DBI inflation : particle creation, backreaction and features

Emeline Cluzel

IPhT CEA Saclay

September 30th 2010, COSMO10

Outline

- Brane inflation introduced in the 2000's

Introduction

- Brane inflation introduced in the 2000's
- Stringy setup

- Brane inflation introduced in the 2000's
- Stringy setup
- $\mathrm{D3} - \overline{\mathrm{D3}}$ scenario

Introduction

- Brane inflation introduced in the 2000's
- Stringy setup
- $\mathrm{D3} - \overline{\mathrm{D3}}$ scenario
- An AdS throat arises from compactification with $f(\phi) = \frac{\lambda}{\phi^4}$

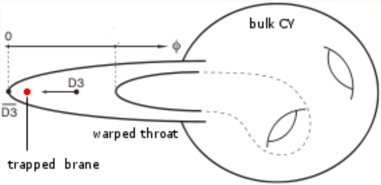
Introduction

- Brane inflation introduced in the 2000's
- Stringy setup
- $\mathrm{D3}-\overline{\mathrm{D3}}$ scenario
- An AdS throat arises from compactification with $f(\phi) = \frac{\lambda}{\phi^4}$
- The inflaton is the radion $\phi = \sqrt{T_3}r$

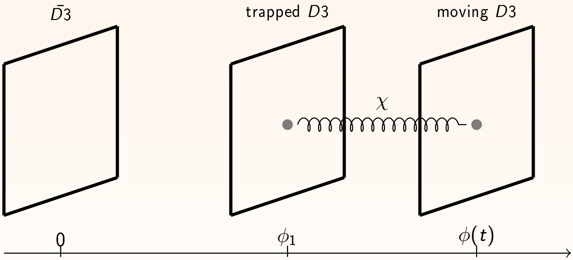
Introduction

- Brane inflation introduced in the 2000's
- Stringy setup
- $\mathrm{D3} - \overline{\mathrm{D3}}$ scenario
- An AdS throat arises from compactification with $f(\phi) = \frac{\lambda}{\phi^4}$
- The inflaton is the radion $\phi = \sqrt{T_3} r$
- Trapped D3-brane(s) in the throat

- Brane inflation introduced in the 2000's
- Stringy setup
- $\mathrm{D3} - \overline{\mathrm{D3}}$ scenario
- An AdS throat arises from compactification with $f(\phi) = \frac{\lambda}{\phi^4}$
- The inflaton is the radion $\phi = \sqrt{T_3}r$
- Trapped D3-brane(s) in the throat



Introduction



$$S = -\frac{1}{g_s}\int \mathrm{d}^4x\sqrt{-g}\ \left(\frac{1}{f(\phi)}\sqrt{1+f(\phi)g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi} - \frac{1}{f(\phi)}\right.$$
$$\left.+g_{\mu\nu}\partial^\mu\chi\partial^\nu\chi + V(\phi) + \frac{g^2}{2}\chi^2|\phi-\phi_1|^2\right) + \int \mathrm{d}^4x\sqrt{-g}\frac{M_P^2}{2}R$$

$V(\phi)$ includes the Coulomb potential, terms coming from the bulk and a mass term

$$\gamma = \frac{1}{\sqrt{1+f(\phi)g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi}} = \frac{1}{\sqrt{1-f\dot{\phi}^2}}.$$

DBI Dynamics

Dynamics very different from the usual slow-roll regime.

Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{\gamma}}{\gamma}\dot{\phi} + \frac{1}{\gamma}\frac{dV}{d\phi} + \frac{1}{\gamma f^2}\frac{df}{d\phi} - \frac{1}{\gamma^2 f^2}\frac{df}{d\phi} - \frac{\dot{\phi}^2}{2f}\frac{df}{d\phi} = 0$$

We know the late-time dynamics, via the Hamilton-Jacobi approach.

Outline

Particle Creation

Particles χ on the trapped brane are coupled to the inflationary brane.

Klein-Gordon equation for the quantum field $\Psi = a\chi$

$$\Psi_k'' + \omega_k^2(\eta)\Psi_k = 0$$

WKB approximation for $\omega_k^2 > 0$

$$\Psi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} \quad e^{-i\int^\eta \omega_k(\eta')\mathrm{d}\eta'} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} \quad e^{i\int^\eta \omega_k(\eta')\mathrm{d}\eta'}$$

Violated when $\left|\frac{\omega_k'}{\omega_k^2}\right| > 1$

2 regimes : $\xi \ll 1$ and $\xi \gg 1$

$$\xi = \frac{H^2}{g|\dot{\phi}|}$$

Particle Creation

Particles χ on the trapped brane are coupled to the inflationary brane.

