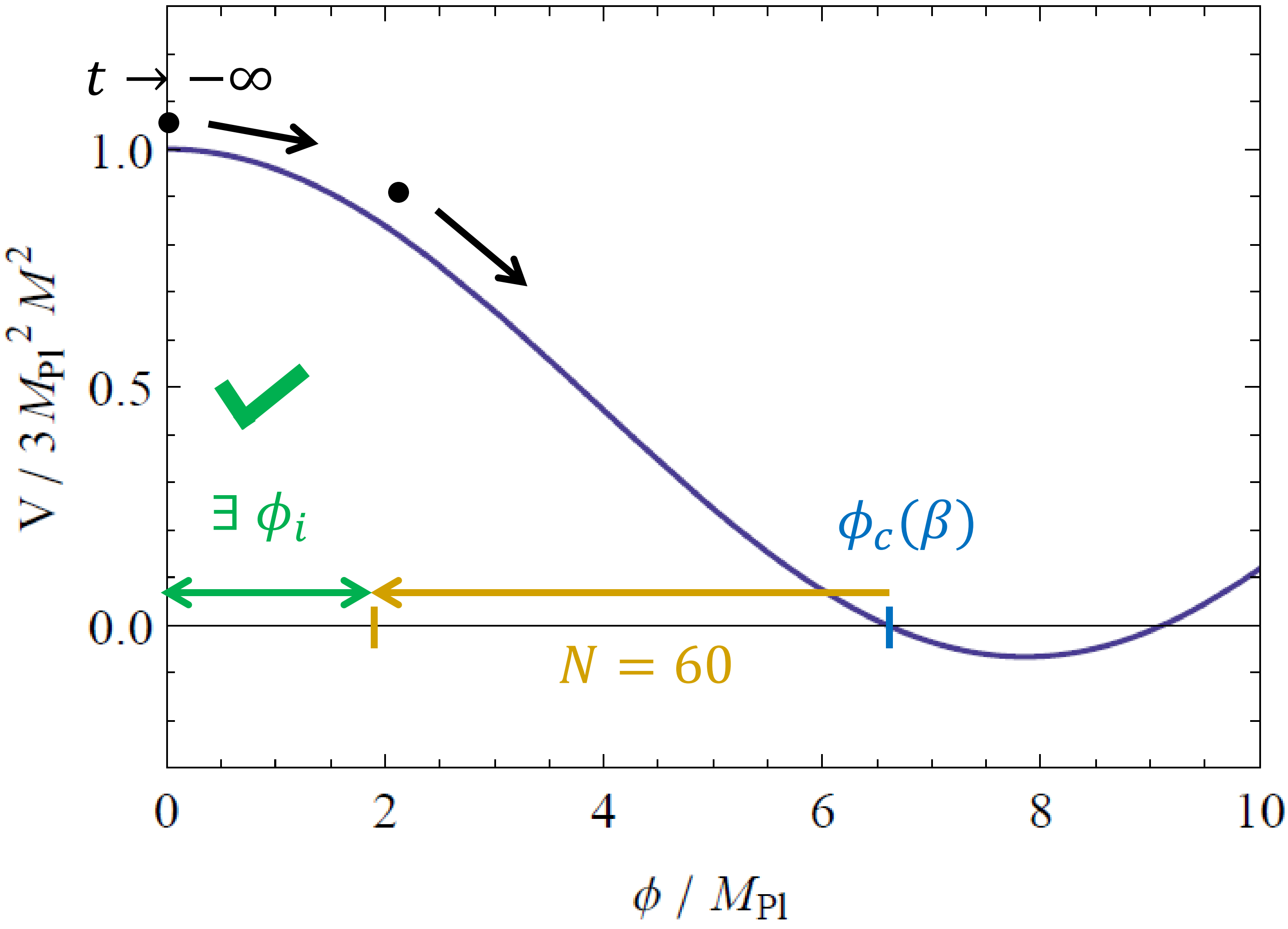
cosh potential $\beta = -3.02$



cos potential $\beta = 0.02$

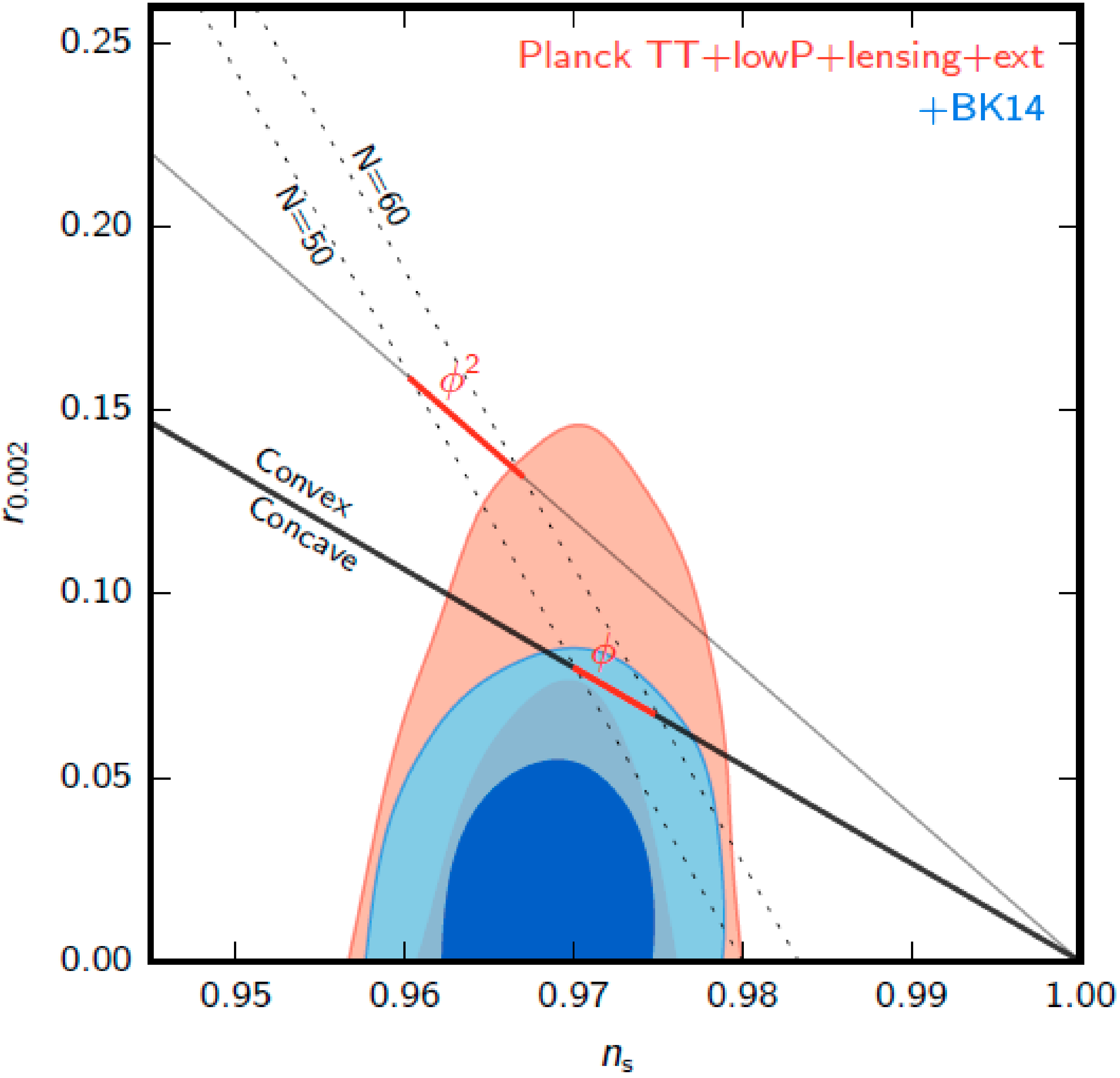


Sufficient number of efolds



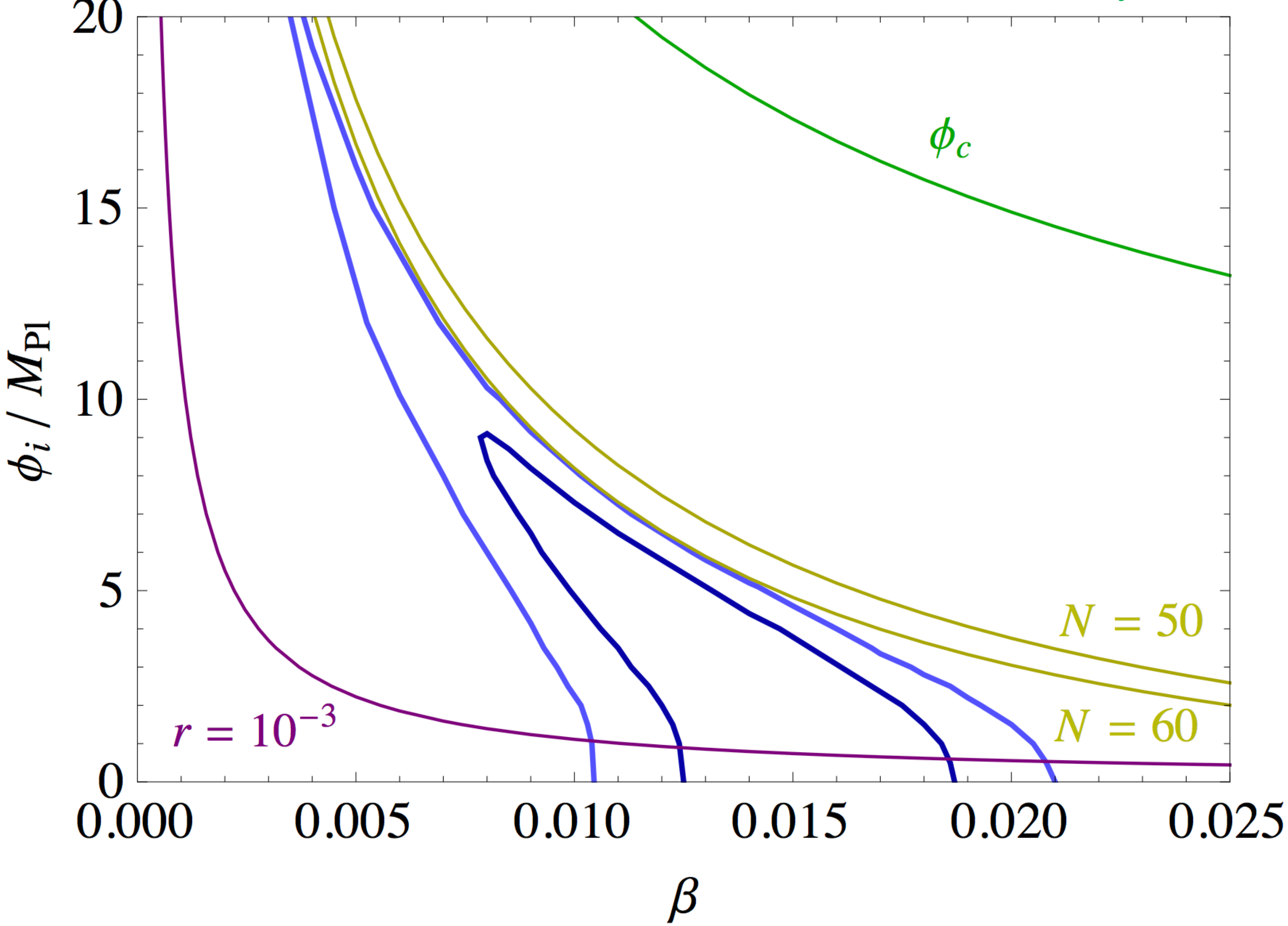
Observational constraint

Ade et al., 1510.09217



Observational constraint

HM, Starobinsky, 1702.05847



$f(R)$ constant-roll inflation

HM, Starobinsky, 1704.08188

$$S = \int d^4x\sqrt{-g}\,\frac{f(R)}{2}$$

EOM

$$F \equiv df/dR$$

$$3FH^2 = \frac{1}{2}(RF - f) - 3H\dot{F}$$

$$2F\dot{H} = -\ddot{F} + H\dot{F} \qquad (M_{Pl} = 1)$$

Constant-roll condition

$$\ddot{F} = \beta H\dot{F}$$

allows one to find analytic solution for

✓ $f(R)$ (parametric form)

