The power spectrum of the magnetic fields generated by the second-order perturbations during the pre-recombination era

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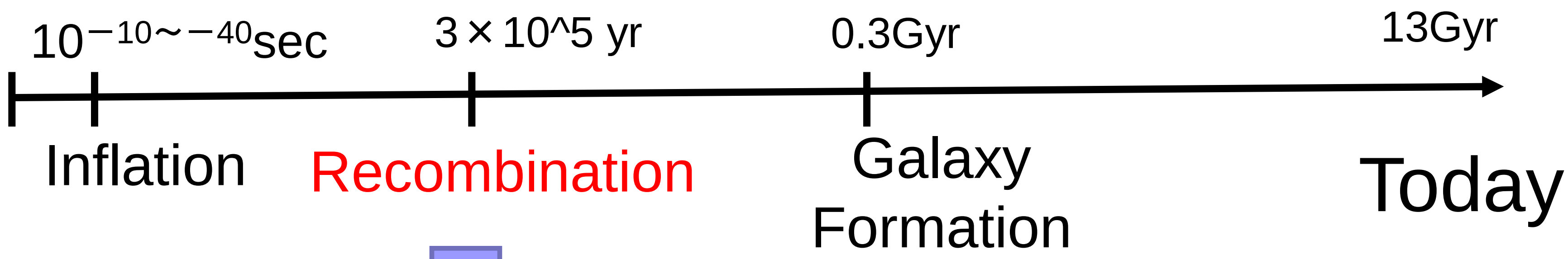
1. Introduction

Evolution of the magnetic field



Generation of the magnetic field

Second-order perturbation + Thomson scattering

$10^{-23\sim-30}\,G$ → Amplified / Dynamo mechanism → μG

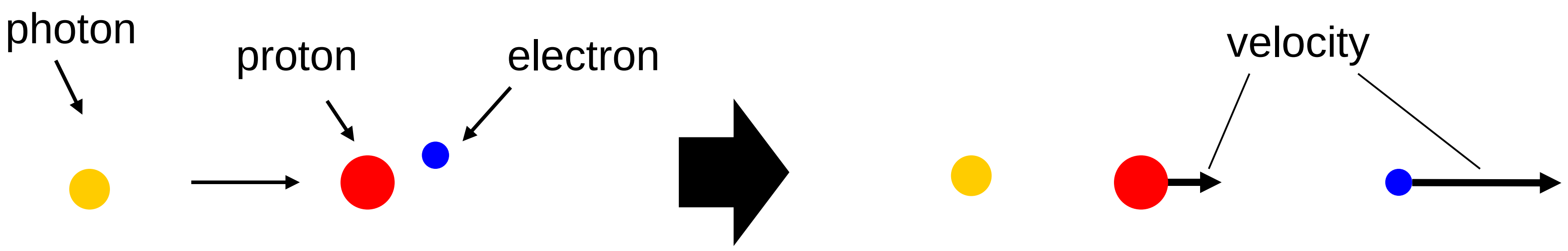
[A.C.Davis, PRD 60, 021301 (1999)]

Generation of the magnetic field at pre-recombination era

•At pre-recombination era

A plasma state of photons, protons and electrons

•Thomson scattering is very important.



The mass of proton and electron is different

→The ”deviation” (the electric field) is generated.

The magnetic fields are generated.

2. Previous work

Previous results

[Maeda et al. ,CQG,26135014,2009]

Evolution equation of the magnetic fields

$$(a^3B^i)' = \frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e}a^2\rho_\gamma^{(0)}\left[2\frac{H}{\alpha^{(0)}}a^2\omega^{(2)i} + \varepsilon^{ijk}\frac{R^{(0)}}{1+R^{(0)}}\partial_j\Delta_b{}^{(I,1)}\delta v_{(\gamma b)k}^{(I,1)}\right] \quad \text{— (1)}$$

