String Gas Cosmology and two-form fluxes

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INTRODUCTION

STRING GAS COSMOLOGY [Brandenberger-Vafa] A cosmological model for the early universe which emphasizes the importance of symmetries and degrees of freedom new to string theory.

T-DUALITY $R \approx \frac{1}{R}$ + OSCILLATORY MODES, WINDING MODES

In this model the UNIVERSE BEGINS with all spatial dimensions on toroidal compactification near string length l_s and it is filled with an ideal gas of fundamental matter (string/brane gas) thermalized at the critical temperature (Hagedorn temperature T_H).

ALL DIMENSIONS COMPACT $\sim T^9$ + GAS OF STRINGS AT THE CRITICAL TEMPERATURE (HAGEDORN T_H)

Why string gas cosmology?

- This model allow us explore the role that new degrees of freedom and symmetries of string theory play in the early stages of the universe.
- Mechanism for dynamical generation of a 4-dimensional space-time.
- Dynamical mechanism for generating 3+1 dimensional space-time.
- Possible alternative to inflation.
- Alternative solution to the Horizon problem $\longrightarrow$ LOITERING PHASE
- Mechanism for generating a scale-invariant spectrum of cosmological perturbations.

Implementation of the model in string theory

EARLY UNIVERSE

BACKGROUND
Low energy effective action (dilaton gravity)
$$S = \frac{1}{2\kappa^2_{d+1}} \int d^{d+1}x\sqrt{-g}e^{-2\phi}\Big(R + 4(\nabla\phi)^2 - V - \frac{1}{12}H^2\Big)$$
+
GAS OF STRINGS
$$\int d^{d+1}x\sqrt{-g}\mathcal{L}_{matter}$$

where R is the scalar curvature, $d+1$ is the dimension of the space-time ($d = 9$ for the superstring), ϕ is the dilaton, $\kappa^2_{d+1} = 8\pi G_{d+1}$ and $H^2 = H_{\mu\nu\lambda}H^{\mu\nu\lambda}$ with $H_{\mu\nu\lambda} = \partial_{[\mu}B_{\nu\lambda]}$ with $B_{\mu\nu}$ a two-form field. This is the model that string theory gives for describing the universe at early stages.

Note: In most of the models for cosmology the two-form potential is neglected $H_{\mu\nu\lambda} = 0$.

OUR OBJECTIVE: study the role that the two-form gauge field $B_{\mu\nu}$ plays in string gas cosmology

- $B_{\mu\nu}$ can provide a mechanism for generating $3+1$ dimensional space time.
- A field gauge can wrap the compact dimensions $\longrightarrow$ what is the effect on the string gas?

Modified string effective action

In order to take into account possible higher order derivative action for the string effective action [Greene et al.] and we couple it to a two-form field potential. In this case we reduce the analysis to a 4-dimensional space time ($d = 3$)

$$S = \int dt\Big[8\pi e^{-\varphi}\Big(\sqrt{1-\dot\varphi^2} - \sqrt{1-d\dot\lambda^2} - U(\lambda)\Big) + L_m\Big]$$

where φ is the shifted dilaton and we have used the homogeneous metric

$$ds^2 = -dt^2 + a(t)^2 dx_i dx^i\,, \qquad i = \{1,2,3\}\,, \qquad a(t) = e^{\lambda(t)}$$

and

$$U(\lambda) \equiv \frac{1}{2}H_0^2(e^{2\lambda} + e^{-2\lambda})^{-d} = \frac{1}{2^{d+1}}H_0^2(\cosh 2\lambda)^{-d}$$

where $U(\lambda)$ is not singular at $\lambda = 0$ and it reduces to the low energy effective action above for large λ, $L_m = -F$ negative of the matter free energy.

