Entanglement of the primordial fluctuation

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Introduction

Inflationary scenario

・Our universe underwent an accelerated expansion in its early days.

・We can explain the origin of the large scale structure in the universe by the quantum fluctuation of a scalar field (inflaton field) in this era.

Problem of classicality of the fluctuation

Quantum variable in inflationary scenario ⟷ Classical stochastic variable in the theory of structure formation

Classical stochastic variable :

Stochastic variables whose averages or variances are calculated by using a normalized positive definite distribution function

Ex. $\langle x^2 \rangle = \int x^2 \rho(x) dx \qquad \int \rho dx = 1 \qquad \rho \geq 0$

Main Problem

Can we use the classical distribution to describe the quantum system?

Bell's theorem (J. S. Bell, 1964; E. G. Cavalcanti et al, 2007)

There does not exist a classical distribution function in the system with quantum correlation (entanglement).

⇨ If the fluctuation has entanglement, they cannot be described by the classical distribution.

⇨ The entanglement must disappear.

⇨ What is the condition for it?

- We are interested in the entanglement between the field on spatially separated two points.
- There are some methods to estimate the entanglement between two points...

- Particle detector model (B. Reznik, 2000; 2005)

  - Two particle detectors coupling to a scalar field
  - They do not interact with each other.
  - We can estimate the entanglement of the field in directly by calculating the entanglement between the two detectors.

I review the basic idea of this model and our current result.

Entanglement

Let us consider a bipartite system consisting of subsystems A, B.

For pure state

Separable state: $$|\Psi\rangle = |\psi\rangle_A \otimes |\phi\rangle_B$$

Entangled state: $$|\Psi\rangle \neq |\psi\rangle_A \otimes |\phi\rangle_B$$

For mixed state

Separable state: $$\rho = \sum_n p_n \rho_A^{(n)} \otimes \rho_B^{(n)} \qquad \sum_n p_n = 1 \quad \text{and} \quad p_n \geq 0$$

Entangled state: $$\rho \neq \sum_n p_n \rho_A^{(n)} \otimes \rho_B^{(n)}$$

Example

Separable state:

$$|\uparrow\rangle_A|\uparrow\rangle_B + |\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B + |\downarrow\rangle_A|\downarrow\rangle_B = (|\uparrow\rangle_A + |\downarrow\rangle_A)(|\uparrow\rangle_B + |\downarrow\rangle_B)$$

Entangled state:

$$|\uparrow\rangle_A|\uparrow\rangle_B + |\downarrow\rangle_A|\downarrow\rangle_B$$

Entanglement measure

For pure state, the entanglement entropy is usually used as the entanglement measure.

Entanglement entropy

$$S := -\mathrm{Tr}[\rho_A \log \rho_A] \qquad \rho_A := \mathrm{Tr}_B[\rho]$$

If the system is separable, S=0.
If the system is entangled, S>0.

⇩

We can distinguish whether the system is entangled or not from the positivity of the entanglement entropy.

However, we cannot use the entanglement entropy for mixed states.

PPT criterion

A negative eigenvalue of ϱ^{T_B} exists. ⟷ The system is entangled.

ϱ: density matrix for the system $\rho^{T_B}_{\mu\alpha,\nu\beta} := \rho_{\mu\beta,\nu\alpha}$

- For two qubits (A. Peres, 1996; R. Horodecki et al, 1996. For continuous variables in Gaussian state, R. Simon, 2000; L. M. Duan et al, 2000.)