Vorticity term

Slip term

Evolution equation of the vorticities

$$(a^2\omega^{(2)i})' + \frac{HR^{(0)}}{1+R^{(0)}}a^2\omega^{(2)i} = \frac{R^{(0)}}{(1+R^{(0)})^2}\alpha^{(0)}\varepsilon^{ijk}\partial_j\Delta_b^{(I,1)}\delta v_{(\gamma b)k}^{(I,1)} \quad \text{— (2)}$$

$\omega^{(2)i}$:photon's vorticity, $\delta v_{(\gamma b)i} = v_{(\gamma)i} - v_{(b)i}$, $\vec{v}_b$: barycentric velocity of baryons

$\alpha^{(0)} \equiv \frac{1+\beta^2}{1+\beta}(1+R^{(0)})\frac{a\sigma_T\rho_\gamma^{(0)}}{m_p}$, $R^{(0)} \equiv 3\rho_b^{(0)}/4\rho_\gamma^{(0)}$, $\beta = m_e/m_p$

$$\delta v_{(\gamma b)i}^{(I,1)} = \frac{1}{\alpha^{(0)}}\left[Hv_i^{(1)} - \frac{1}{4}\frac{\partial_i\delta\rho_\gamma^{(1)}}{\rho_\gamma^{(0)}}\right], \quad \Delta_b{}^{(I,1)} = \int d\eta\,\partial_k\delta v_{(\gamma b)}^{(I,1)k}$$

Derivation of the power spectrum of the magnetic fields

- Procedure of the calculation

→Solve the linear Einstein equation

→Substitute into the equation of the magnetic field

→Calculation $\langle B_i B^i \rangle$

The equation of the power spectrum

$$\frac{2\pi^2}{k^3}P_B(k) = \left(\frac{1-\beta^3}{1+\beta}\frac{\sigma_T\rho_{\gamma 0}}{ea^3}\right)^2 (2\pi^2)^2 \int d^3p|\vec{k}\times\vec{p}|^2 \frac{P_\psi(p)}{p^3}\frac{P_\psi(|\vec{k}-\vec{p}|)}{|\vec{k}-\vec{p}|^3}\times$$
$$\times \int_0^\eta d\eta_1 \int_0^\eta d\eta_2 a^{-2}(\eta_1)a^{-2}(\eta_2)\{g(\vec{k},\vec{p},\eta_1)+f(\vec{k},\vec{p},\eta_1)\}\times$$
$$\times\{g(\vec{k},\vec{p},\eta_2)-g(\vec{k},\vec{k}-\vec{p},\eta_2)+f(\vec{k},\vec{p},\eta_2)-f(\vec{k},\vec{k}-\vec{p},\eta_2)\}$$

from Vorticity

From the gradient of density perturbation

We evaluate the term from the gradient of density perturbation.

$$f(\vec{k},\vec{p},\eta) = \frac{1}{(2\pi)^{3/2}}\frac{\bar{R}^{(0)}}{1+\bar{R}^{(0)}}\frac{1}{\bar{\alpha}^{(0)}}\frac{\eta^2}{4}p^4|\vec{k}-\vec{p}|^2\frac{j_1(y_2)}{y_2}\int_0^\eta d\eta'\left(\frac{(\eta')^2}{\bar{\alpha}^{(0)}}\frac{j_1(y_1')}{y_1'}\right),$$

$$g(\vec{k},\vec{p},\eta) = \frac{1}{4\eta\bar{\alpha}^{(0)}}\frac{1}{(2\pi)^{3/2}}e^{-\int_0^\eta P(\eta')d\eta'}p^4|\vec{k}-\vec{p}|^2\times$$
$$\times\int_0^\eta d\eta'\frac{\bar{R}^{(0)}}{(1+\bar{R}^{(0)})^2}(\eta')^2\frac{j_1(y_2')}{y_2'}\int_0^{\eta'} d\eta''\left(\frac{(\eta'')^2}{\bar{\alpha}^{(0)}}\frac{j_1(y_1'')}{y_1''}\right)e^{\int_0^{\eta'}P(\eta'')d\eta''}$$

$$y_1 = p\eta/\sqrt{3},\ y_2 = |\vec{k}-\vec{p}|\eta/\sqrt{3}$$

3. Estimation of the power spectrum

A range of integral

As we do not consider the Silk damping, The integral diverges at small scale.