Klein-Gordon equation for the quantum field $\Psi = a\chi$

$$\Psi_k'' + \omega_k^2(\eta)\Psi_k = 0$$

WKB approximation for $\Omega_k^2 = -\omega_k^2 > 0$ (tachyonic regime)

$$\Psi_k(\eta) = \frac{a_k(\eta)}{\sqrt{2\Omega_k(\eta)}} \quad e^{-\int^\eta \Omega_k(\eta')\mathrm{d}\eta'} + \frac{b_k(\eta)}{\sqrt{2\Omega_k(\eta)}} \quad e^{\int^\eta \Omega_k(\eta')\mathrm{d}\eta'}$$

Violated when $\left|\frac{\Omega_k'}{\Omega_k^2}\right| > 1$

2 regimes : $\xi \ll 1$ and $\xi \gg 1$

$$\xi = \frac{H^2}{g|\dot{\phi}|}$$

2 regimes

$\xi \ll 1$: Parametric Resonance

$$n_k = |\beta_k|^2 = \exp\left(-\pi \frac{\mathcal{K}^2}{g|\dot{\phi}_1|}\right)$$

Region of interaction of size $\Delta\phi \sim \sqrt{|\dot{\phi}|/g}$ $\quad (\Delta t < 1/H)$

Cut-off at $\mathcal{K} = \frac{k}{a} \leq \sqrt{g|\dot{\phi}|}$

$\xi \gg 1$: Tachyonic Instability

$$n_k = |\beta_k|^2 = \exp\left(\pi \frac{2H^2 - \mathcal{K}^2}{g|\dot{\phi}_1|}\right)$$

Region of interaction of size $\Delta\phi \sim H/g$ $\quad (\Delta t > 1/H)$

Cut-off at $\mathcal{K} = \frac{k}{a} \leq H$

Outline

Backreaction

the energy density of χ particles adds a linear term in the effective potential

$$\rho_\chi \approx \frac{1}{(2\pi)^3}\frac{a_s^3}{a^3} y(\xi) H^3 g|\phi - \phi_1| = \frac{\rho_0 A(\phi)}{a^3}$$

with :

$$\begin{aligned} A(\phi) &= 0 \text{ for } \phi > \phi_1 \\ &= \frac{\phi_1 - \phi}{\phi_1} \text{ for } \phi < \phi_1 \end{aligned}$$

$\xi \ll 1$ (parametric)

$$y(\xi) \approx \xi^{-3/2}$$

$\xi \gg 1$ (tachyonic)

$$y(\xi) \approx 9\xi^{-3/2} e^{2\pi\xi}$$

Scalar-tensor theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\frac{M_P^2}{2}R + \int \mathrm{d}^4x\sqrt{-g}\mathcal{P}(\phi, X) + \int \mathrm{d}^4x\mathcal{L}_m(\chi, \tilde{g}_{\mu\nu})$$

$X = \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$, $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ and $\mathcal{L}_m = \sqrt{-\tilde{g}}\left(-\frac{(\partial\chi)^2}{2} - g^2\phi_1^2\chi^2\right)$

Chameleonic potential with a time-dependant minimum

$$V_{eff} = m^2\phi^2 + \rho_\chi$$

Scalar-tensor theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\frac{M_P^2}{2}R + \int \mathrm{d}^4x\sqrt{-g}\mathcal{P}(\phi, X) + \int \mathrm{d}^4x\mathcal{L}_m(\chi, \tilde{g}_{\mu\nu})$$

$X = \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$, $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ and $\mathcal{L}_m = \sqrt{-\tilde{g}}\left(-\frac{(\partial\chi)^2}{2} - g^2\phi_1^2\chi^2\right)$

Chameleonic potential with a time-dependant minimum

$$V_{eff} = m^2\phi^2 + \rho_\chi$$

$$m_\chi = gA(\phi)\phi_1 = g|\phi - \phi_1|$$

$$m_\chi(\phi_s) = H/\xi \gg H \Rightarrow \mathrm{CDM}$$

$$\dot{\rho}_{\mathrm{tot}} = \frac{\dot{A}\rho_0}{a^3} - 3H(\rho_{\mathrm{tot}} + p)$$