Entanglement measure (Negativity): $N(\varrho)$

$$N(\rho) := ||\rho^{T_B}|| - 1$$

※ ||A||: Sum of the absolute values of the eigenvalues of A

The system is entangled ↔ There exists a negative eigenvalue of ϱ^{T_B}

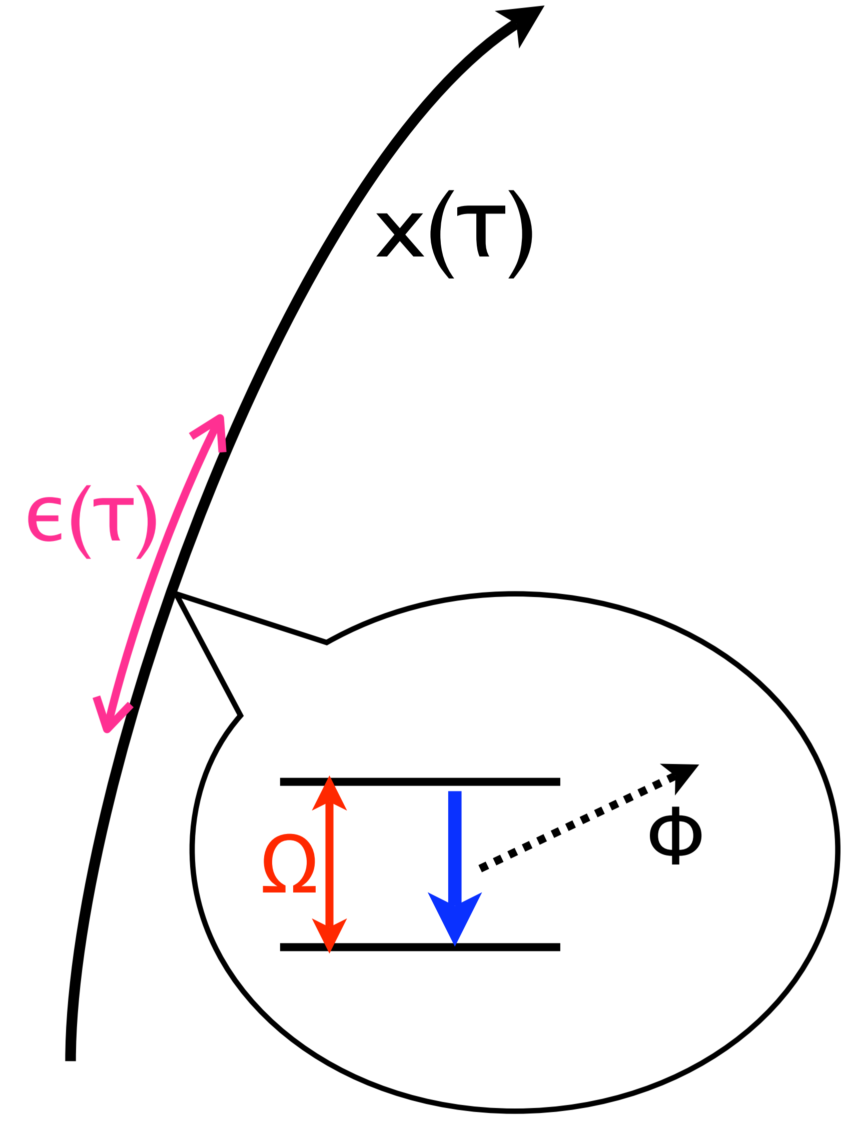
$$\leftrightarrow ||\varrho^{T_B}|| > 1 \leftrightarrow N(\varrho) > 0$$

De Witt detector model

- Moving in spacetime along classical geodesics
- 2 level system spanned by $|\uparrow\rangle, \ |\downarrow\rangle$

$$H_{\rm int} = \varepsilon(\tau)M(\tau)\phi(x(\tau),\tau)$$

$$M(\tau) = e^{i\Omega\tau}\sigma_+ + e^{-i\Omega\tau}\sigma_-$$



Φ: operator of the free real scalar field
τ: proper time of the detector
x(τ): world line of the detector
M(τ): operator of the detector
Ω: energy gap between two levels
σ_±: raising and lowering operator
ϵ(τ): window function giving switch on/off (<1)

How to detect the entanglement (Basic idea)

B. Reznik, 2000; 2005

For two detectors not directly interacting with each other...

t_{ini}		t
Two detectors: Separable + field: vacuum	→ time evolution	Two detectors: Entangled (?) + field: not vacuum

Entanglement cannot be produced by local unitary operation.
→If the final state of the detectors are entangled,
we can regard it as the entanglement
initially possessed by the field!

