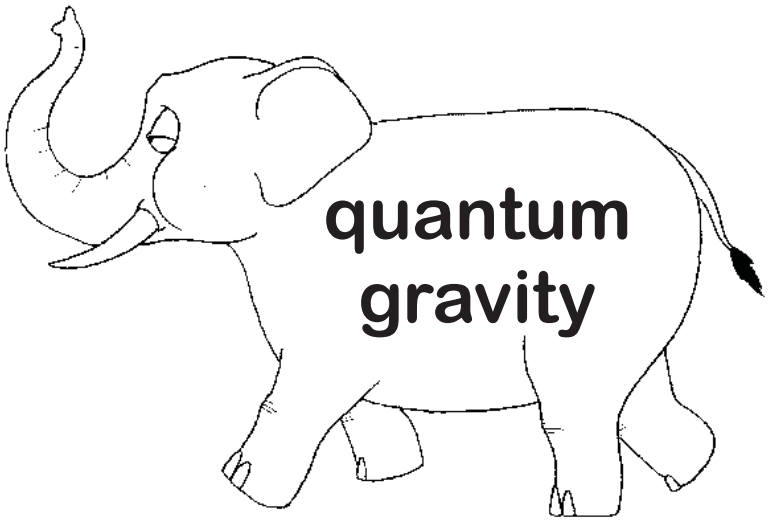
Polymer inflation

Sanjeev Seahra (with G Hossain and V Husain)

September 30, 2010

Quantum gravity



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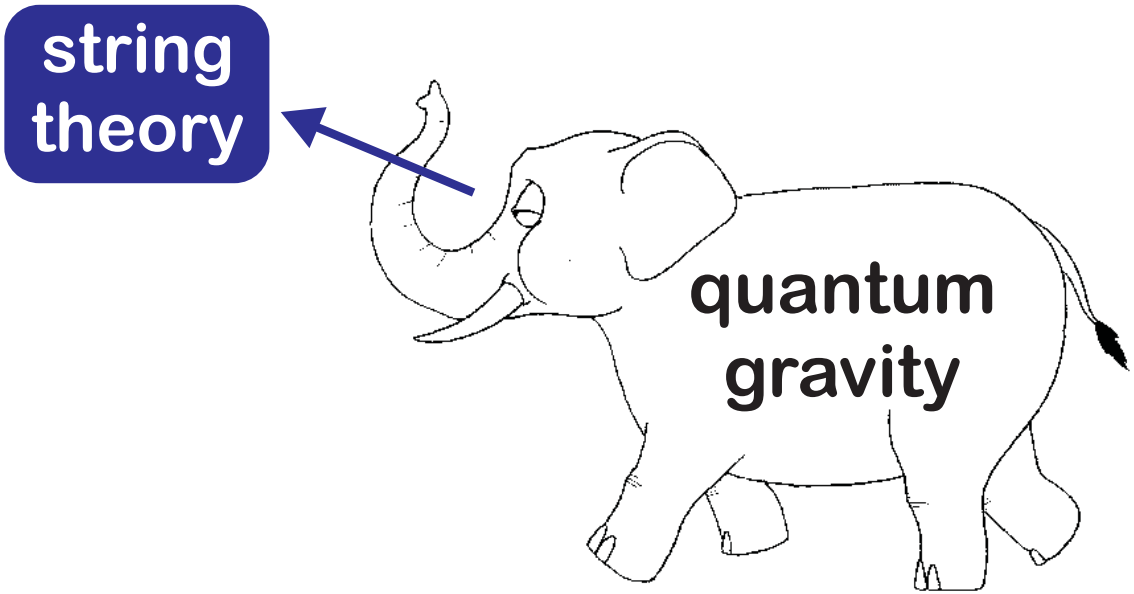

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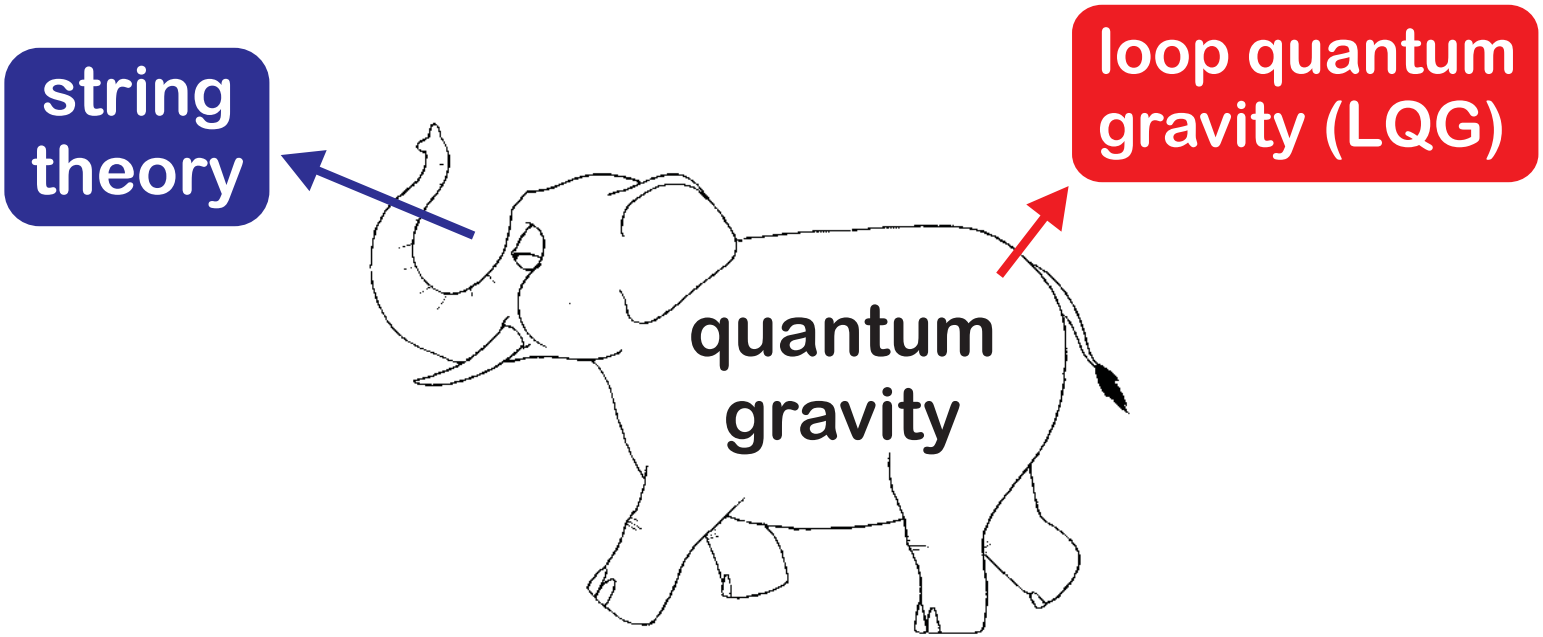

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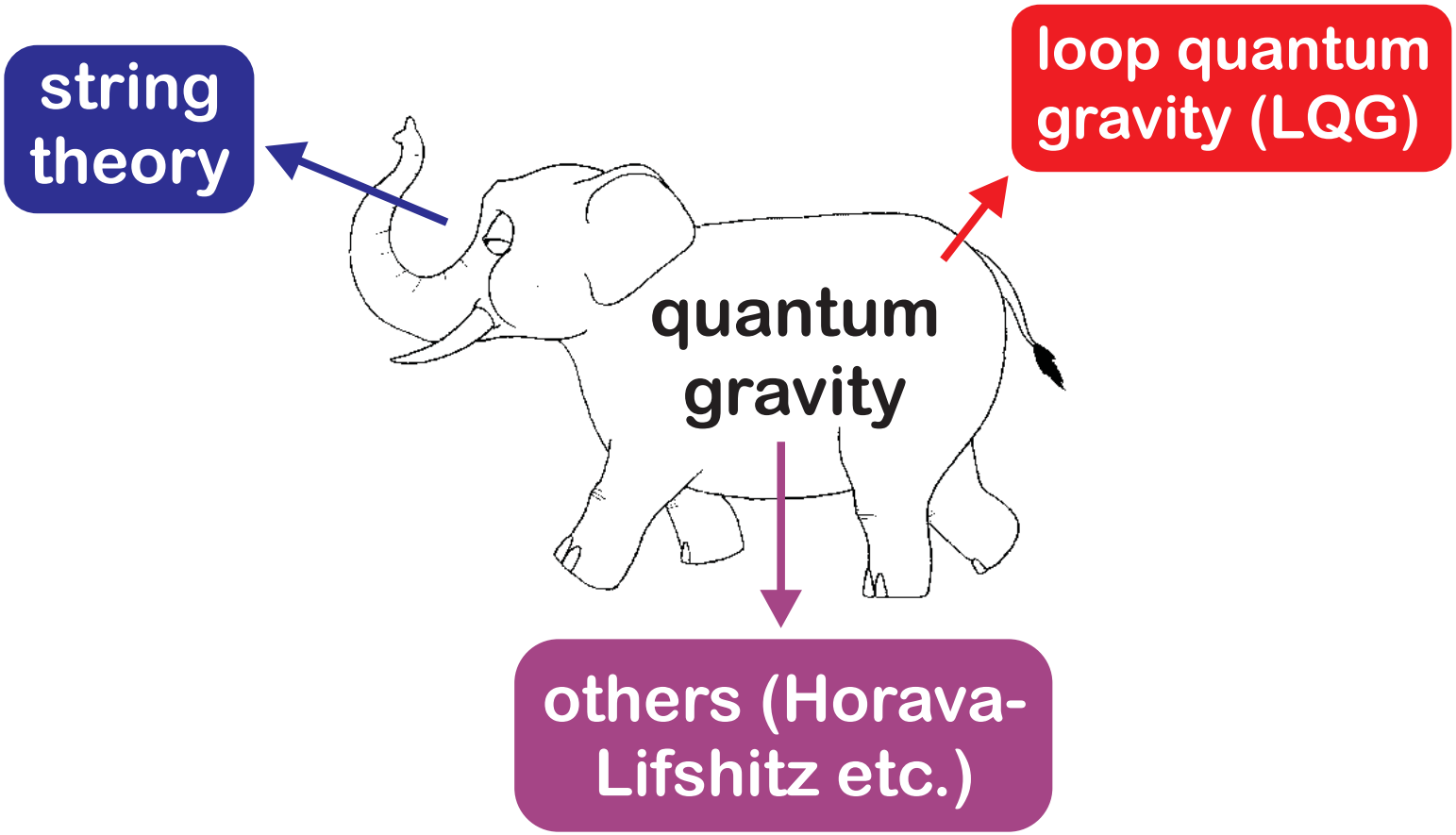