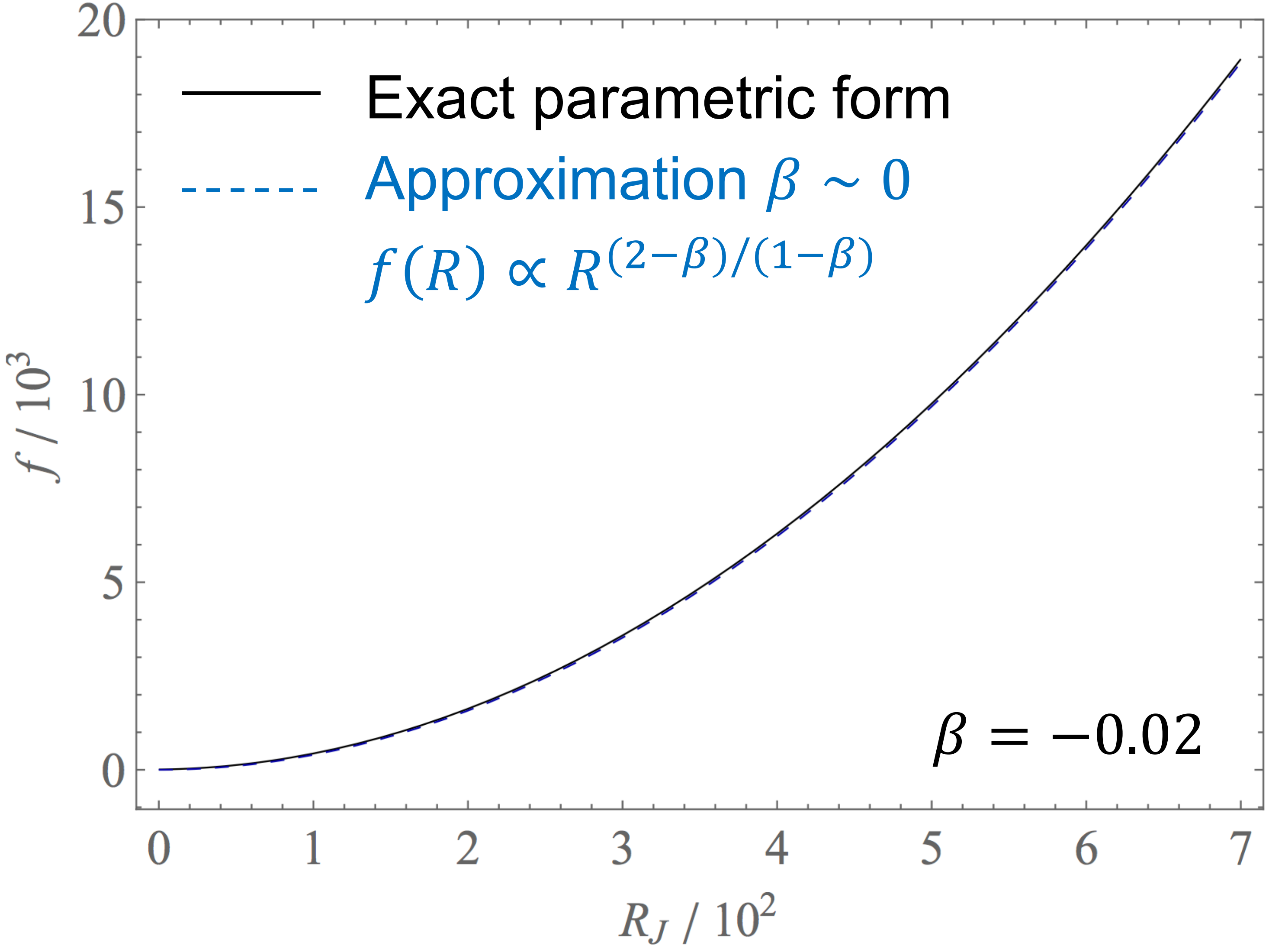
✓ $V(\phi)$

✓ $\phi(t_E)$

} Einstein frame $g^E_{\mu\nu} = F g^J_{\mu\nu},\ F = e^{\sqrt{\frac{2}{3}}\phi}$