We take the matter as follows

- $E = E_W + E_K + E_{\rm dust} + V$
- Winding modes (W_i =winding number: $E_{Wi} = 2dW_i e_i^{\lambda}$
- Momentum modes K_i =momentum number: $E_{Ki} = 2dK_i e^{-\lambda_i}$
- Oscillator modes: $E_{\rm dust}$
- Pressures: $P_\varphi = \frac{\partial E}{\partial\varphi} = \frac{\partial V}{\partial\varphi}$, $P_{\lambda i} = -\frac{1}{d}\frac{\partial E}{\partial\lambda_i} = 2K_i e^{-\lambda_i} - 2W_i e^{\lambda_i}$

Equations of motion

$$\dot\gamma_\varphi = \dot\varphi(\gamma_\varphi - \gamma_\lambda^{-1} - U(\lambda)) + \frac{1}{8\pi^2}\dot\varphi e^{\varphi}P_\varphi\,, \quad \gamma_\varphi \equiv \frac{1}{\sqrt{1-\dot\varphi^2}}$$
$$\dot\gamma_\lambda = \dot\varphi(\gamma_\lambda - \gamma_\lambda^{-1}) - \lambda\frac{\partial U}{\partial\lambda} + \frac{1}{8\pi^2}d\dot\lambda e^{\varphi}P_\lambda\,, \quad \gamma_\lambda \equiv \frac{1}{\sqrt{1-d\dot\lambda^2}}$$

Hamiltonian constraint
$$\gamma_\varphi - \gamma_\lambda - U(\lambda) = \frac{1}{8\pi^2}Ee^{\varphi}\,,$$

λ (size of the universe) is subjected to an effective potential produced by the two-form field and the energy E of the string gas. $U(\lambda)$ pushes the universe towards expansion while E makes the universe to remain near $\lambda = 0$ [Campos].

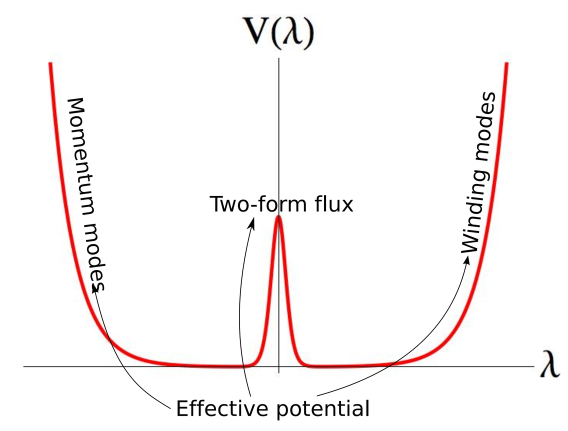


Figure: Effective potential.

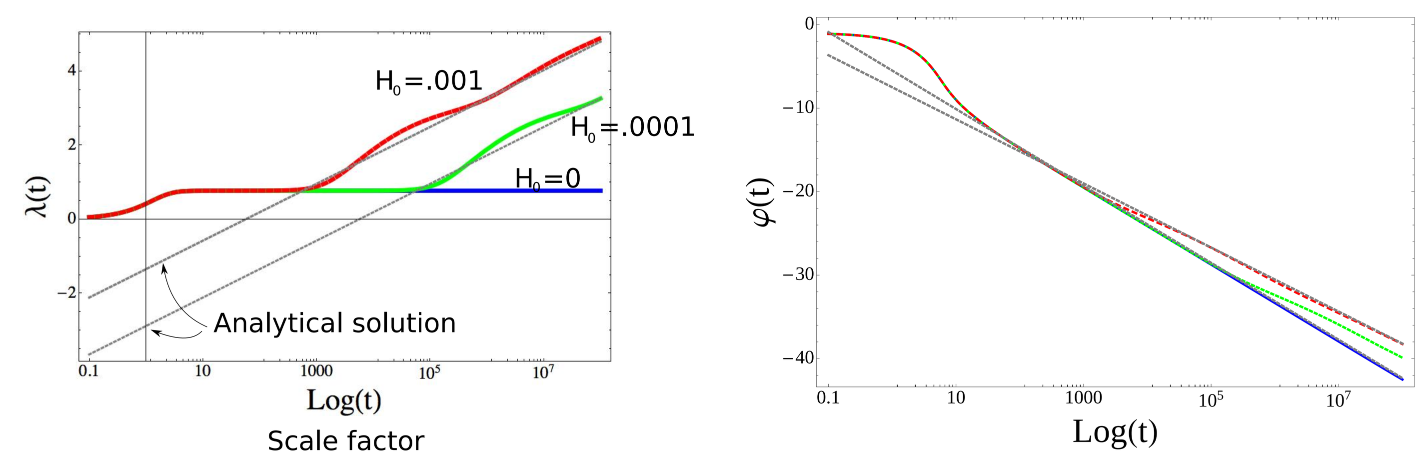
RESULTS (d=3)

