

ABSTRACT : We study quantum tunneling from false vacuum in multi-field system. Tunneling is often treated by using method of instanton. It was not known whether multi-field instanton exists or not when multi-field interaction is considered, so we study about it. We found multi-field instanton exists in the potential of open inflation model. But, in certain potential there is no instanton. We discuss physical interpretation on this. Method of instanton corresponds to WKB approximation in Quantum Mechanics, so there is a range of application. We have to study when method of instanton can be applied.

Motivations

- Multi-field tunneling occurs on string-landscape
- Method of instanton is often used to describe quantum tunneling
- It was not known whether multi-field instanton exists when multi-field interaction is considered



We study whether multi-field instanton exists.

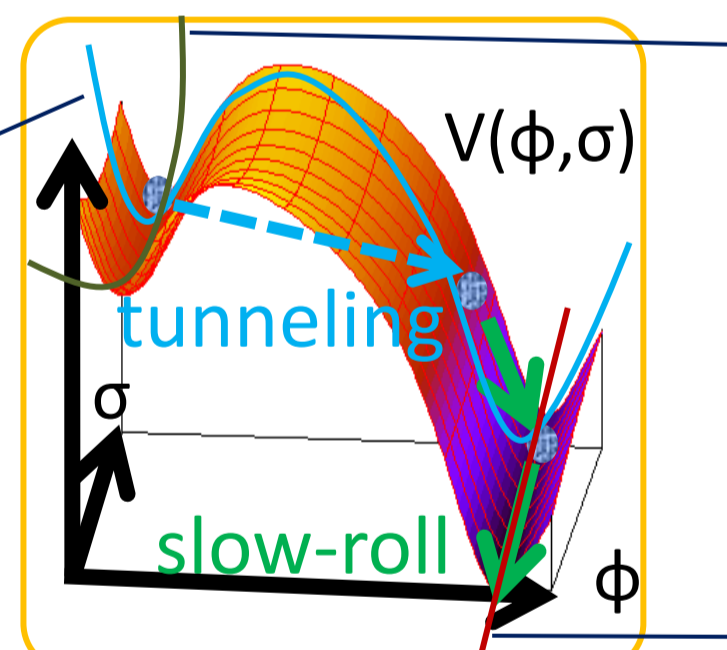
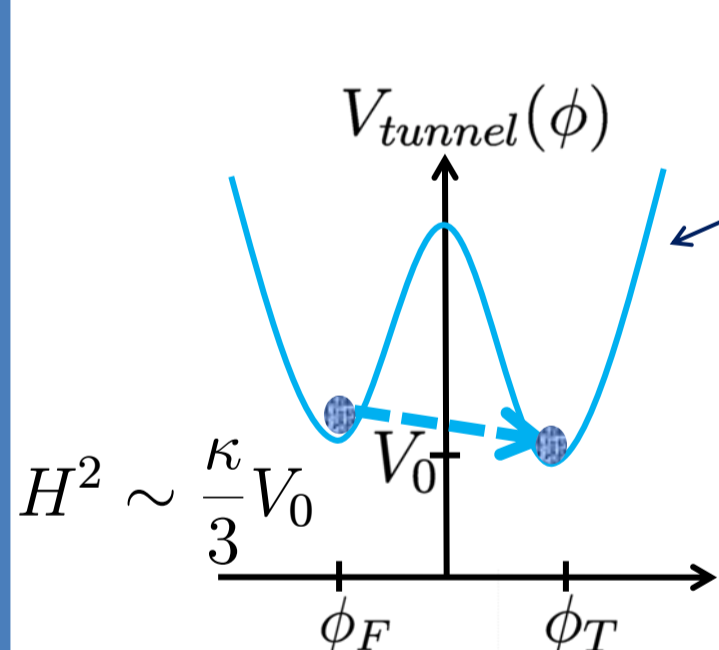
- No instanton does not directly mean tunneling does not occur
- But, in that case, it is necessary to go beyond instanton method

Model & Methods

We study quantum tunneling in the potential of **multi-field Open Inflation model** (Linde-Mezhlumian 1995)

tunneling of tunneling field ϕ from false vacuum
 \Rightarrow potential which slow-roll field σ feels changes

$$V(\phi, \sigma) = V_{\text{tunnel}}(\phi) + V_{\text{int}}(\phi, \sigma)$$



confined by mass potential

$$\frac{m^2}{2} \sigma^2 + V_{\text{int}}(\phi_F, \sigma)$$

slow-rolls on linear potential

$$\lambda m^3 \sigma + V_{\text{int}}(\phi_T, \sigma)$$

σ : slow-roll field
 λ : tilt after tunneling

ϕ : tunneling field

We can treat quantum tunneling by using method of **instanton**

assuming O(4)-symmetry $ds^2 = d\tau^2 + a^2(\tau)d\Omega^2$

$$\text{Euclidean action } S_E = \int d\tau d^3x a^3 \left(\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 + V(\phi, \sigma) \right) + \frac{3}{\kappa} (a^2 \ddot{a} + a \dot{a}^2 - a)$$

instanton is a solution which satisfies following EOM and B.C. (Boundary Condition)

Friedman eq. **EOM**

$$\left(\frac{a'}{a} \right)^2 - \frac{1}{a^2} = \frac{\kappa}{3} \left(\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 - V \right)$$

$$\phi'' + \frac{3a'}{a} \phi' - V_{\phi}(\phi, \sigma) = 0$$

$$\sigma'' + \frac{3a'}{a} \sigma' - V_{\sigma}(\phi, \sigma) = 0$$

friction term

2-Dim particle mechanics form

B.C.