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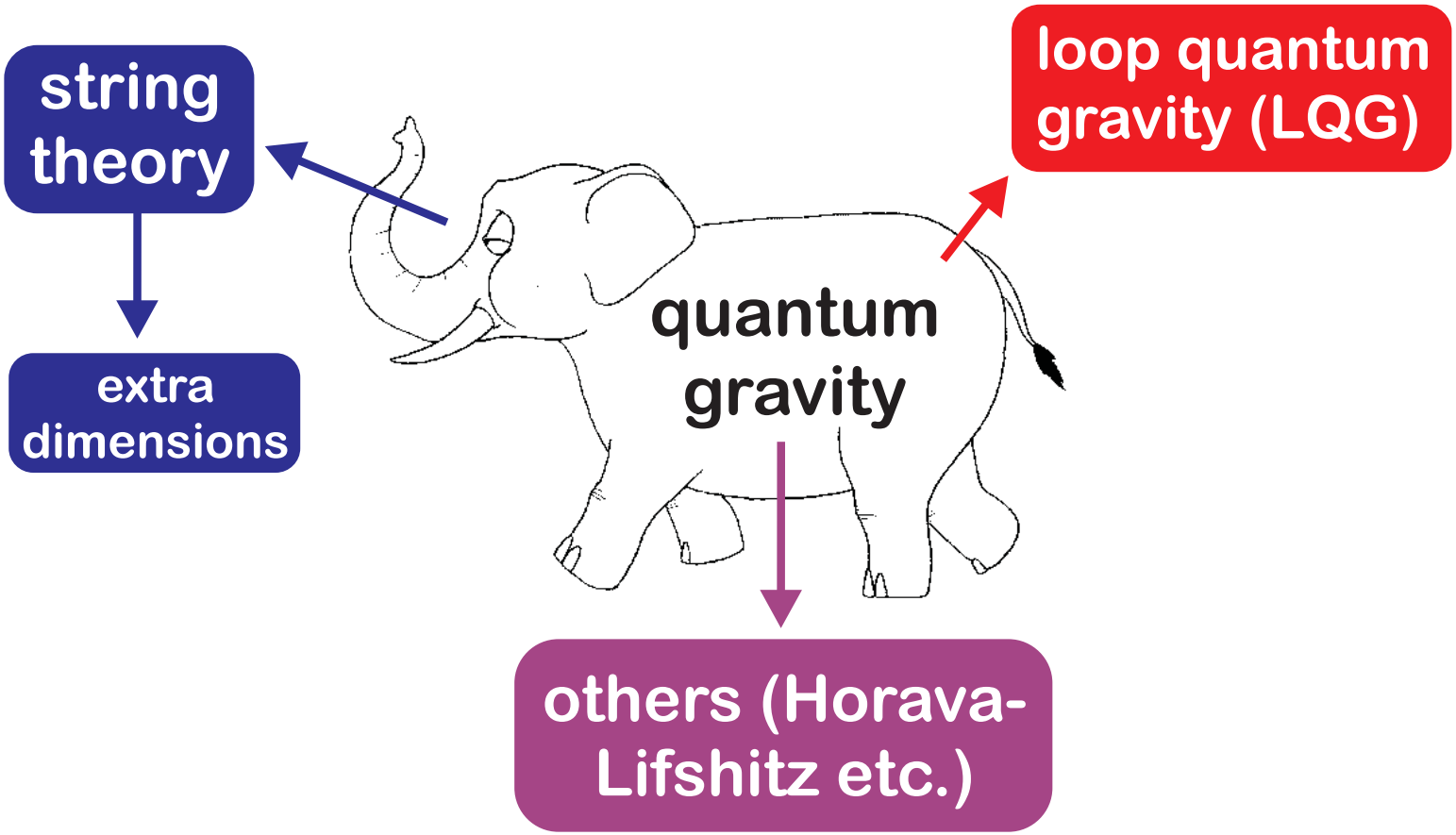

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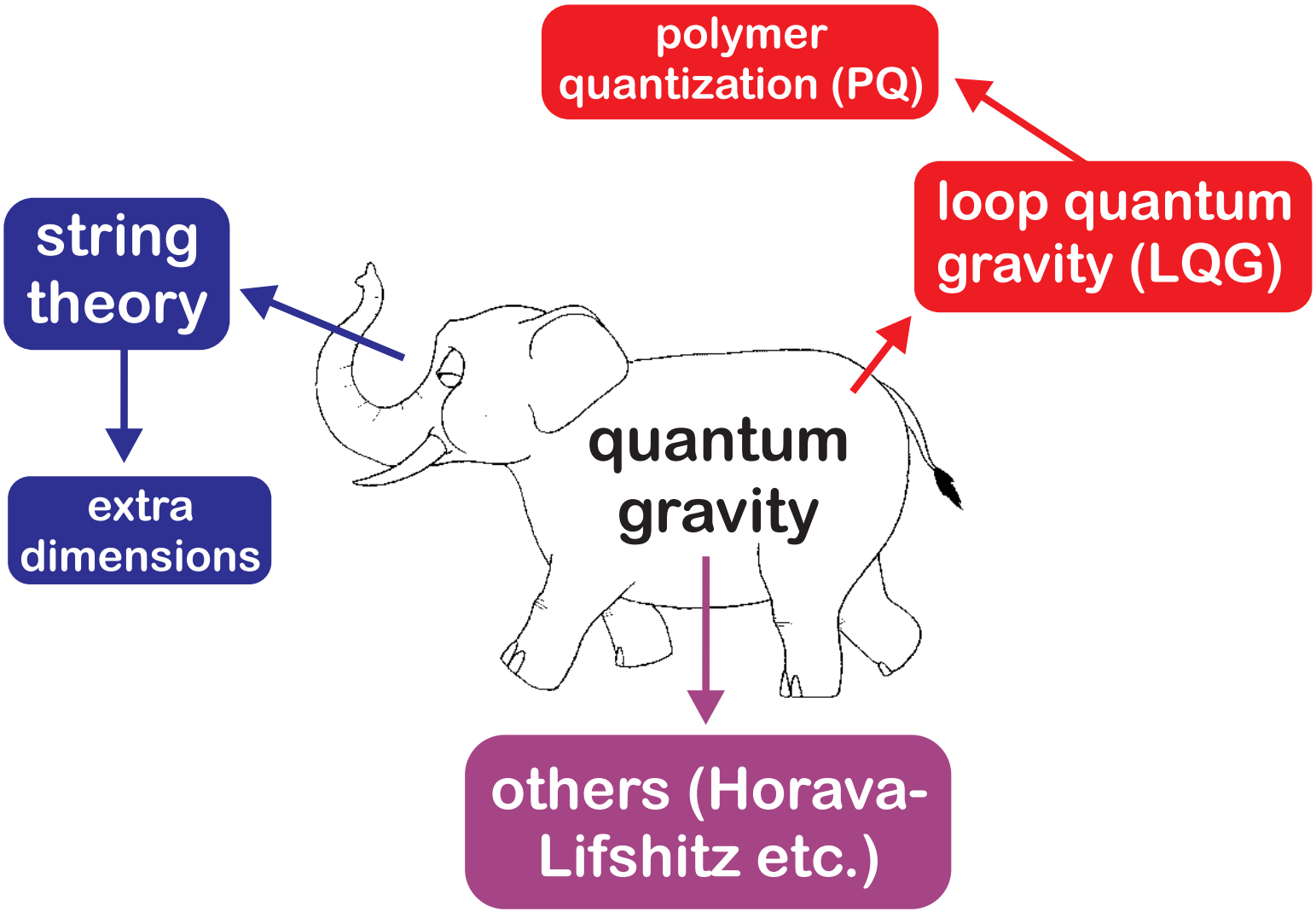


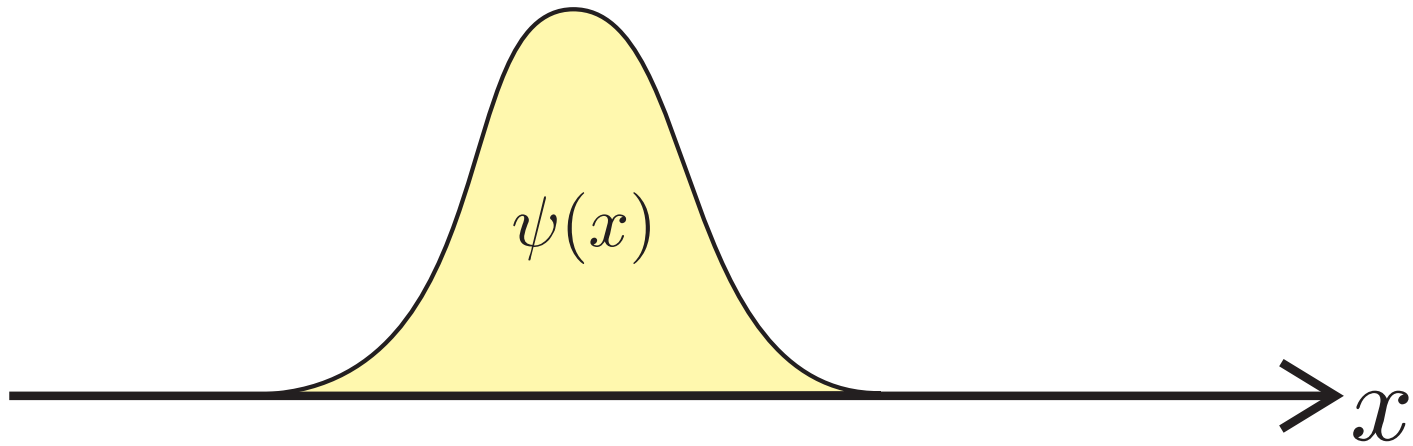
what are the effects of polymer quantization in the early universe?