$P_\lambda = 0$

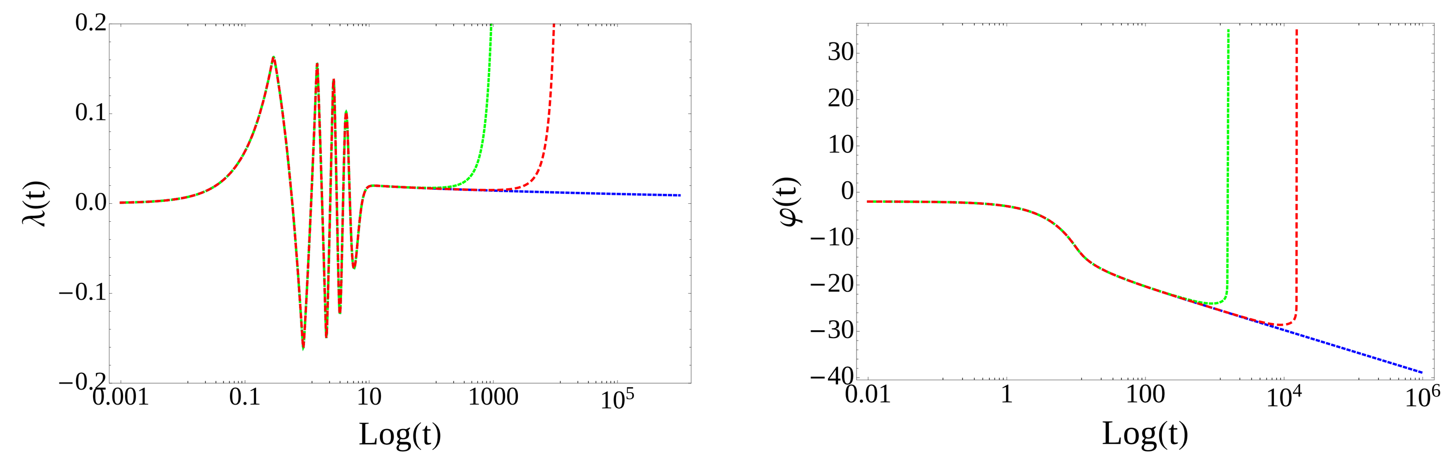
A representative solution when $P_\lambda = 0$ and we go to late times $\dot\lambda \to 0,\ \dot\varphi \to 0,\ |\lambda| \to \infty,\ \varphi \to -\infty$, is given by

$$\varphi = \Big(-2 + \frac{1}{d}\Big)\log t + B\,, \qquad \lambda = \frac{1}{d}\log t + \frac{1}{2d}\log\Big(\frac{H_0^2 d^2}{d-1}\Big)$$

where H_o determines the moment at which the universe begins to grow large. The numerical solutions are plotted in colour and the gray straight lines represent the solutions for the analytical solution above. Here we observe also that the two-form potential drives the universe to become large. As can be seen in the figure, in the case when H_o vanishes the size of the universe remains small while it eventually grows when the two-form flux is present.