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization
- are slowed down

Selection mechanism

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization
- are slowed down $=$ "Brane bremsstrahlung" *(Brax & Cluzel 09, arXiv:0912.0806)*

Selection mechanism

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization
- are slowed down $=$ "Brane bremsstrahlung" *(Brax & Cluzel 09, arXiv:0912.0806)*

Inflationary branes in the $\xi \ll 1$ regime (parametric) :

Selection mechanism

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization
- are slowed down $=$ "Brane bremsstrahlung" *(Brax & Cluzel 09, arXiv:0912.0806)*

Inflationary branes in the $\xi \ll 1$ regime (parametric) :

- can satisfy the COBE normalization

Selection mechanism

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization
- are slowed down $=$ "Brane bremsstrahlung" *(Brax & Cluzel 09, arXiv:0912.0806)*

Inflationary branes in the $\xi \ll 1$ regime (parametric) :

- can satisfy the COBE normalization
- are not affected by a stack of trapped branes

Selection mechanism

Inflationary branes in the $\xi \gg 1$ regime (tachyonic) :

- can't satisfy the COBE normalization
- are slowed down $=$ "Brane bremsstrahlung" *(Brax & Cluzel 09, arXiv:0912.0806)*

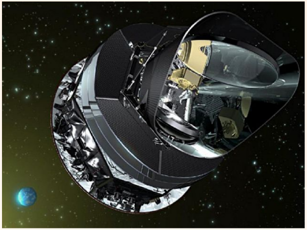
Inflationary branes in the $\xi \ll 1$ regime (parametric) :

- can satisfy the COBE normalization
- are not affected by a stack of trapped branes
- The background is not affected but what happens at the perturbation level ? Features ?

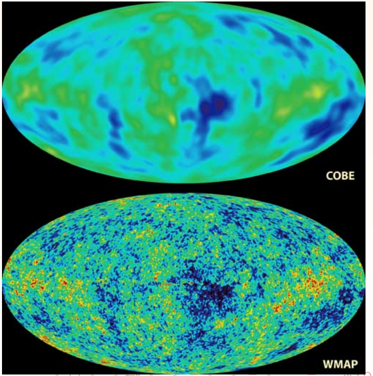
Outline

Observations

Precision cosmology enables testing stringy models.



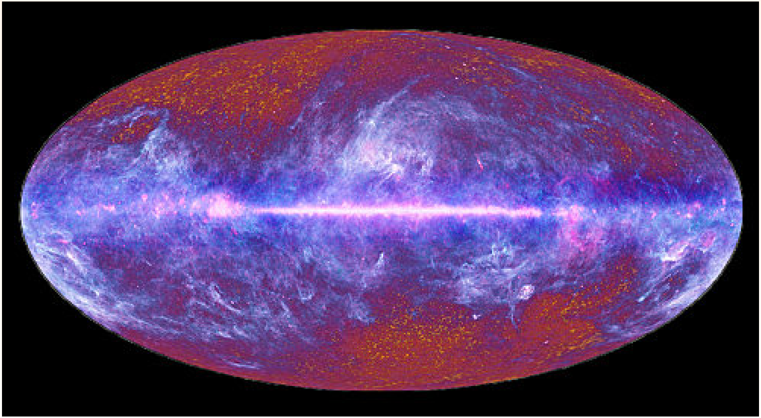
Planck satellite : CMB spectrum of perturbations



Observational signatures of the model :

- features
- spectral index (scale variance)
- non-gaussianities
- detection of gravitational waves

Observations



Outline

Modified perturbation equation

$$v_k'' + \Bigg[c_s^2 k^2 \frac{\rho + p}{\rho_\phi + p} - \frac{z_A''}{z_A} - \left(\frac{A'\rho_0}{a}\left(\frac{4\pi G}{3\mathcal{H}^2} - \frac{c_s^2}{\rho_\phi + p}\right)\right)'$$

$$+\frac{A'\rho_0}{a}\left(\frac{4\pi G}{3\mathcal{H}^2} - \frac{c_s^2}{\rho_\phi + p}\right)\frac{\left(\frac{c_s^2\mathcal{H}k^2}{4\pi Ga(\rho_\phi+p)} + \frac{A'\rho_0}{a^2}\left(\frac{4\pi G}{3\mathcal{H}^2} - \frac{c_s^2}{\rho_\phi+p}\right)\right)'}{\frac{c_s^2\mathcal{H}k^2}{4\pi Ga(\rho_\phi+p)} + \frac{A'\rho_0}{a^2}\left(\frac{4\pi G}{3\mathcal{H}^2} - \frac{c_s^2}{\rho_\phi+p}\right)}\Bigg] v_k = 0$$