$$\rho_{\rm detec} = {\rm Tr}_{\rm field}\left[|\psi_{\rm fin}(t)\rangle\langle\psi_{\rm fin}(t)|\right]$$

|ψ_fin(t)>:
final state of the total system

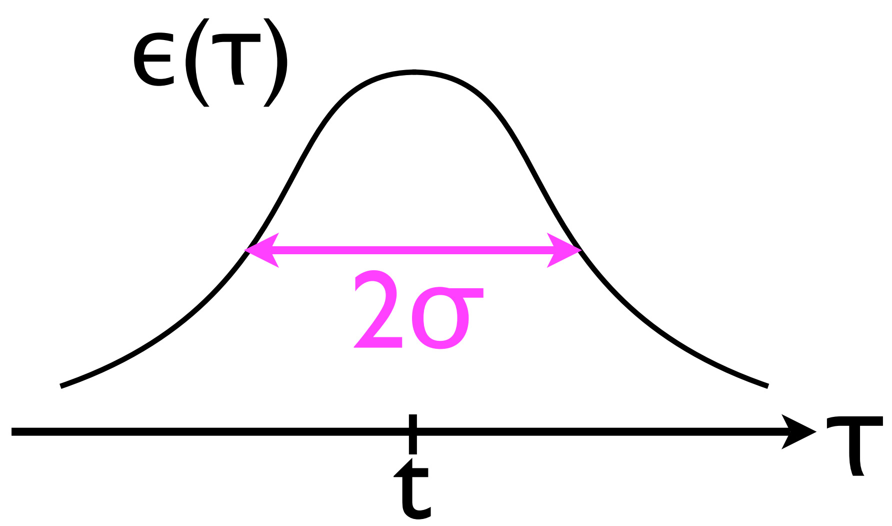
⇩

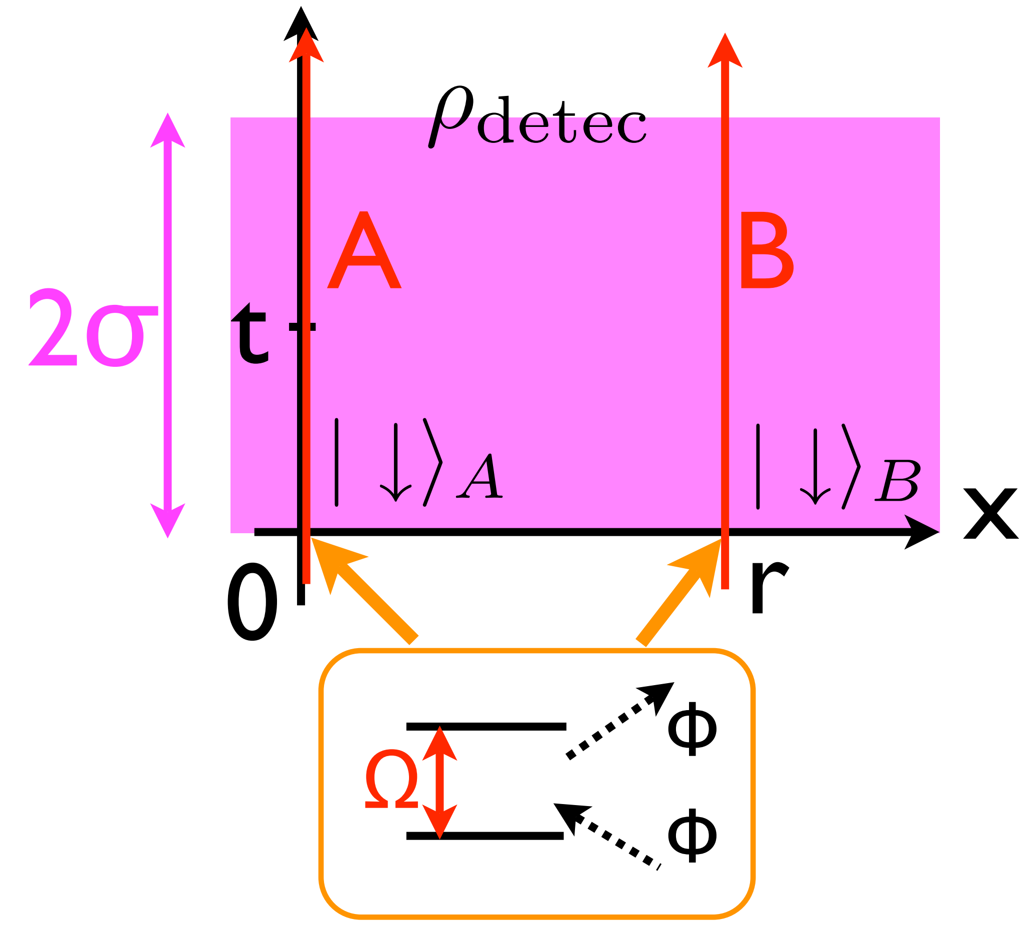
$$N(\rho_{\rm detec})$$: entanglement between the detectors
= entanglement of the field