$$\phi'(0) = \phi'(\tau_f) = 0$$

$$\sigma'(0) = \sigma'(\tau_f) = 0$$

zero velocity at $\tau=0, \tau_f$

Using these eqs., we search for instanton for the above potential **numerically** (shooting method) and **analytically** (approximated with $V_{\text{int}} \ll V_{\text{tunnel}}$)

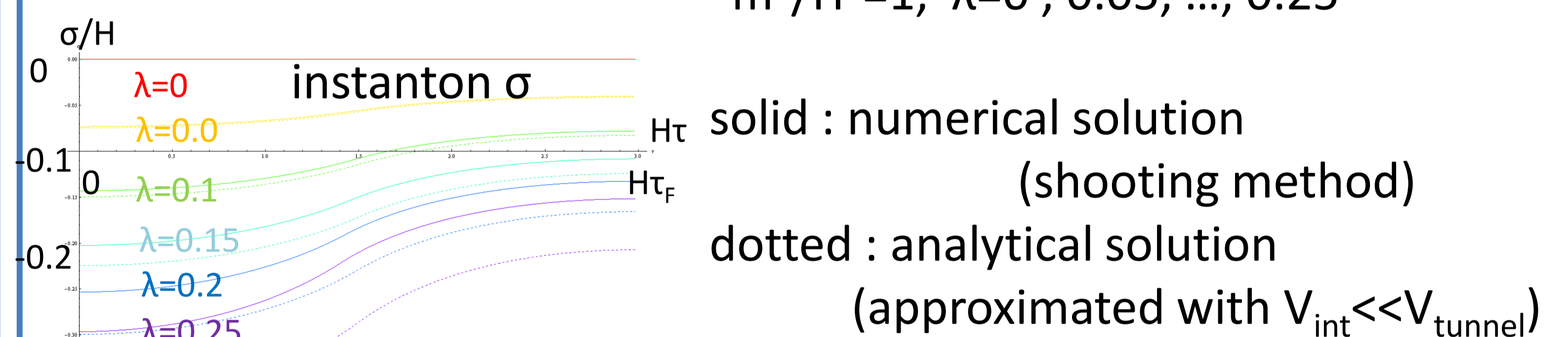
tunneling rate is given using instanton $\bar{a}(\tau), \bar{\phi}(\tau), \bar{\sigma}(\tau)$

$$\Gamma/V \propto \exp[-S_E[\bar{a}(\tau), \bar{\phi}(\tau), \bar{\sigma}(\tau)]/\hbar]$$

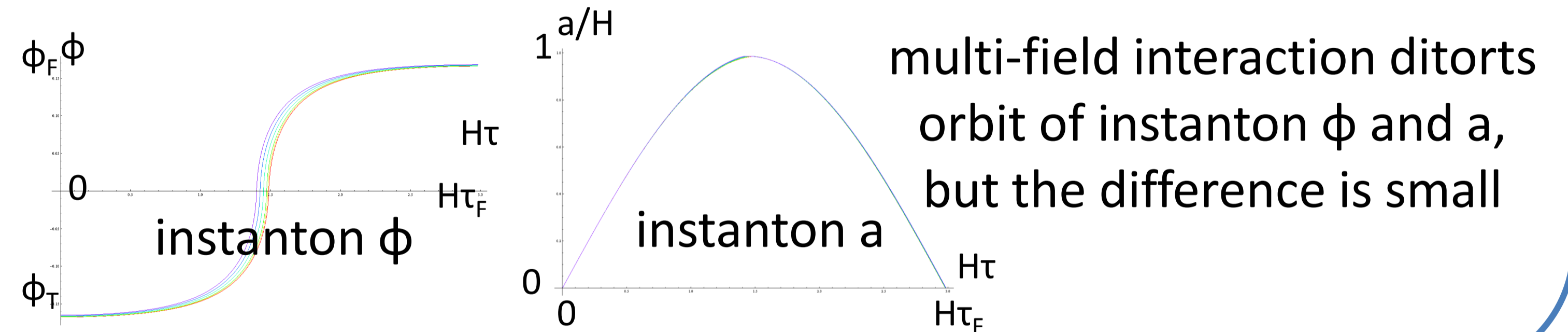
Results

we found multi-field instanton does **exist** in this potential
 these are examples of instanton when parameters are

$$m^2/H^2=1, \lambda=0, 0.05, \dots, 0.25$$



- $\lambda=0 \Rightarrow \sigma$ is not dynamical because of $(\sigma \leftrightarrow -\sigma)$ symmetry
- $\lambda \uparrow \Rightarrow$ dynamical range of σ becomes larger



Conclusions & Discussions

- **Multi-field instanton exists** when potential for σ is mass potential in false vacuum and linear after tunneling. (potential of multi-field open inflation model)
- In this case, we can naturally extend method of instanton to multi-field tunneling

- But, if potential at false vacuum is like the figure, there may exist **no instanton**
- We can understand this as follows

EOM and B.C. of instanton is like particle mechanics

$$\tau=0 \Rightarrow \begin{matrix} 0 < \tau < \tau_f \\ v=0 \end{matrix} \Rightarrow \begin{matrix} \text{particle moves feeling} \\ \text{potential } -V \text{ and friction } a'/a \end{matrix} \Rightarrow \begin{matrix} \tau=\tau_f \\ v=0 \end{matrix}$$

In such potential, potential gradient outside false vacuum can be always greater than potential gradient at false vacuum. Then the particle cannot stop at $\tau=\tau_f$. This means there exists no instanton.

There are some possible physical interpretations on **no instanton** in this potential

1. Even if instanton does not exist, quantum tunneling may occur. WKB approximation may be broken. It is necessary to check first WKB order action is by far larger than second WKB order action.
2. In such potential, scalar field σ is not confined at the false vacuum. σ may spread broadly from beginning, because high k-modes don't feel mass potential.

Anyway, we have to study **when method of instanton can be applied**