With :

$$z_A = e^{\frac{1}{2}\int \frac{A'\rho_0}{a\gamma^3\phi'^2}\mathrm{d}\eta}\frac{a\gamma^{3/2}\phi'/\mathcal{H}}{\sqrt{1 + \frac{A'\rho_0\gamma^3\phi'^2(4\pi G)^2}{3a\mathcal{H}^3k^2}}}$$

Perturbation equation

$$\frac{d^2 v_k}{dx^2} + \left(1 - \frac{2}{x^2} + \hat{u}\delta(x - x_1) + b\delta'(x - x_1)\right) v_k = 0$$

$$x = kc_s\eta, \; c_s = 1/\gamma \approx cst, \; \hat{u} = u/kc_s$$

Perturbation equation

$$\frac{d^2 v_k}{dx^2} + \left(1 - \frac{2}{x^2} + \hat{u}\delta(x - x_1) + b\delta'(x - x_1)\right) v_k = 0$$

$$x = kc_s\eta,\ c_s = 1/\gamma \approx cst,\ \hat{u} = u/kc_s$$

$$b = \frac{1}{2} \frac{\frac{\phi'}{\phi_1}\frac{\rho_0}{a^3}\left(\frac{4\pi G}{3\mathcal{H}} - \frac{\mathcal{H}}{\gamma^3 \phi'^2}\right)}{\frac{k^2\mathcal{H}^2}{4\pi G\gamma^3\phi'^2 a^2} - \frac{\phi'\rho_0}{\phi_1 a^3}\left(\frac{4\pi G}{3\mathcal{H}} - \frac{\mathcal{H}}{\gamma^3\phi'^2}\right)}\Big|_{\eta_1}$$

Perturbation equation

$$\frac{d^2 v_k}{dx^2} + \left(1 - \frac{2}{x^2} + \hat{u}\delta(x - x_1) + b\delta'(x - x_1)\right) v_k = 0$$

$$x = kc_s\eta,\ c_s = 1/\gamma \approx cst,\ \hat{u} = u/kc_s$$

$$b = \frac{1}{2} \frac{\frac{\phi'}{\phi_1}\frac{\rho_0}{a^3}\left(\frac{4\pi G}{3\mathcal{H}} - \frac{\mathcal{H}}{\gamma^3\phi'^2}\right)}{\frac{k^2\mathcal{H}^2}{4\pi G\gamma^3\phi'^2 a^2} - \frac{\phi'\rho_0}{\phi_1 a^3}\left(\frac{4\pi G}{3\mathcal{H}} - \frac{\mathcal{H}}{\gamma^3\phi'^2}\right)}\Big|_{\eta_1}$$

$$b \underset{k\to\infty}{\longrightarrow} 0$$

$$b \underset{k\to 0}{\longrightarrow} -1/2$$

Perturbation equation

$$\frac{d^2 v_k}{dx^2} + \left(1 - \frac{2}{x^2} + \hat{u}\delta(x - x_1) + b\delta'(x - x_1)\right) v_k = 0$$

$$x = kc_s\eta,\ c_s = 1/\gamma \approx cst,\ \hat{u} = u/kc_s$$

$$b = \frac{1}{2}\frac{\frac{\phi'}{\phi_1}\frac{\rho_0}{a^3}\left(\frac{4\pi G}{3\mathcal{H}} - \frac{\mathcal{H}}{\gamma^3\phi'^2}\right)}{\frac{k^2\mathcal{H}^2}{4\pi G\gamma^3\phi'^2 a^2} - \frac{\phi'\rho_0}{\phi_1 a^3}\left(\frac{4\pi G}{3\mathcal{H}} - \frac{\mathcal{H}}{\gamma^3\phi'^2}\right)}\Big|_{\eta_1}$$

$$b \underset{k\to\infty}{\longrightarrow} 0$$

$$b \underset{k\to 0}{\longrightarrow} -1/2$$

$$u \underset{k\to\infty}{\longrightarrow} 0$$

$$u \underset{k\to 0}{\longrightarrow} -\mathcal{H}_1/4$$

before the feature

$$v_k^- = A\left(i + \frac{1}{x}\right)e^{-ix}$$

after the feature

$$v_k^+ = \alpha\left(i + \frac{1}{x}\right)e^{-ix} + \beta\left(-i + \frac{1}{x}\right)e^{+ix}$$