Example: Minkowski vacuum

Detector:

- Two detectors separated by distance r
- Both of them are static against the coorinate system.
- Their energy gaps Ω are taken to be the same magnitude.
- Initial state: |↓↓> (separable state).
- Window function ϵ(τ): Gaussian with variance σ





Integral form of the negativity

In the first order perturbation...

$$N(\rho_{\mathrm{detec}}) = |Y| - X$$

$$Y := -2\int_{-\infty}^{\infty}\int_{-\infty}^{t_1} \varepsilon(t_1)\varepsilon(t_2)e^{i\Omega(t_1+t_2)}\underline{\langle 0|\phi(x_A,t_1)\phi(x_B,t_2)|0\rangle} dt_2 dt_1$$

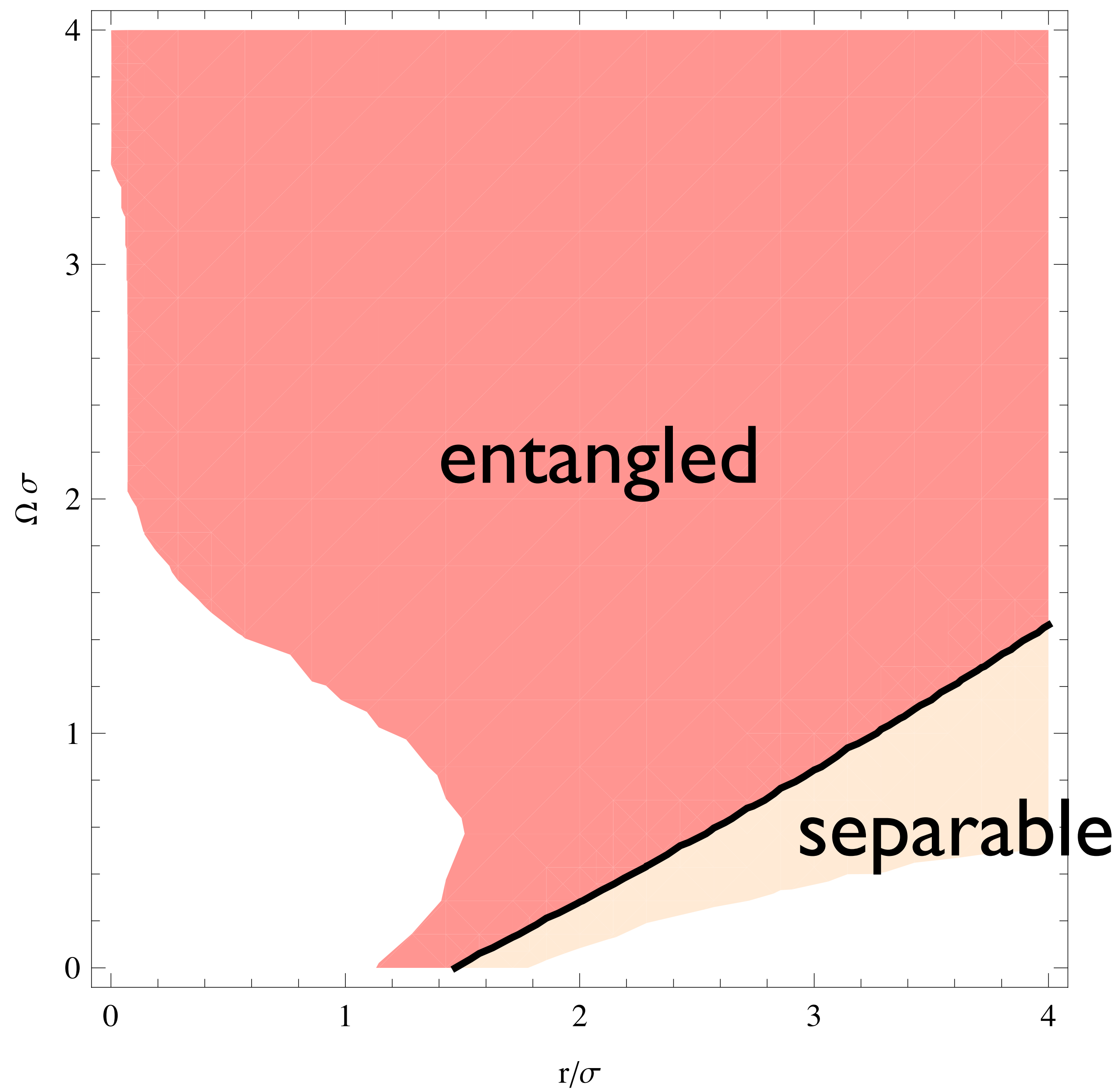
two point funcion: propagation of the quantum