what is the difference between polymer and Schrodinger quantization?

consider a particle moving on a line:

Polymer quantization

in Schrödinger quantum mechanics physical states are delocalized in x



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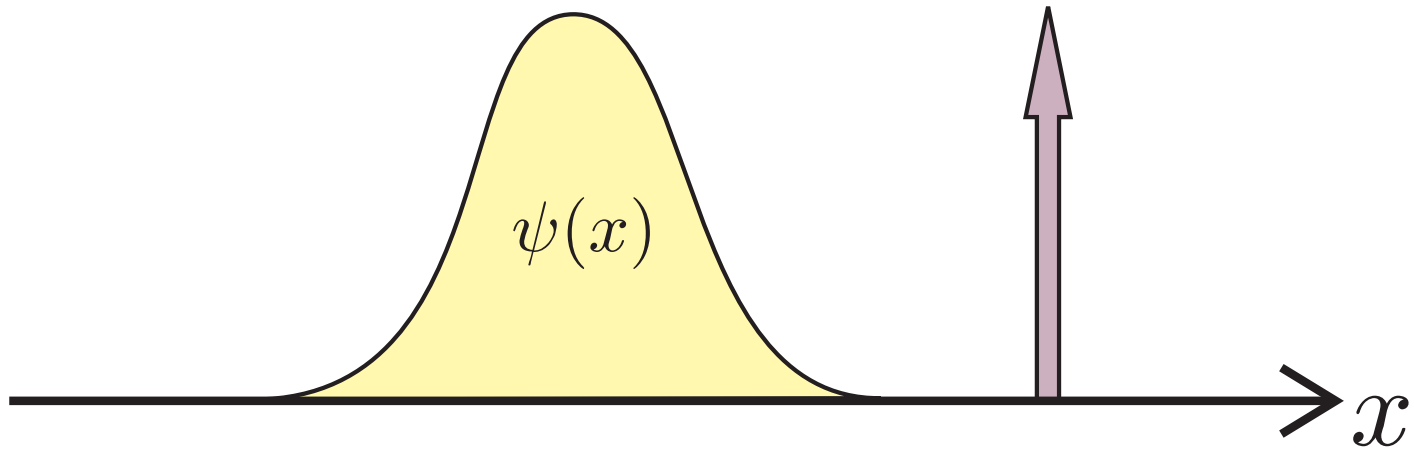

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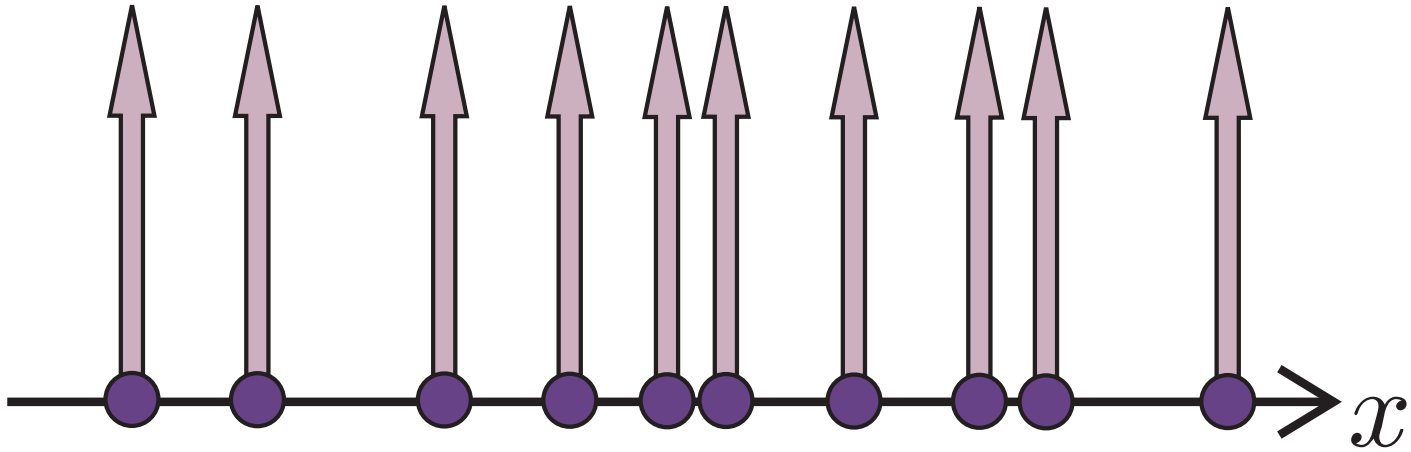

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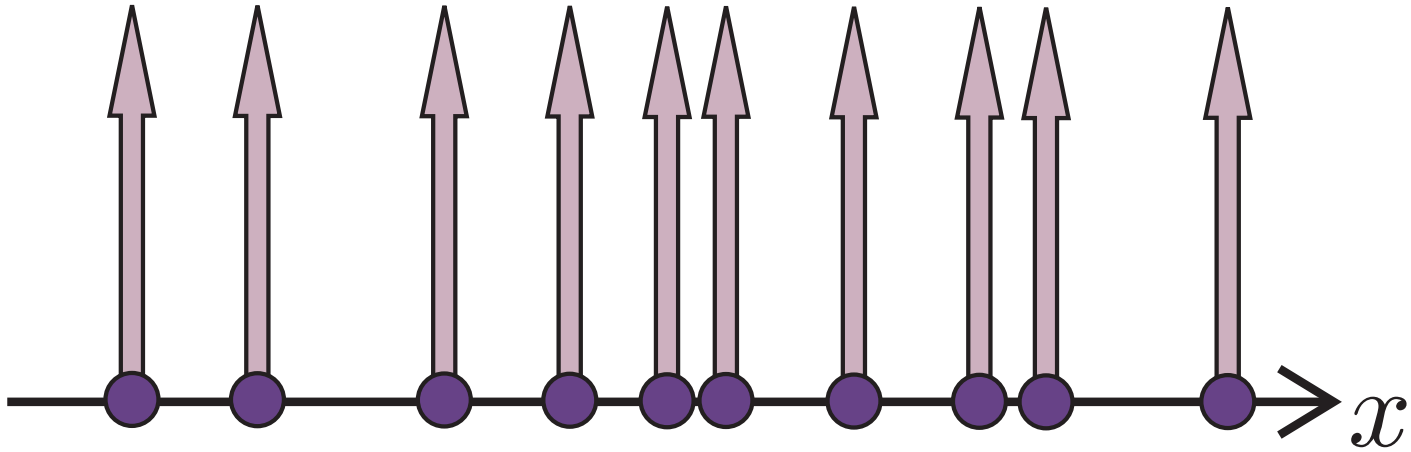
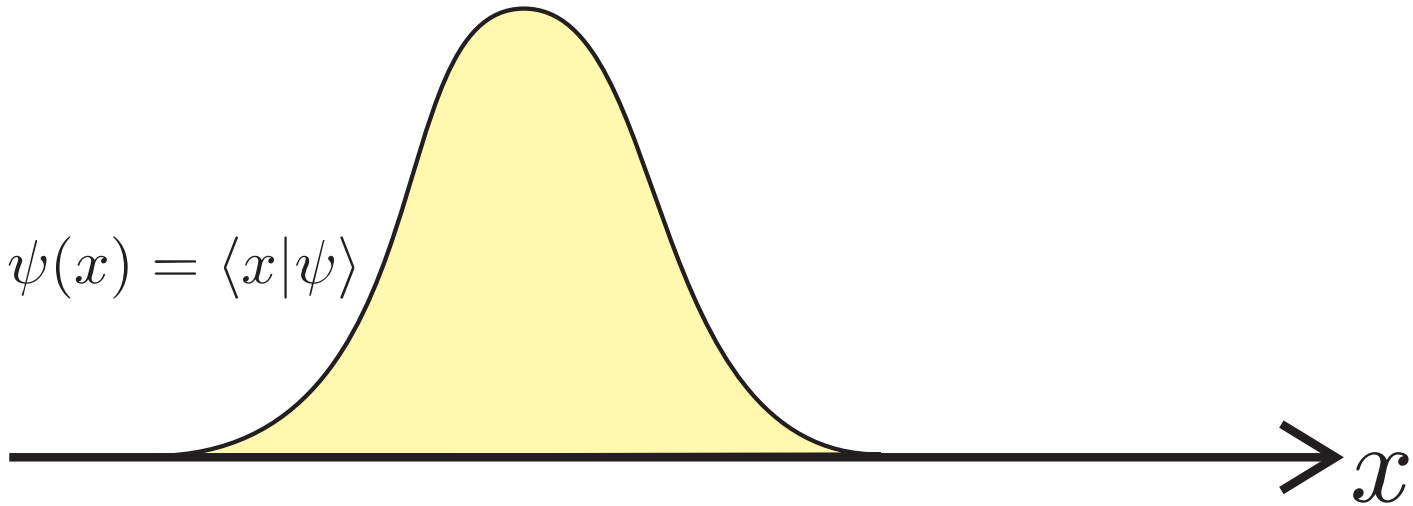


for each x there is a normalizable basis state $|x\rangle$ $\Rightarrow$ notion of fundamental discreteness