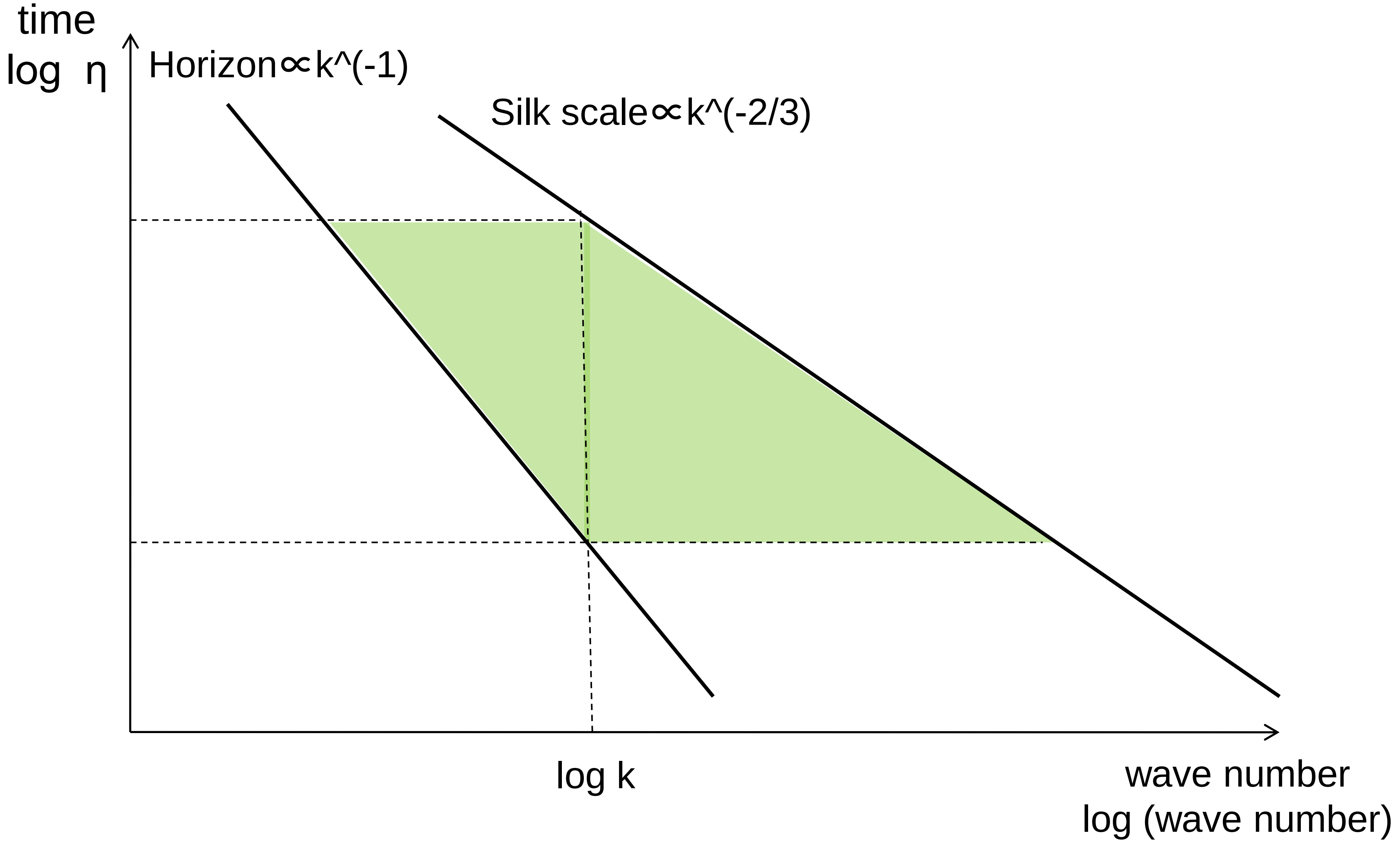
↓

We take the Silk scale as the cutoff scale.