$$X := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon(t_1)\varepsilon(t_2)e^{-i\Omega(t_1-t_2)}\underline{\langle 0|\phi(x_A,t_1)\phi(x_A,t_2)|0\rangle} dt_2 dt_1$$

autocorrelation

$$\langle\phi(x_1)\phi(x_2)\rangle = \frac{1}{r^2 - (t_1 - t_2 - i\varepsilon)^2}$$

Plot for Minkowski spacetime



Ω: Energy gap of detectors (~ frequency observed by detectors)
σ: Duration of observation
r: Distance between detectors

Expanding universe

Back ground spacetime: de Sitter spacetime

$$ds^2 = -dt^2 + e^{2Ht}d\vec{x}^2 = e^{2Ht}(-d\eta^2 + d\vec{x}^2)$$

$$(-\infty < \eta < 0 \quad \Leftrightarrow \quad -\infty < t < \infty)$$

Field: minimal coupling massless test scalar field

$$\phi = \frac{1}{(2\pi)^{3/2}a(t)} \int \left(a_{\vec{k}}\varphi_k e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}\right) dk^3$$

$$\varphi_k = \sqrt{-\eta}(H^{(1)}_{3/2}(-k\eta)C_1 + H^{(1)}_{3/2}(-k\eta)C_2) \qquad [a_{\vec{k}}, a^\dagger_{\vec{l}}] = \delta^{(3)}(\vec{k} - \vec{l})$$

Boundary condition: $(1/\sqrt{2k})\exp(-k\eta)$ in the limit $\eta \to -\infty$

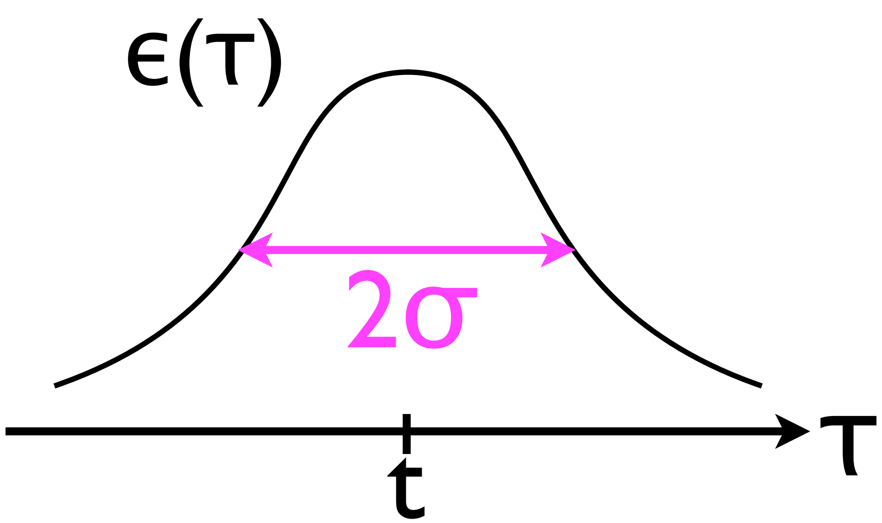
$$\varphi_k = \sqrt{-\eta}H^{(1)}_{3/2}(-k\eta)$$