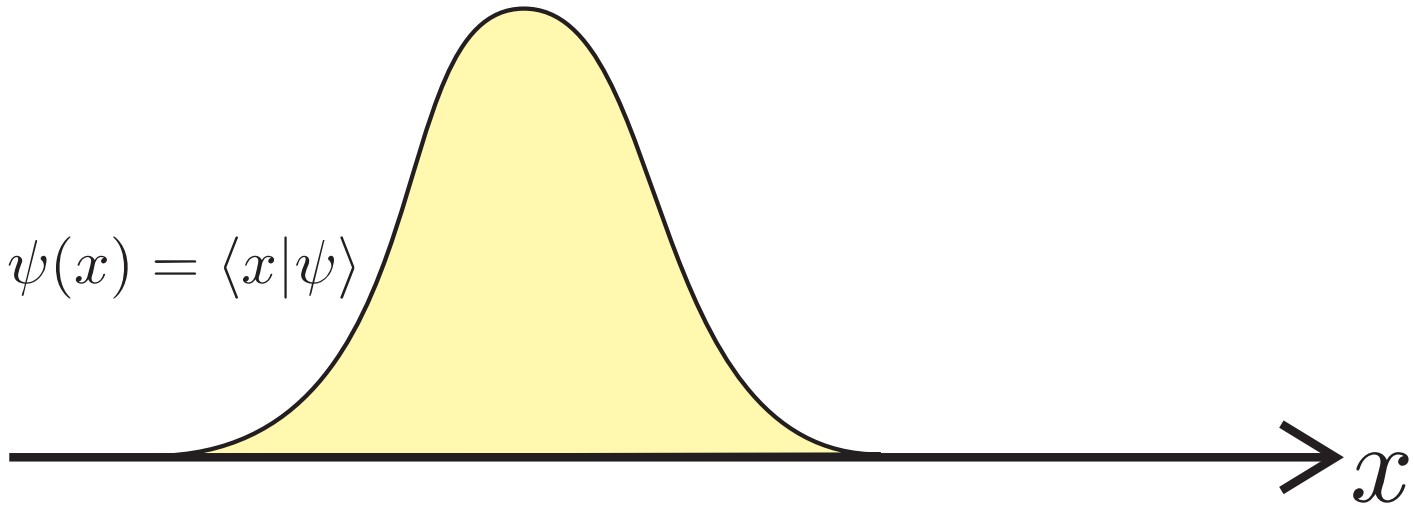
what are the basic operators in polymer quantum mechanics?

Schrödinger QM:



Basic operators

Schrödinger QM: action of momentum $\hat{p}$ and translation $\hat{U}_\lambda$ well defined



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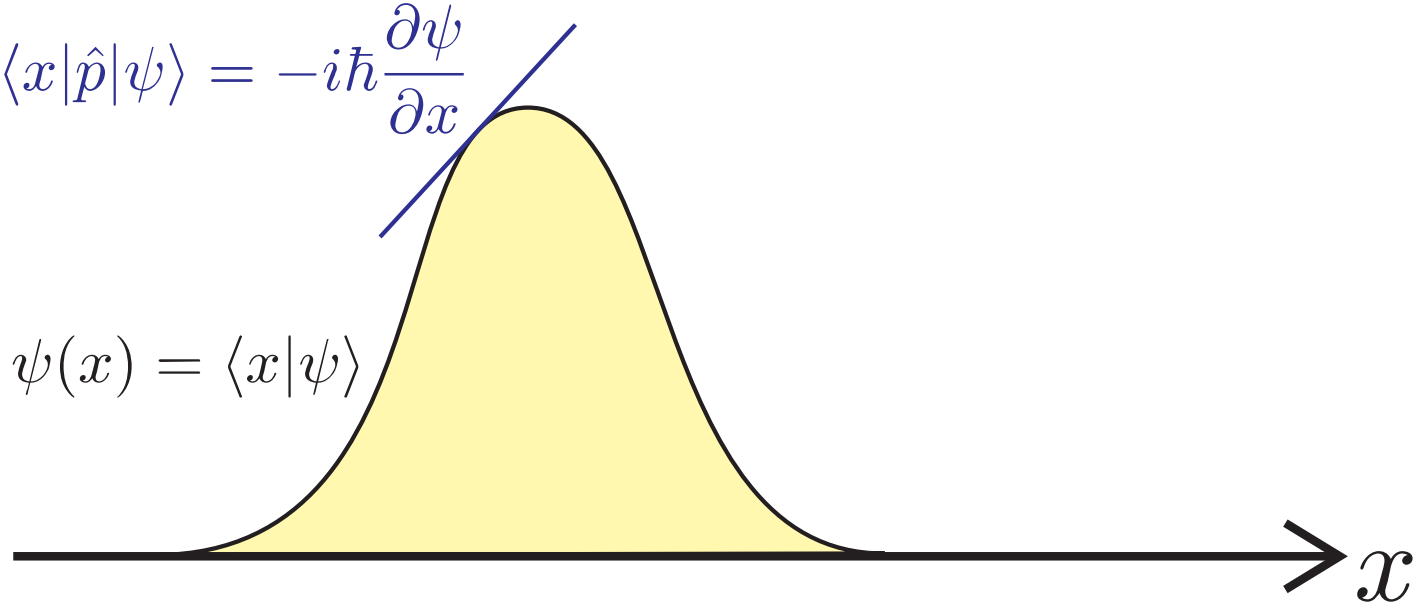
$$\langle x|\hat{p}|\psi\rangle = -i\hbar\frac{\partial\psi}{\partial x}$$

$$\psi(x) = \langle x|\psi\rangle$$



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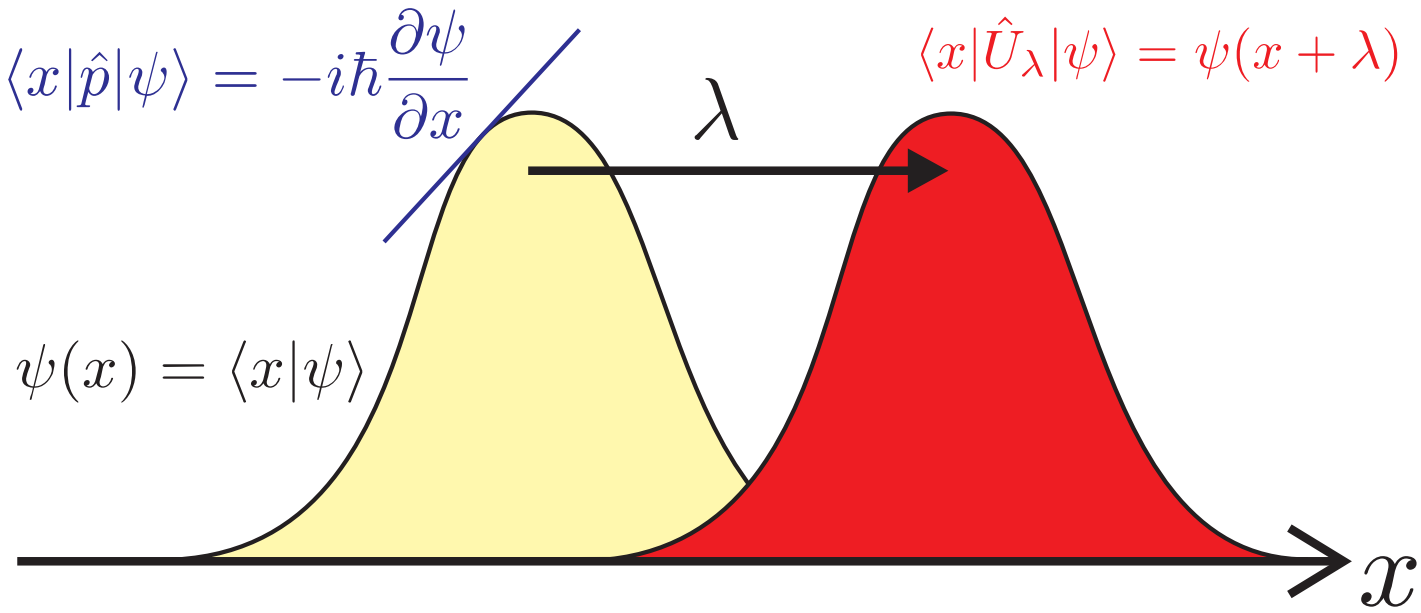

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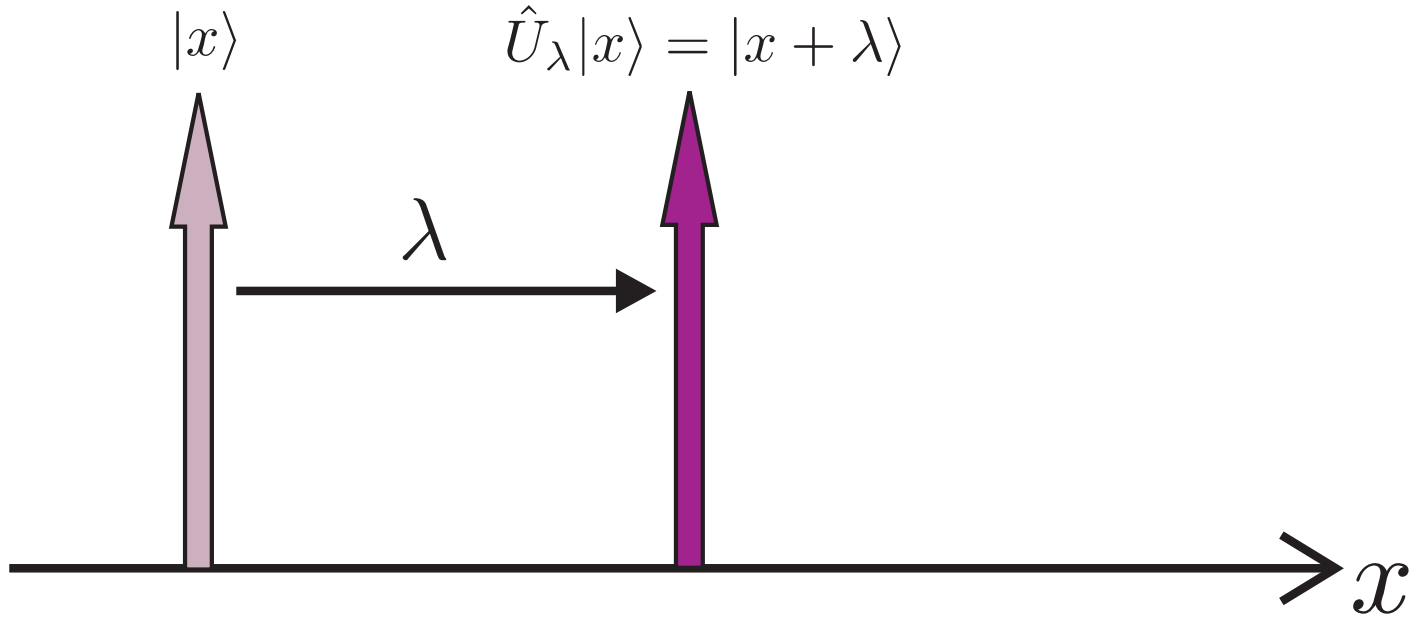