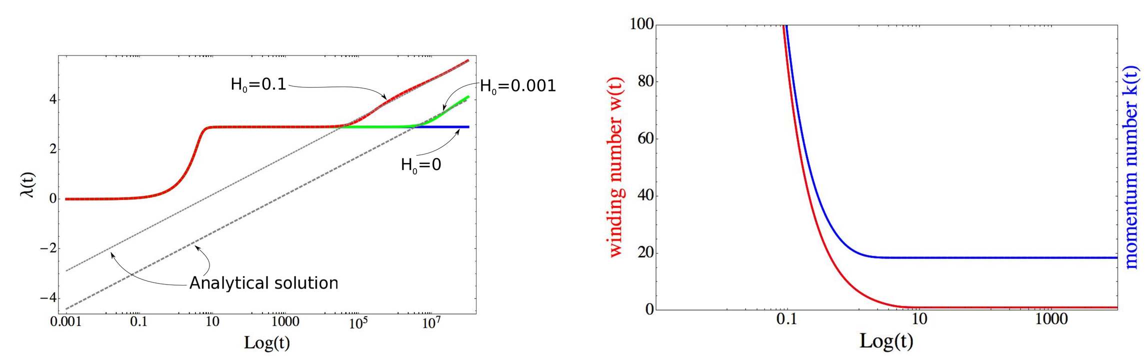
Winding modes, momentum modes and flux



Three dimensional scale factor grows large!!

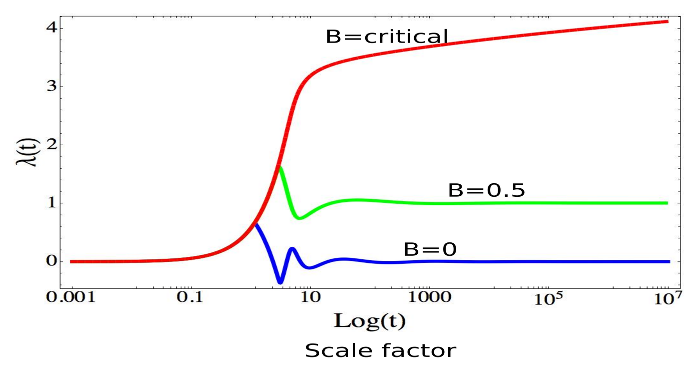
Boltzmann equations (weak string coupling)

$$\dot W = -\frac{e^{2\lambda+\varphi}}{\pi}(W^2 - \langle W\rangle^2)\,, \qquad \dot K = -\frac{e^{-2\lambda+\varphi}}{\pi}(K^2 - \langle K\rangle^2)\,. \tag{1}$$



Constant $B_{\mu\nu}$ ($H_{\lambda\mu\nu} = 0$)

- Constant flux around compact dimension $\Rightarrow$ winding string energy spectrum is modified
- Consider $B \equiv B_{01} = B_{02} = B_{03}$, then $E_W = 2d(1-\sqrt{3}B)We^{\lambda}$. For critical value of B the energy of the string vanishes.



Conclusions and outlook

- We introduced a modified potential in order to make the model compatible with T-duality
- The introduction of a two-form field gives allows for the decompactification of 3 large spatial dimensions.
- String gas Matter dominance: at late times two-form field is not significant and late solutions reduce to those found in dilaton cosmology (radiation).
- Two-form potential dominance: vanishing string pressure and $H_{\mu\nu\lambda} \neq 0 \Rightarrow$ expanding universe. Match numerical solution.
- For constant $B_{\mu\nu}$ there is a critical value thar makes the energy of the winding strings vanish $\rightarrow$ universe is allowed to expand in 3+1 dimensions.
- It remains to see if the extra dimensions and the dilaton ϕ can be stabilized simultaneously.

References

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