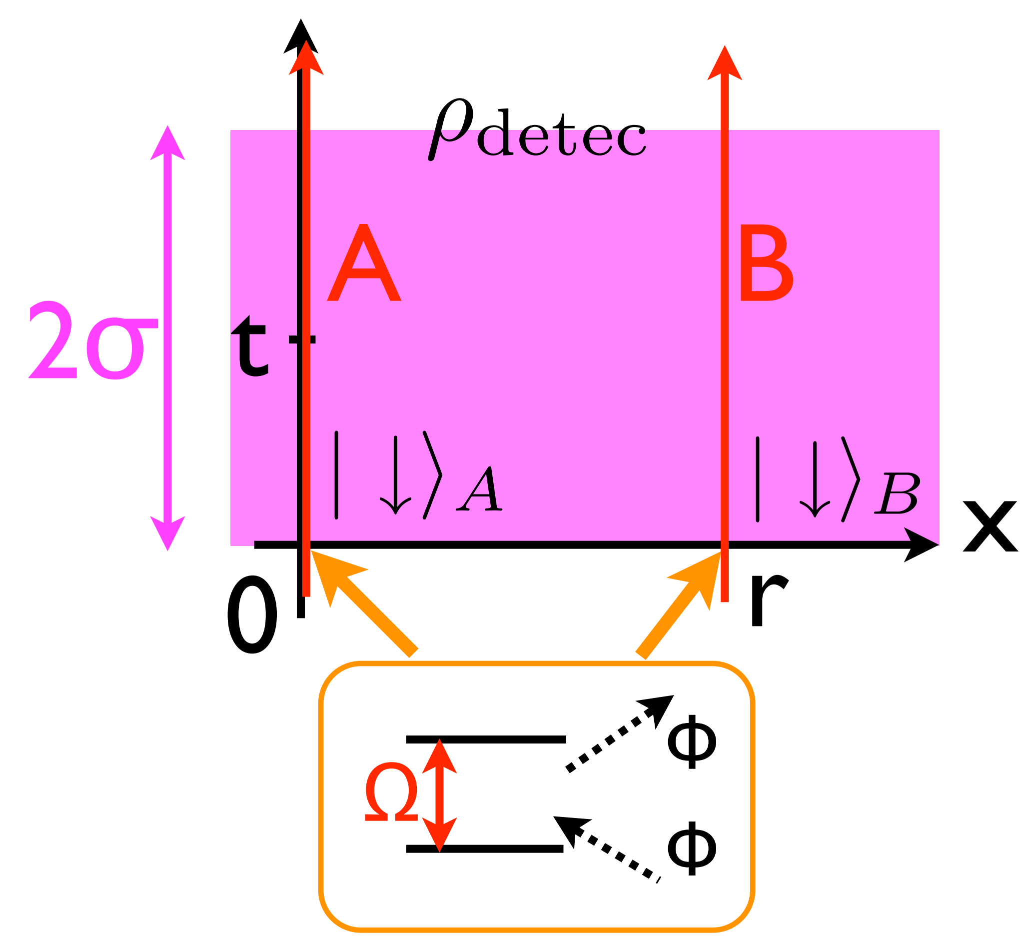
State: Bunch-Davies vacuum

$$a_{\vec{k}}|0\rangle = 0$$

Detector:

- Two detectors separated by comoving distance r
- They fly away from each other by the cosmological expansion.
- Their energy gaps, Ω are taken to be the same magnitude.
- Initial state: $|\downarrow\downarrow\rangle$ (separable state).
- Window function $\epsilon(\tau)$: Gaussian with variance σ
- The time parameter τ in the window function is the proper time of the detector (= cosmic time).





Negativity

$$N(\rho_{\mathrm{detec}}) = |Y| - X$$

$$Y := -2\int_{-\infty}^{\infty}\int_{-\infty}^{t_1} \varepsilon(t_1)\varepsilon(t_2)e^{i\Omega(t_1+t_2)}\underline{\langle 0|\phi(x_A,t_1)\phi(x_B,t_2)|0\rangle} dt_2 dt_1$$

two point funcion: propagation of the quantum

$$X := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon(t_1)\varepsilon(t_2)e^{-i\Omega(t_1-t_2)}\underline{\langle 0|\phi(x_A,t_1)\phi(x_A,t_2)|0\rangle} dt_2 dt_1$$