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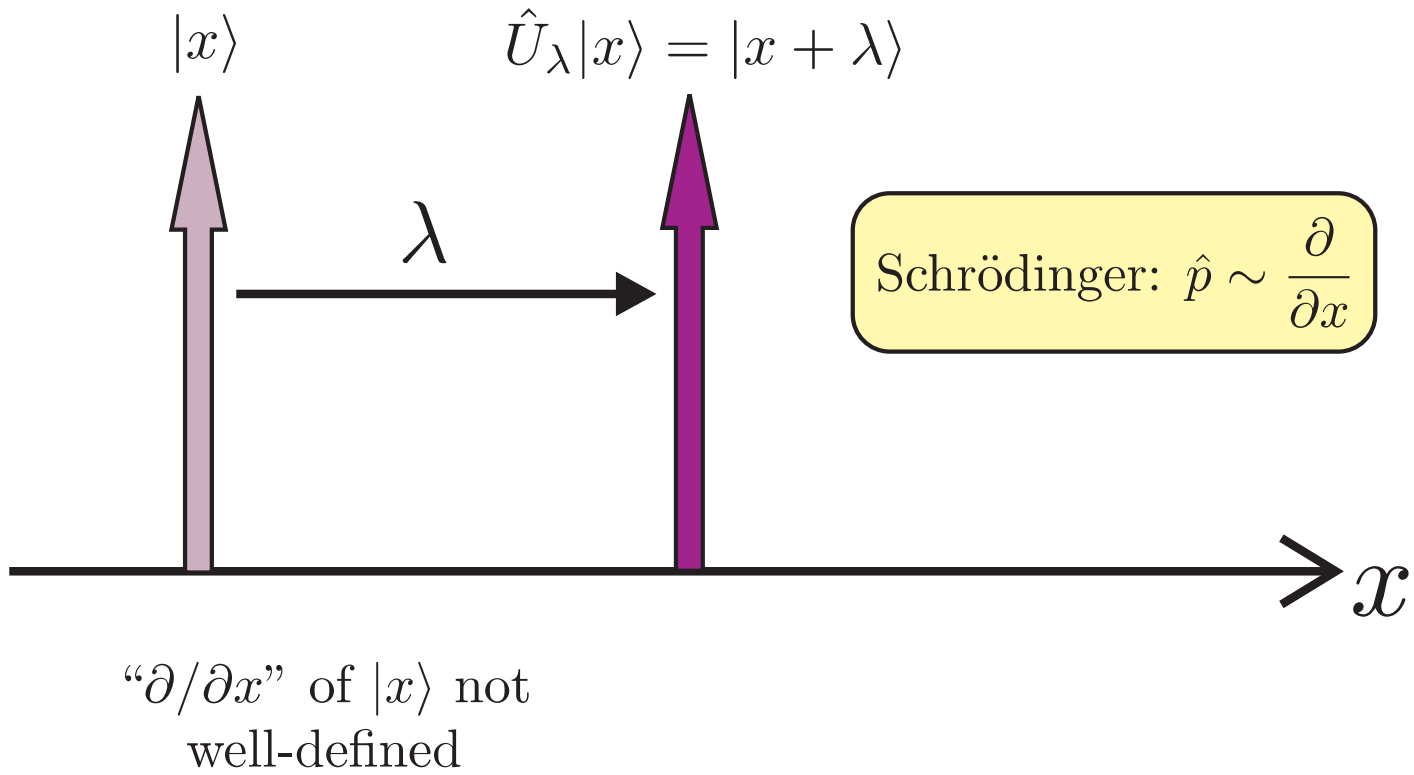

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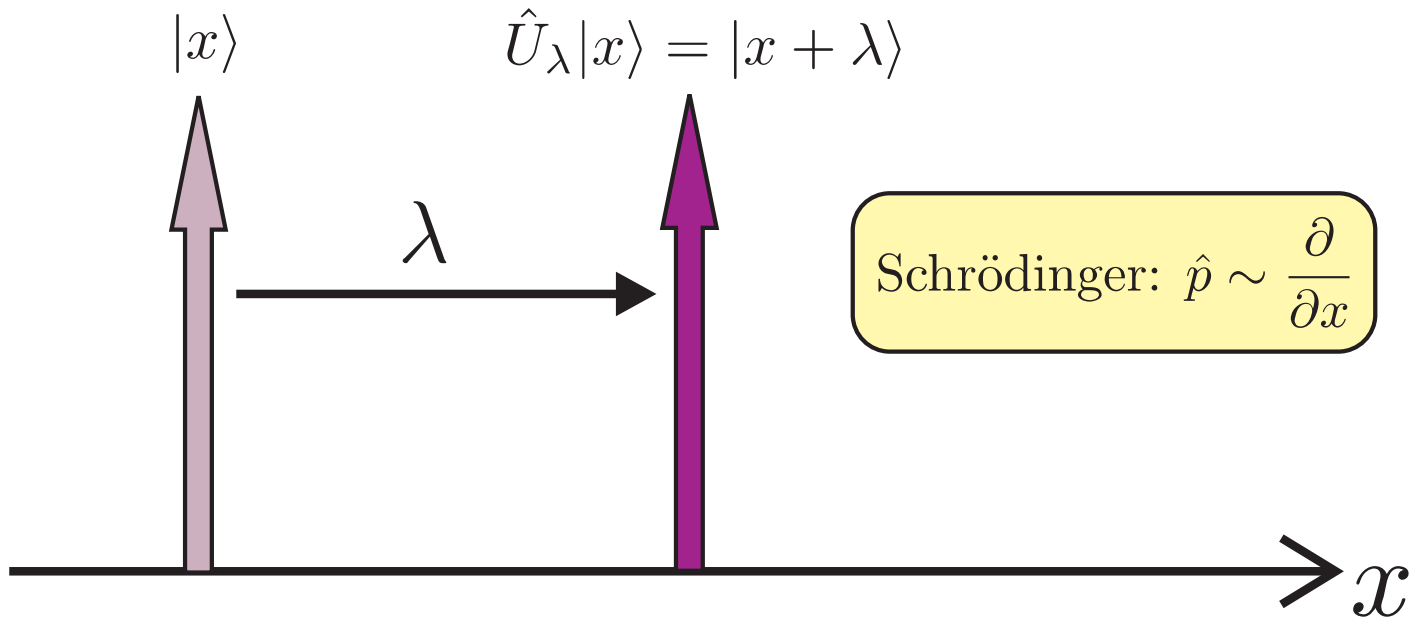

Polymer momentum

operationally, polymer QM defined by:

$$\hat{p} \mapsto \hat{p}_{\lambda_\star} \equiv i\hbar \left(\frac{\hat{U}_{\lambda_\star} - \hat{U}^\dagger_{\lambda_\star}}{2\lambda_\star} \right)$$

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NB: not a unique choice of $\hat{p}$ and $M_\star$ determined by experiment

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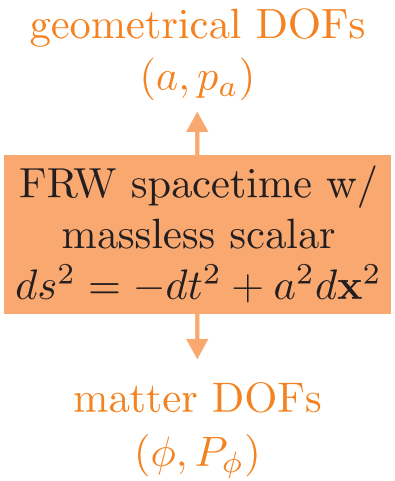
we want to apply polymer methods to FRW models... what has been done before?