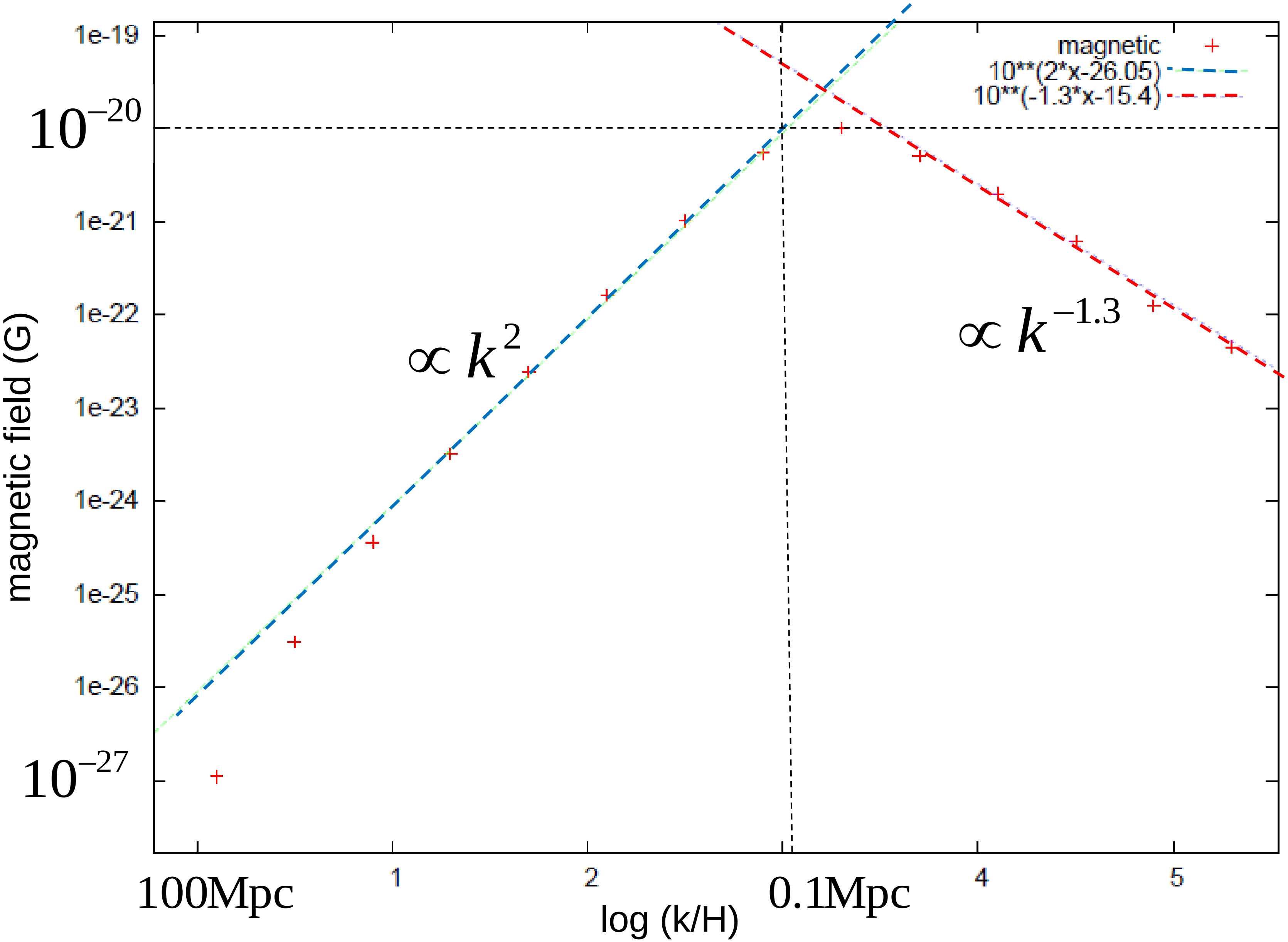
$$k_{cutoff} \approx k_{Silk} \propto \eta^{-2/3}$$

And we suppose that the magnetic field is generated when the perturbations enter into the horizon.