Bogoliubov coefficients

$$\alpha = \frac{A}{(2-b)i}\left(2i + \frac{b}{x_1^3} + \hat{u}\left(1 + \frac{1}{x_1^2}\right)\right)$$

$$\beta = Ae^{-2ix_1}\frac{i + \frac{1}{x_1}}{-i + \frac{1}{x_1}}\left(1 - \frac{1}{(2-b)i}\left(2i + \frac{b}{x_1^3} + \hat{u}\left(1 + \frac{1}{x_1^2}\right)\right)\right)$$

Power spectrum

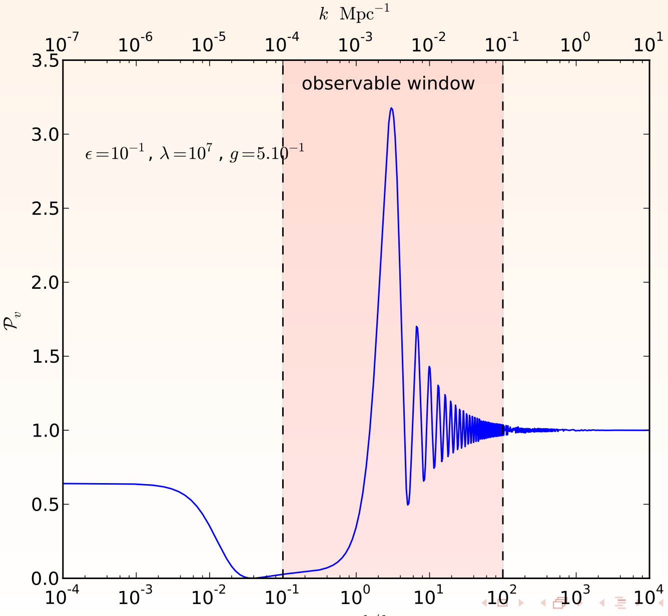
evaluate at sound horizon crossing

$$v_k \approx \frac{\alpha+\beta}{x_*}$$

$$(\alpha+\beta) \underset{k\to\infty}{\longrightarrow} A$$

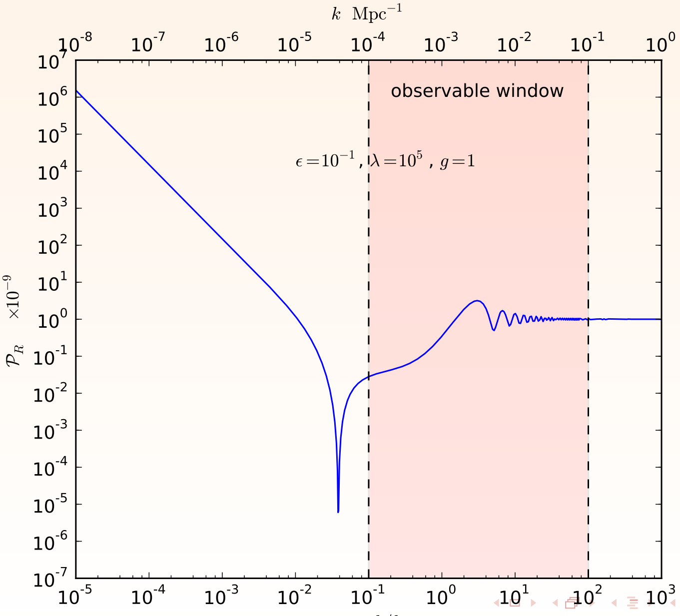
$$(\alpha+\beta) \underset{k\to 0}{\longrightarrow} A\left[1+\frac{2(b_0+u_0)}{3(2-b_0)}\right] \approx \frac{4A}{5}$$