Quantum cosmologies

FRW spacetime w/ massless scalar

$$ds^2 = -dt^2 + a^2 d\mathbf{x}^2$$

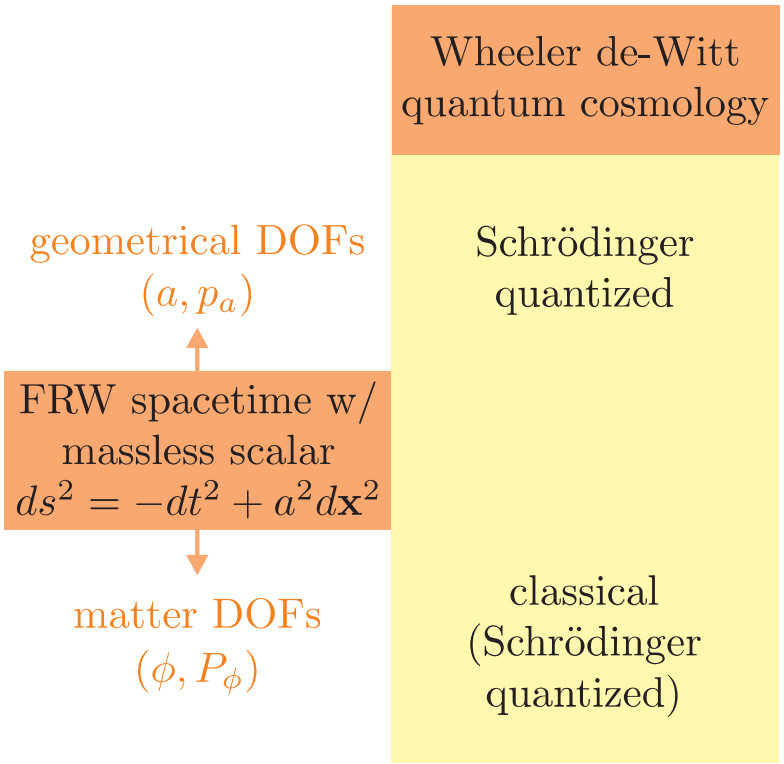
Quantum cosmologies



$a =$ scale factor $\qquad \phi =$ scalar field amplitude

$p_a =$ momentum conjugate to $a \qquad P_\phi =$ momentum conjugate to ϕ

Quantum cosmologies



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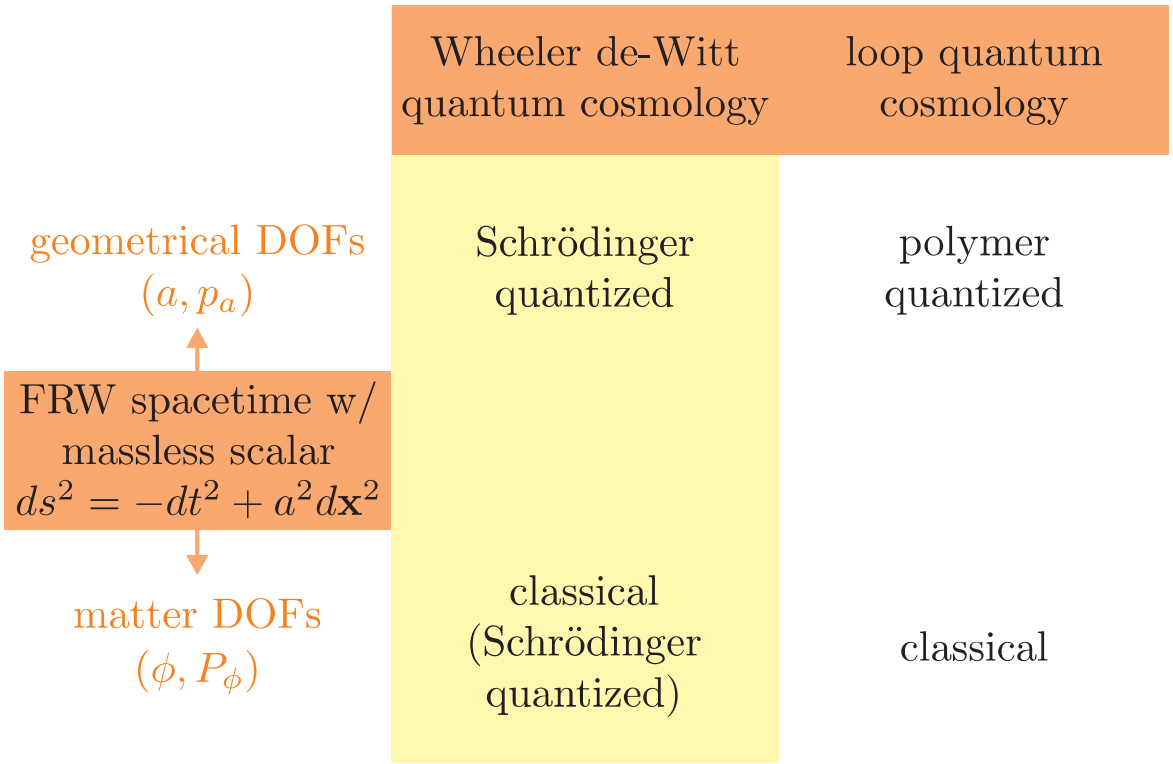

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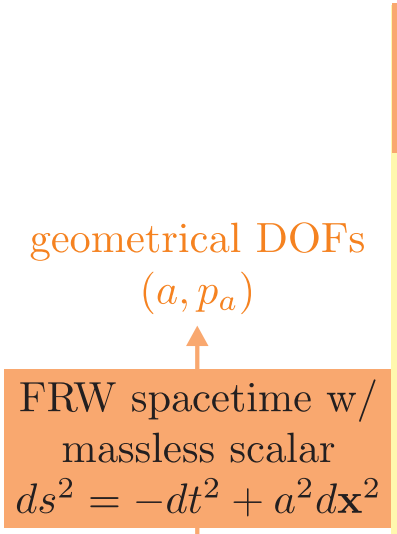

	Wheeler de-Witt quantum cosmology	loop quantum cosmology	this work
geometrical DOFs (a, p_a)	Schrödinger quantized	polymer quantized	classical
matter DOFs (ϕ, P_ϕ)	classical (Schrödinger quantized)	classical	polymer quantized

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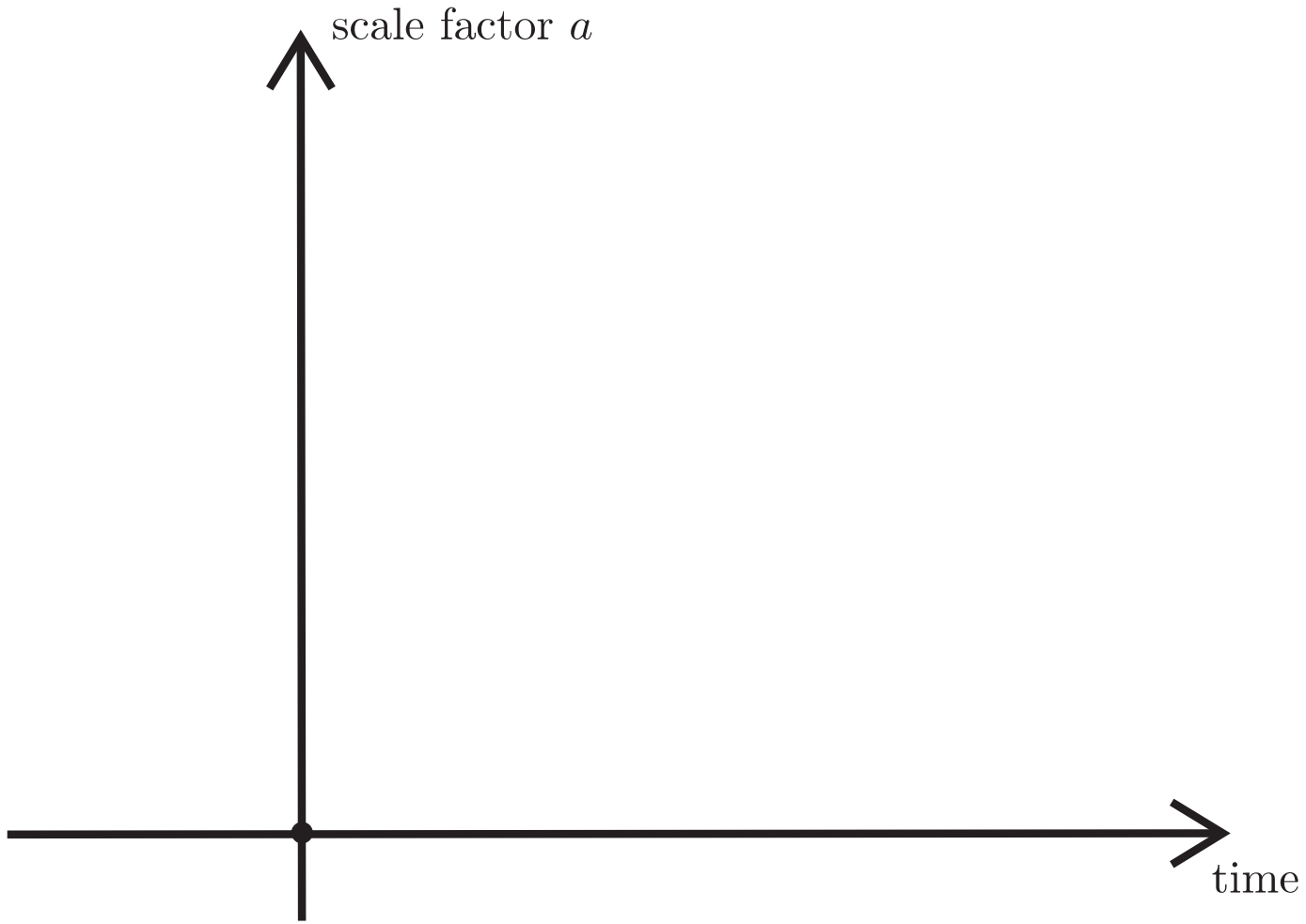
Avoiding the big bang

what are the effective cosmological dynamics for prior quantum cosmologies?