A range of integration



Results



magnetic
10**(2*x-26.05)
10**(-1.3*x-15.4)
1e-19
10^{-20}
1e-21
1e-22
1e-23
1e-24
1e-25
1e-26
10^{-27}
magnetic field (G)
$\propto k^2$
$\propto k^{-1.3}$
100Mpc
1
2
0.1Mpc
4
5
log (k/H)

4. Simple estimation of the power law

Simple estimation of the power(1/3)

$$P_B \simeq k^3 \int d^3p |\vec{k}\times\vec{p}|^2 \frac{1}{p^3}\frac{1}{|\vec{k}-\vec{p}|^3}\int_0^\eta d\eta_1 a^{-2}(\eta_1) f(\vec{k},\vec{p},\eta_1)\int_0^\eta d\eta_2 a^{-2}(\eta_2)\left\{f(\vec{k},\vec{p},\eta_2)-f(\vec{k},\vec{k}-\vec{p},\eta_2)\right\}$$

- $$f(\vec{k},\vec{p},\eta) \simeq \frac{\bar{R}^{(0)}}{1+\bar{R}^{(0)}}\frac{1}{\bar{\alpha}^{(0)}}\frac{\eta^2}{4}p^4|\vec{k}-\vec{p}|^2\frac{j_1(y_2)}{y_2}\int_0^\eta d\eta'\left(\frac{(\eta')^2}{\bar{\alpha}^{(0)}}\frac{j_1(y_1')}{y_1'}\right)$$

$$\left(y_1 = p\eta/\sqrt{3},\ y_2 = |\vec{k}-\vec{p}|\eta/\sqrt{3}\right)$$

$$\bar{R}^{(0)} \propto \eta, 1+\bar{R}^{(0)} \sim 1, \bar{\alpha}^{(0)} \propto \eta, z = \left|\vec{k}-\vec{p}\right|/p$$

$$\begin{aligned}
f(\vec{k},\vec{p},\eta) &\simeq \eta^5 p^3 |\vec{k}-\vec{p}| j_1(|\vec{k}-\vec{p}|\eta)\int_0^\eta d\eta'(\eta')^4 j_1(p\eta') \\
&= \eta^5 p^4 z j_1(p\eta z)\int_0^\eta d\eta'(\eta')^4 j_1(p\eta') \\
&\simeq p\eta^7 \cos(pz\eta)\sin(p\eta) \\
&= \frac{p\eta^7}{2}\left(\sin(p\eta+pz\eta)+\sin(p\eta-pz\eta)\right)
\end{aligned}$$

$$p\eta >> 1$$

Simple estimation of the power(2/3)

And

$$\int d\eta_1 \frac{1}{(\eta_1)^2} f(\vec{k},\vec{p},\eta_1) \simeq p\int d\eta_1 \eta_1^5 \Big(\sin(p\eta_1 + pz\eta_1) + \sin(p\eta_1 - pz\eta_1)\Big)$$
$$\simeq \eta^5 \left[\frac{1}{1+z}\cos(p\eta + pz\eta) + \frac{1}{1-z}\cos(p\eta - pz\eta)\right]$$

Another term is

$$\int d\eta_2 \frac{1}{(\eta_2)^2}\left[f(\vec{k},\vec{p},\eta_2) - f(\vec{k},\vec{k}-\vec{p},\eta_2)\right] \simeq \eta^5 \left[\frac{1-z}{1+z}\cos(p\eta + pz\eta) - \frac{1+z}{1-z}\cos(p\eta - pz\eta)\right]$$