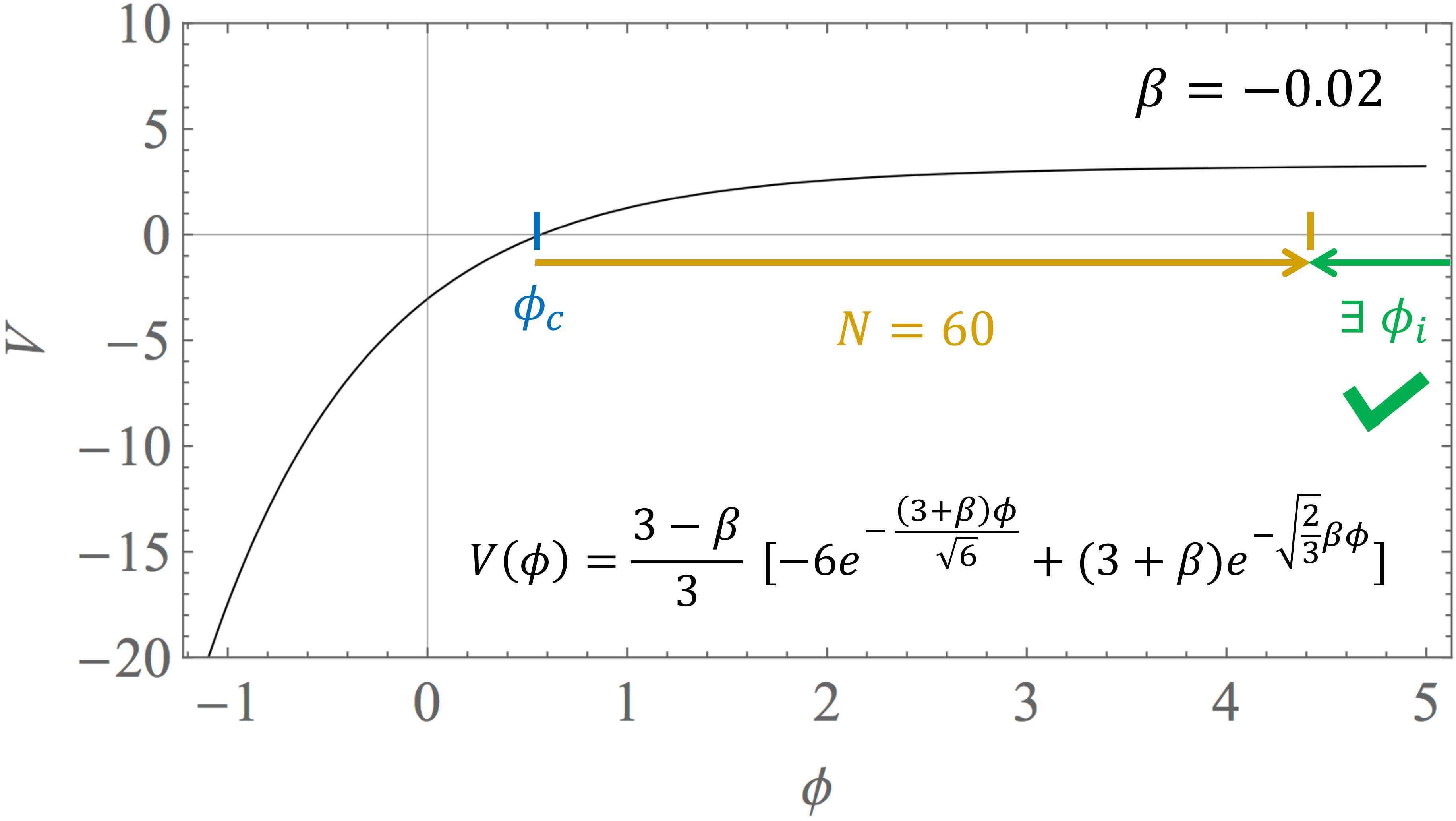
$f(R)$

HM, Starobinsky, 1704.08188



Einstein frame potential

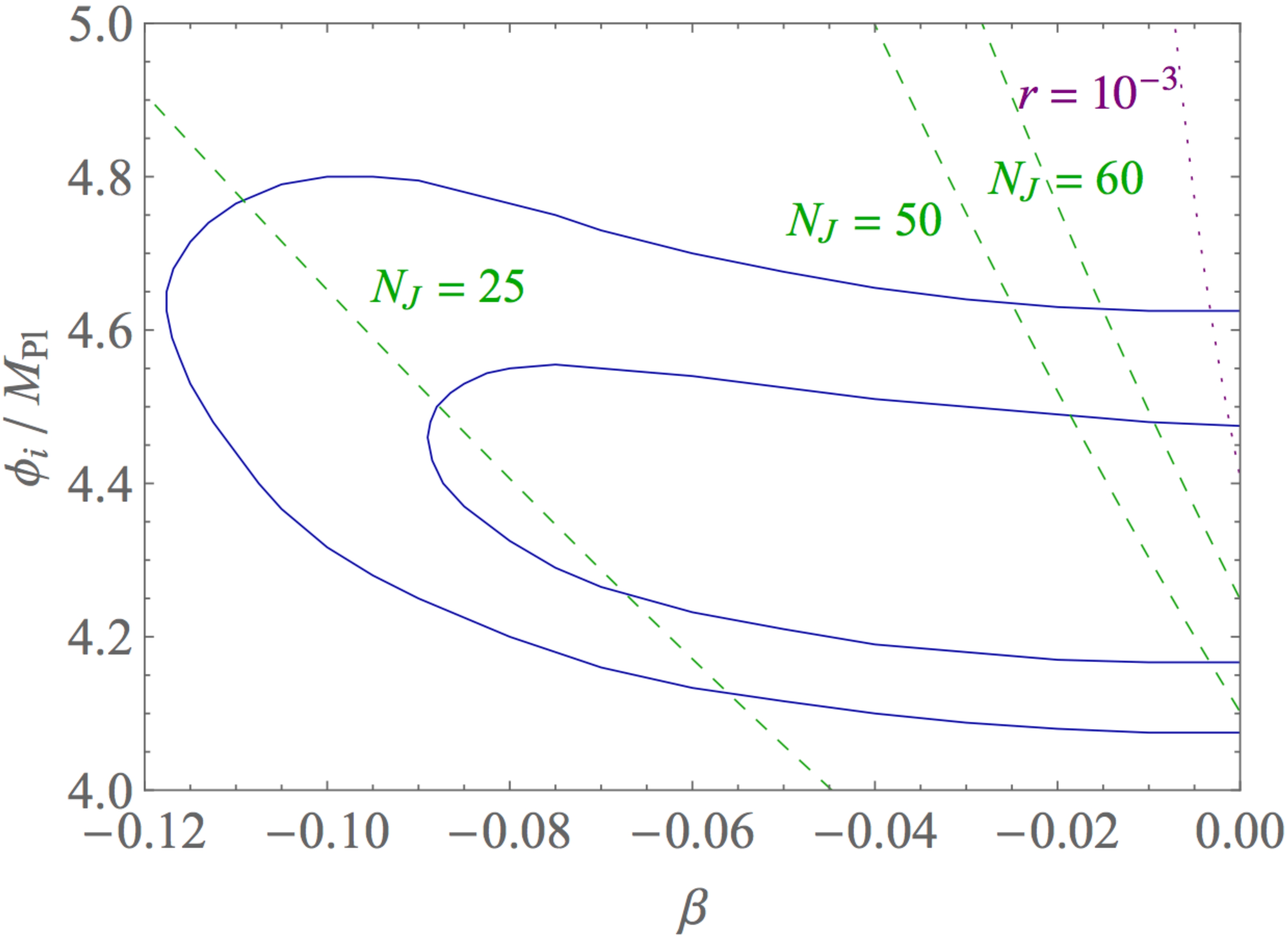
HM, Starobinsky, 1704.08188



$$V(\phi) = \frac{3-\beta}{3}\left[-6e^{-\frac{(3+\beta)\phi}{\sqrt{6}}} + (3+\beta)e^{-\sqrt{\frac{2}{3}}\beta\phi}\right]$$

Observational constraint

HM, Starobinsky, 1704.08188



Summary

Constant-roll condition

$$\ddot{\phi} = \beta H \dot{\phi} \quad \text{or} \quad \ddot{F} = \beta H \dot{F}$$

allows one to derive analytic solution of

$$V(\phi), \phi(t), H(t), a(t) \quad \text{and} \quad f(R)$$

for which the condition is satisfied throughout evolution.

Constant-roll inflation

✓ Attractor

✓ ζ_k conserved

✓ $N \sim 60$

✓ n_s & r