Avoiding the big bang



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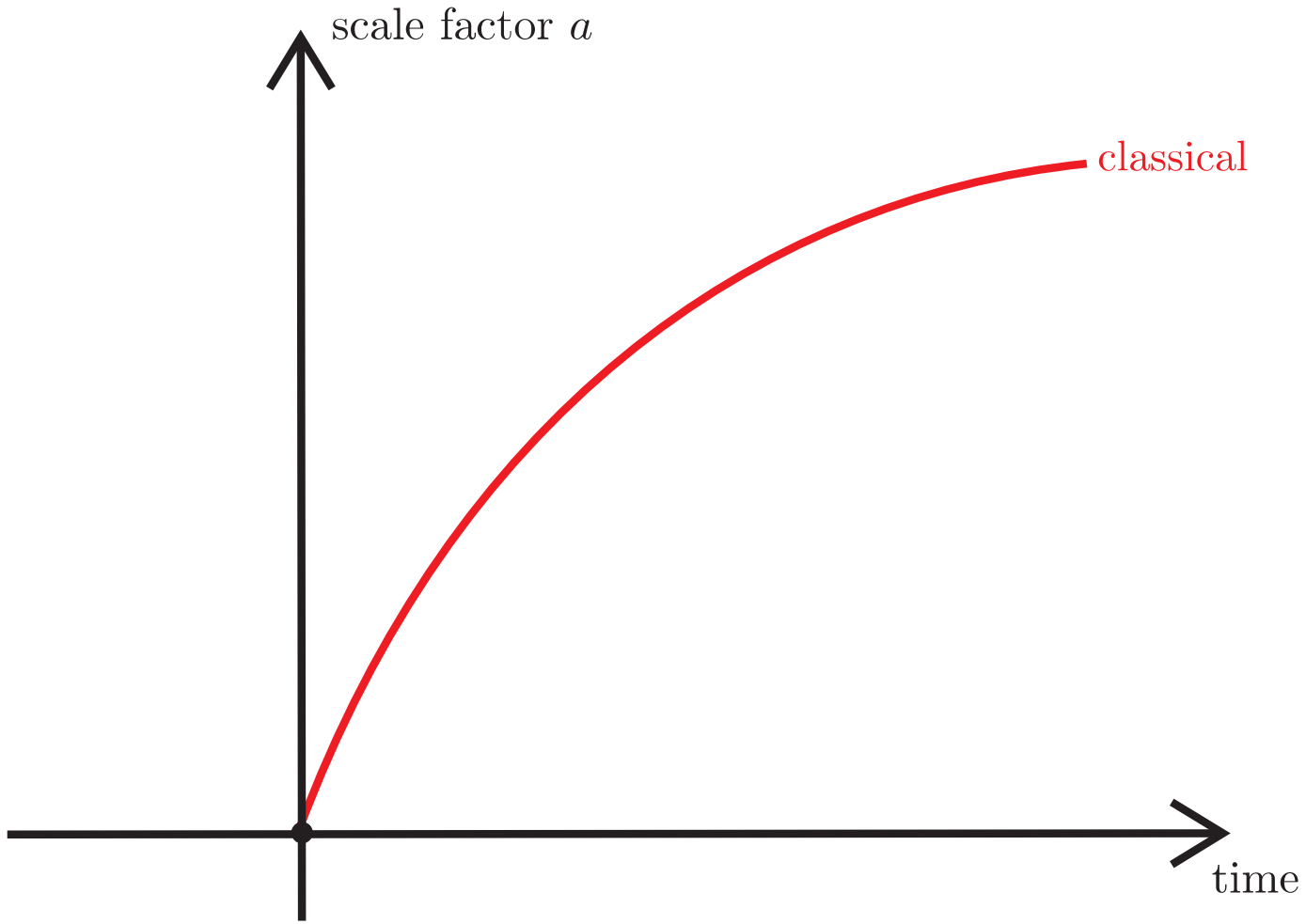

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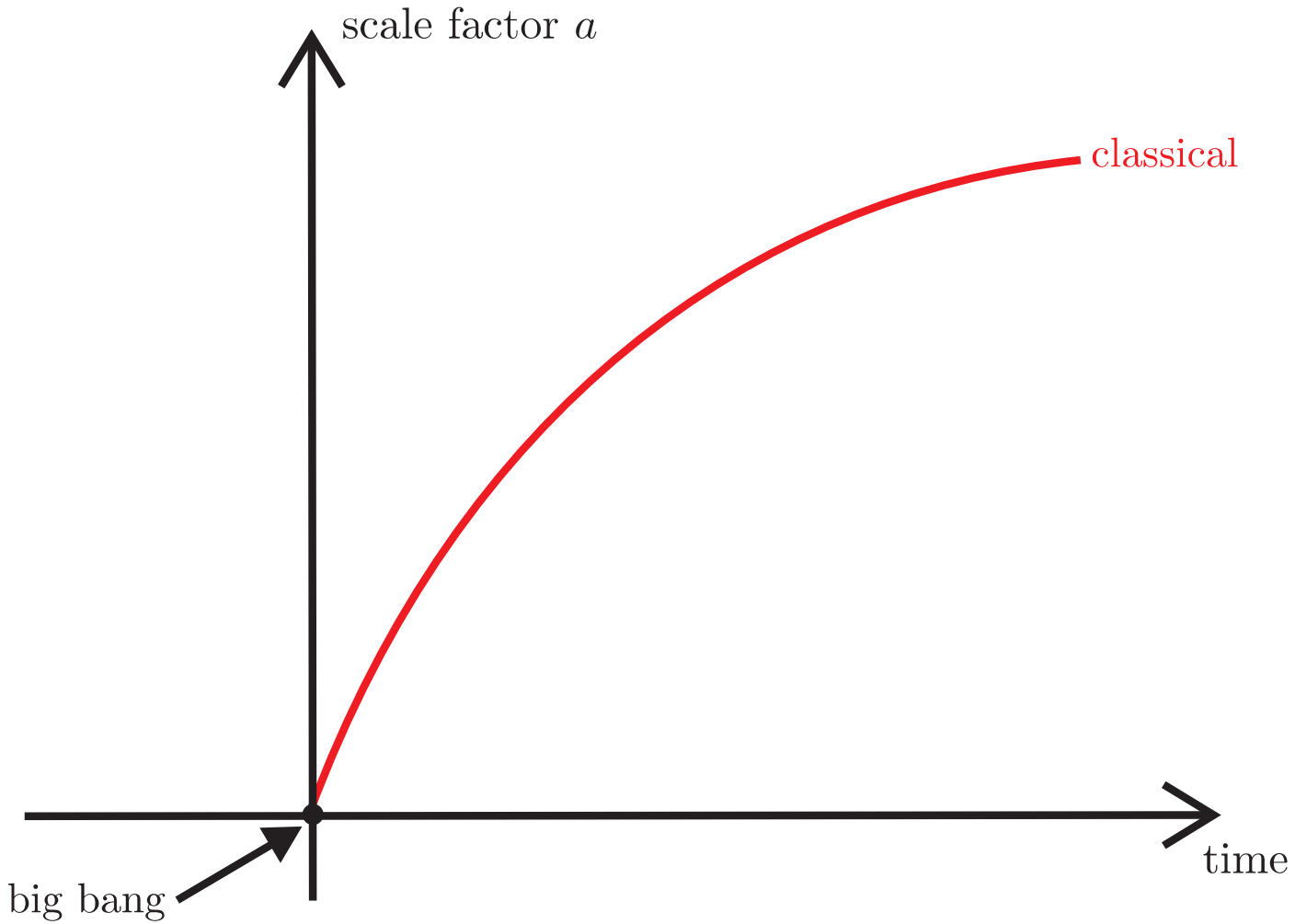

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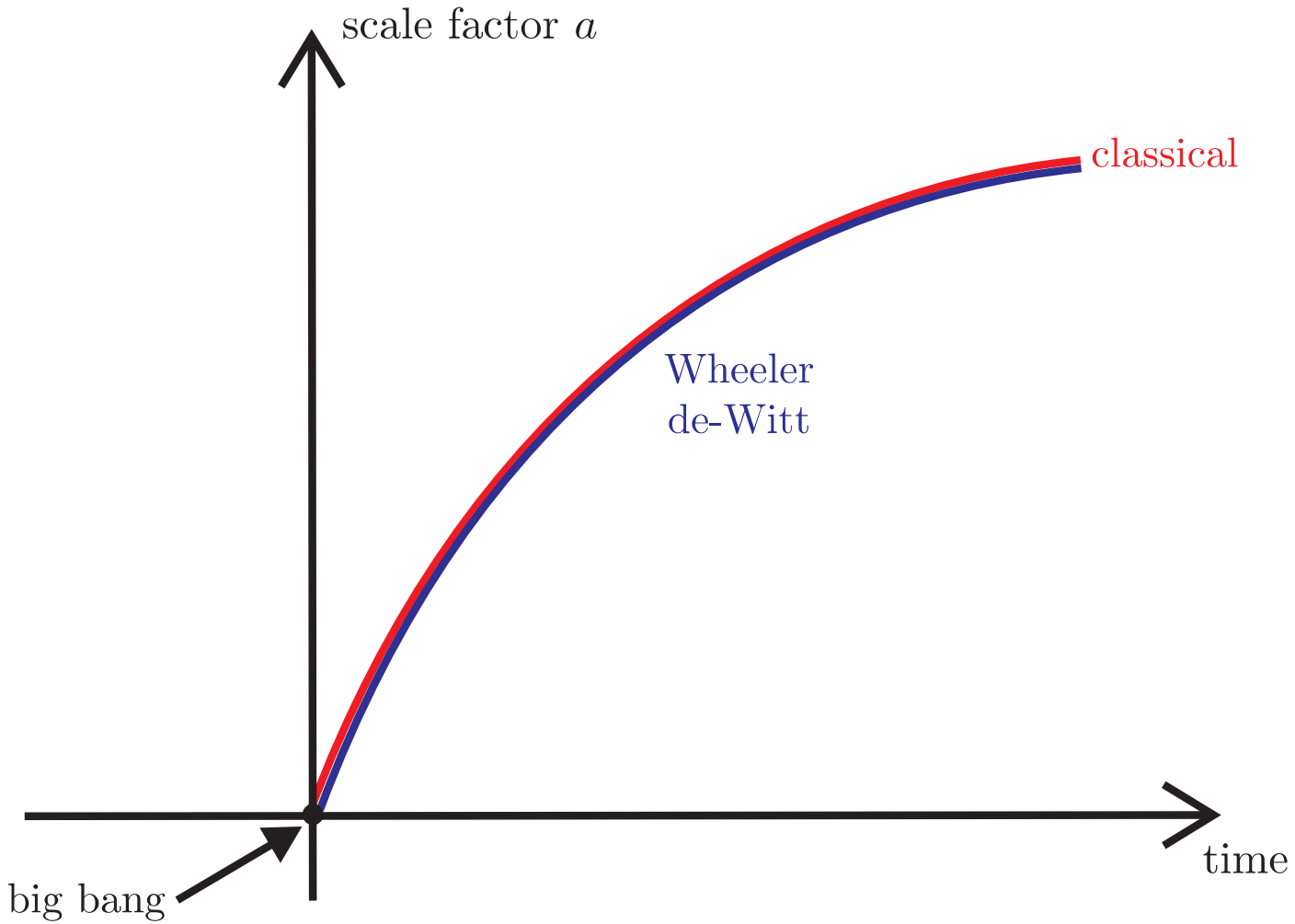