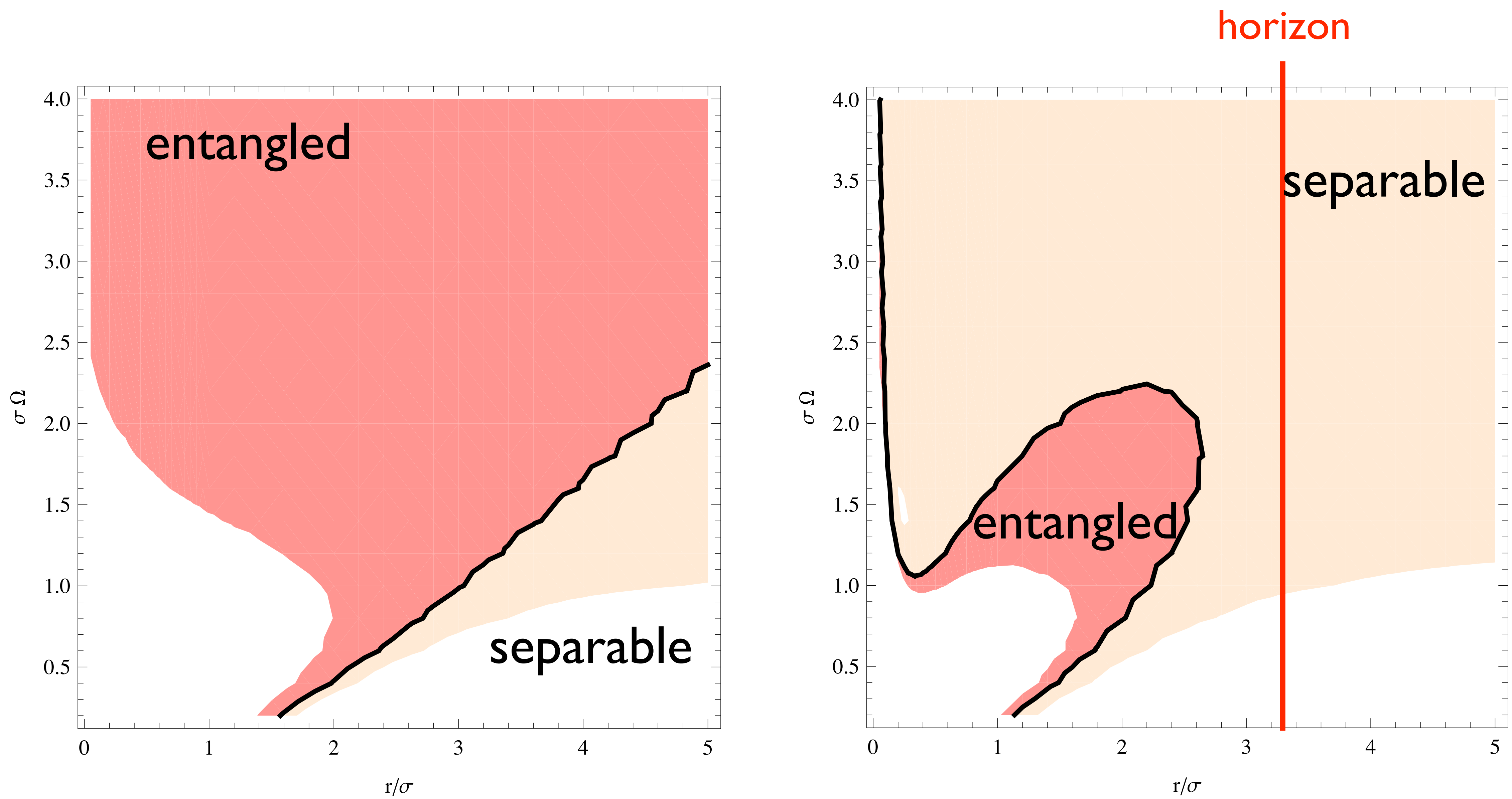
autocorrelation

$$\langle\phi(x_1)\phi(x_2)\rangle = \frac{H^2}{(2\pi)^2}\left[1 - \gamma - \ln\frac{k_0}{H} - \frac{\eta_1\eta_2}{(\eta_1-\eta_2-i\varepsilon)^2 - r^2} - \frac{1}{2}\ln H^2(r^2 - (\eta_1-\eta_2-i\varepsilon)^2)\right]$$

k_0: infra red cut off

The negativity converses in the limit of k_0 → 0.