$\Rightarrow$ jump

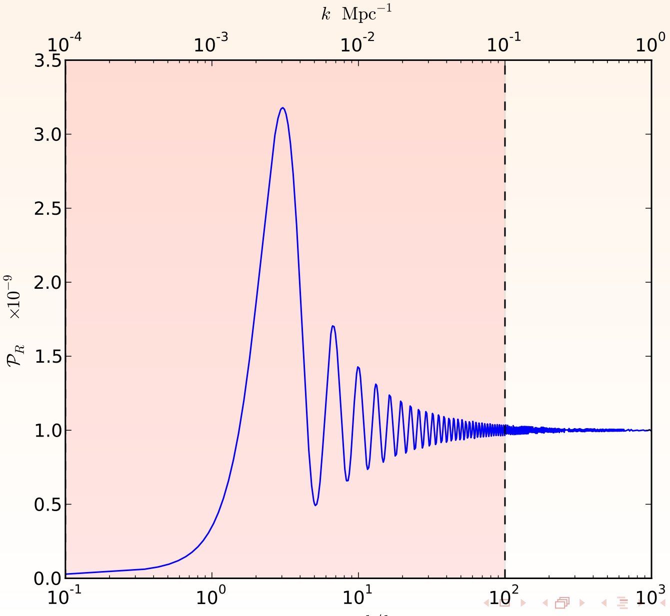


k Mpc^{-1}
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
observable window
$\epsilon = 10^{-1}$, $\lambda = 10^{7}$, $g = 5.10^{-1}$
$\mathcal{P}_v$
3.5 3.0 2.5 2.0 1.5 1.0 0.5 0.0
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4}

Power spectrum : plots



Power spectrum : plots



Outline

$$v'' + (c_s^2 k^2 - \frac{a''}{a} + u\delta(\eta - \eta_1))v = 0$$
$$\Rightarrow \frac{v(k \to 0)}{v(k \to \infty)} = 1 - \frac{u}{3c_s k_1} \Rightarrow \text{jump}$$

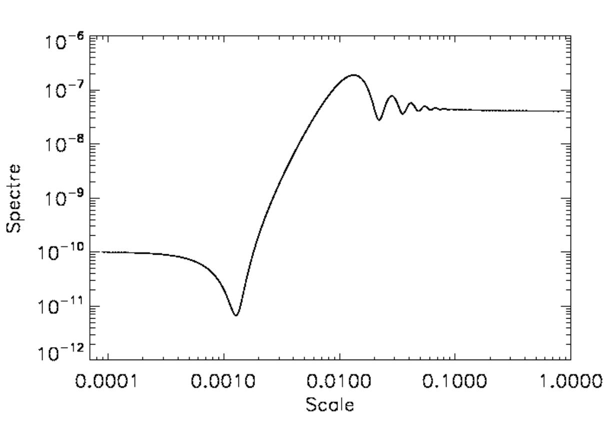
Comparison with Starobinsky model in canonical inflation

Linear inflaton potential with a sudden change in the slope : SR disrupted

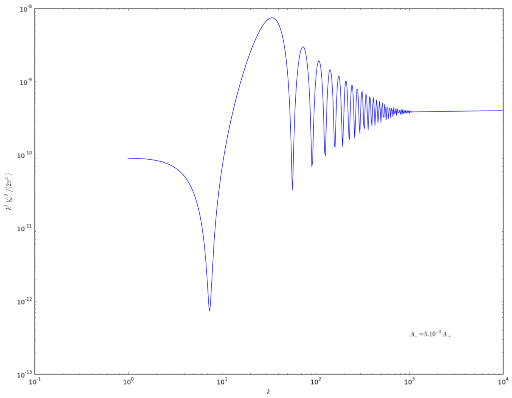
$$\begin{aligned} V(\phi) &= V_0 + A_+(\phi - \phi_0) \text{ for } \phi > \phi_0 \\ &= V_0 + A_-(\phi - \phi_0) \text{ for } \phi < \phi_0 \end{aligned}$$

recover Starobinsky's results with $c_s = 1$ and $u = -3\mathcal{H}_1 \left(\frac{A_-}{A_+} - 1\right)$

DBI $\Rightarrow$ *Brax, Cluzel, Martin* (in prep) example for $A_- = 5.10^{-2} A_+$



example for $A_- = 5.10^{-2} A_+$ and $\gamma \sim 10$



Outline

Conclusion and prospects

Through **non minimal coupling to matter**,

particles are created when crossing **trapped branes**,

this **does not affect the background**,

but it **affects drastically the perturbations**,

and can leave **features in the power spectrum**

Brax & Cluzel 10 (next week)

Future work :

- *bispectrum*
- *numerical study of different featureful potentials in DBI*
- *quantitative study of features*