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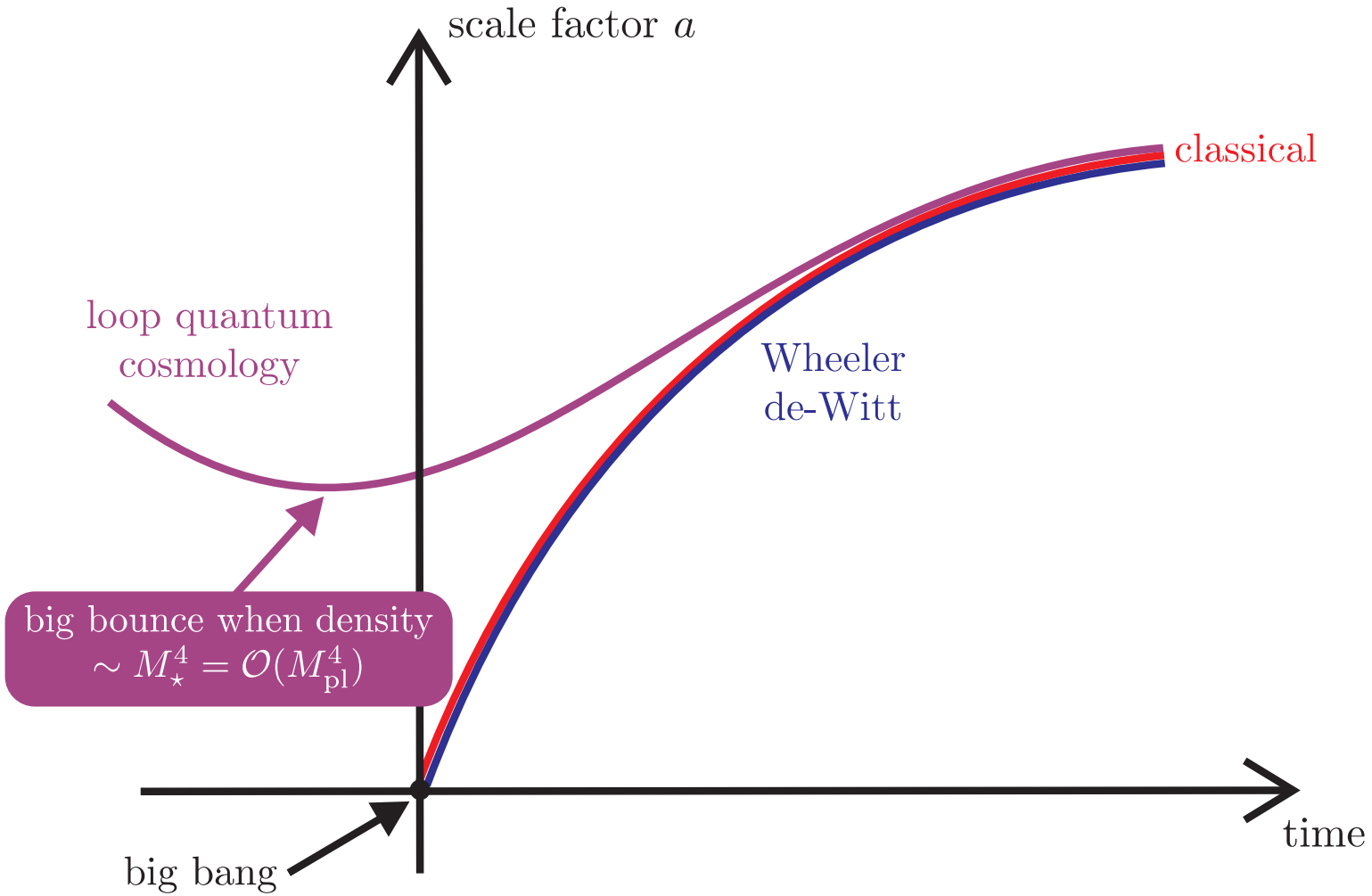

Semiclassical approximation

to derive cosmological dynamics with polymer matter DOFs, we use a semiclassical approximation...

Semiclassical approximation

$$\text{Hamiltonian constraint: } 0 = H_g(a, p_a) + H_\phi(a, P_\phi)$$

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$$\hat{P}_\phi = \frac{a^3}{2i\lambda_\star}(\hat{U}_\star - \hat{U}_\star^\dagger)$$

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semiclassical Hamiltonian constraint $\rightarrow$ effective Friedmann equation

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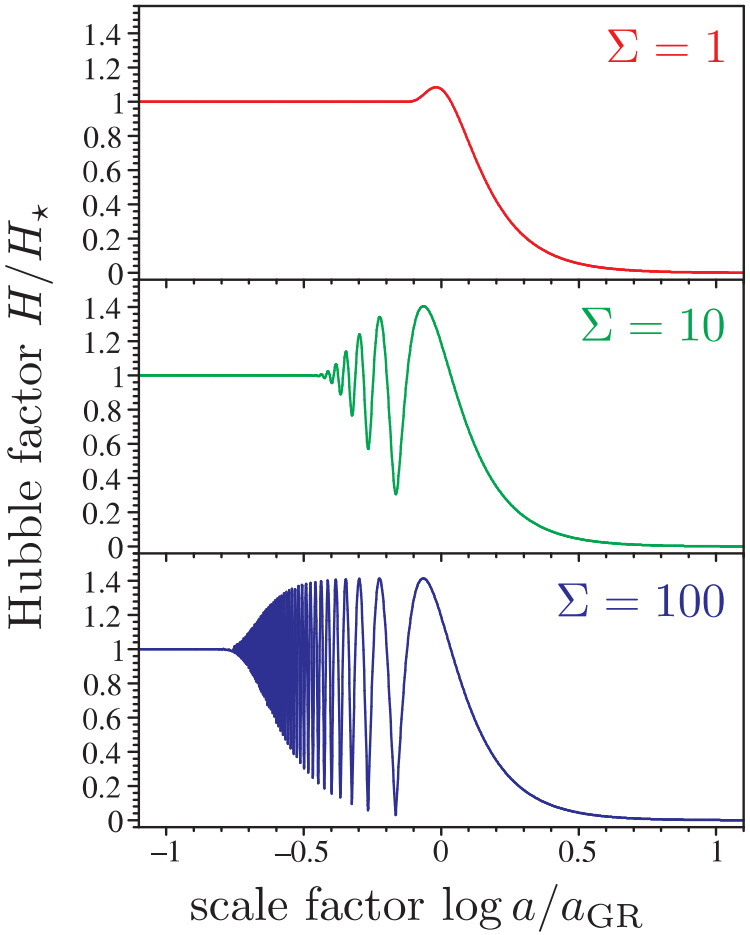
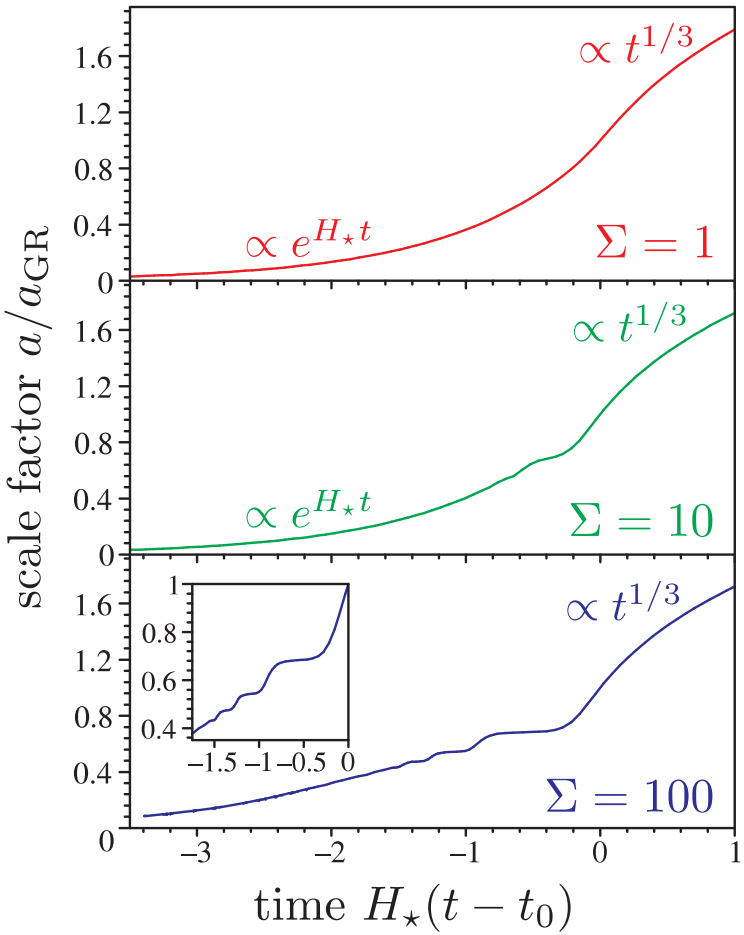
$$\text{low density } \rho_{\rm cl} \lesssim M_\star^4 \Rightarrow \rho_{\rm eff} \sim \rho_{\rm cl} \text{ (recover classical)}$$

$$\text{high density } \rho_{\rm cl} \gtrsim \Sigma^2 M_\star^4 \Rightarrow \rho_{\rm eff} \sim \text{constant (de Sitter inflation)}$$

Numerical results





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  - simultaneous PQ of gravity and matter DOFs