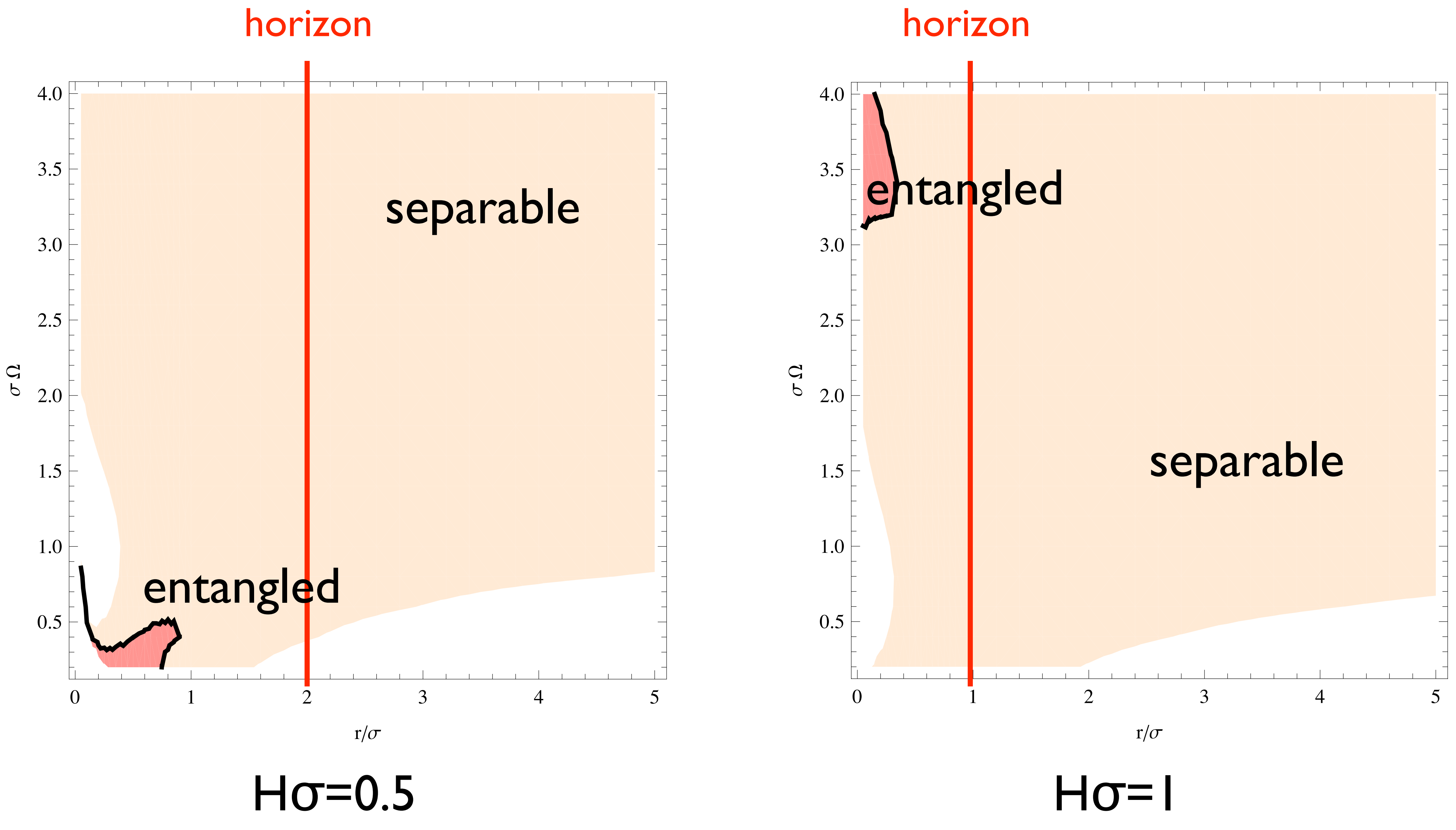
Result



Hσ=0.1

Hσ=0.3

Result


horizon
separable
entangled
σ Ω
r/σ
Hσ=0.5
horizon
entangled
separable
σ Ω
r/σ
Hσ=1

Summary

- The entanglement of the primordial fluctuation between spatial two points must disappear.
- We estimated the entanglement of the field by using the particle detectors on the accelerating universe model.
- The field is not entangled beyond the super horizon scale.