Finally

$$P_B \simeq \frac{k^5\eta^{10}}{pz^3}\left[\frac{1-z}{(1+z)^2}\cos^2(p\eta + pz\eta) - \frac{2z}{(1+z)(1-z)}\cos(p\eta + pz\eta)\cos(p\eta - pz\eta) - \frac{1+z}{(1-z)^2}\cos^2(p\eta - pz\eta)\right]$$

Take leading term →

$$P_B \propto k^5 p^{-1} \times \eta^{10}(p^2 k^{-2}) \sim k^3 p \eta^{10}$$

Simple estimation of the power(3/3)

$$P_B \propto k^3 p \eta^{10}$$

- The power of the magnetic fields

$$B \sim \sqrt{P_B} \sim k^{3/2} p^{1/2} \eta^5$$

(i) $k < k_{silk}$ ⟶ $p \sim k, \ \eta \sim Const.$

$$B \sim k^2$$

(ii) $k > k_{silk}$ ⟶ $p \sim k, \ \eta \sim k^{-2/3}$

$$B \sim k^{-1.3}$$

5. Summary

Summary

- We calculate the power spectrum of the magnetic fields from the gradient of the density perturbation.
- The power is k^2 at the large scale and $k^{-1.3}$ at small scale.
- The amplitude of the magnetic fields is about $10^{-20}\,\mathrm{Gauss}$.

The residual terms

We evaluate also the residual terms from vorticity in the following.

$$\frac{2\pi^2}{k^3}P_B(k) = \left(\frac{1-\beta^3}{1+\beta}\frac{\sigma_T\rho_{\gamma 0}}{ea^3}\right)^2 (2\pi^2)^2 \int d^3p |\vec{k}\times\vec{p}|^2 \frac{P_\psi(p)}{p^3}\frac{P_\psi(|\vec{k}-\vec{p}|)}{|\vec{k}-\vec{p}|^3} \times$$
$$\times \int_0^\eta d\eta_1 \int_0^\eta d\eta_2 a^{-2}(\eta_1)a^{-2}(\eta_2)\{g(\vec{k},\vec{p},\eta_1) + f(\vec{k},\vec{p},\eta_1)\} \times$$
$$\times \{g(\vec{k},\vec{p},\eta_2) - g(\vec{k},\vec{k}-\vec{p},\eta_2) + f(\vec{k},\vec{p},\eta_2) - f(\vec{k},\vec{k}-\vec{p},\eta_2)\}$$

$$f(\vec{k},\vec{p},\eta) = \frac{1}{(2\pi)^{3/2}}\frac{\bar{R}^{(0)}}{1+\bar{R}^{(0)}}\frac{1}{\bar{\alpha}^{(0)}}\frac{\eta^2}{4}p^4|\vec{k}-\vec{p}|^2\frac{j_1(y_2)}{y_2}\int_0^\eta d\eta' \left(\frac{(\eta')^2}{\bar{\alpha}^{(0)}}\frac{j_1(y_1')}{y_1'}\right),$$

$$g(\vec{k},\vec{p},\eta) = \frac{1}{4\eta\bar{\alpha}^{(0)}}\frac{1}{(2\pi)^{3/2}}e^{-\int_0^\eta P(\eta')d\eta'}p^4|\vec{k}-\vec{p}|^2 \times$$
$$\times \int_0^\eta d\eta' \frac{\bar{R}^{(0)}}{(1+\bar{R}^{(0)})^2}(\eta')^2\frac{j_1(y_2')}{y_2'}\int_0^{\eta'} d\eta'' \left(\frac{(\eta'')^2}{\bar{\alpha}^{(0)}}\frac{j_1(y_1'')}{y_1''}\right)e^{\int_0^{\eta'} P(\eta'')d\eta''}$$