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G-inflation

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Based on work with:
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arXiv:1008.0603

G-inflation = Inflation driven by the Galileon field

The Galileon field

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = (\nabla\phi)^2$$

$$\mathcal{L}_3 = (\nabla\phi)^2 \,\Box\phi$$

$$\mathcal{L}_4 = (\nabla\phi)^2 \Big[2(\Box\phi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2 - \frac{R}{2}(\nabla\phi)^2\Big]$$

$$\mathcal{L}_5 = (\nabla\phi)^2 \left[(\Box\phi)^3 + \cdots\right]$$

Field equations are 2nd order

Galilean shift symmetry in flat space

$$\partial_\mu\phi \rightarrow \partial_\mu\phi + b_\mu$$

$$\mathcal{L}_n \sim \partial^{2(n-1)}\phi^n$$

Nicolis et al. '09;
Deffayet et al. '09

Our inflaton Lagrangian

$$\mathcal{L} = \frac{R}{2} + K(\phi, X) - F(\phi, X)\Box\phi$$

where $X := -\frac{1}{2}(\nabla\phi)^2$

Field equations are 2nd order

Deffayet, Pujolas, Sawicki, Vikman 1008.0048;
TK, Yamaguchi, Yokoyama 1008.0603

Simple motivation

The Galileon field has been used to explain current cosmic acceleration......

Chow, Khoury '09; Silva, Koyama '09;
TK, Tashiro, Suzuki '09; **TK** '10;
Gannouji, Sami '10;
De Felice, Tsujikawa '10; De Felice, Mukohyama, Tsujikawa '10; ...

Simple motivation

Why don't we use the Galileon field to drive inflation in the early universe?

Talk plan

I. Introduction

II. G-inflation

III. Primordial perturbations

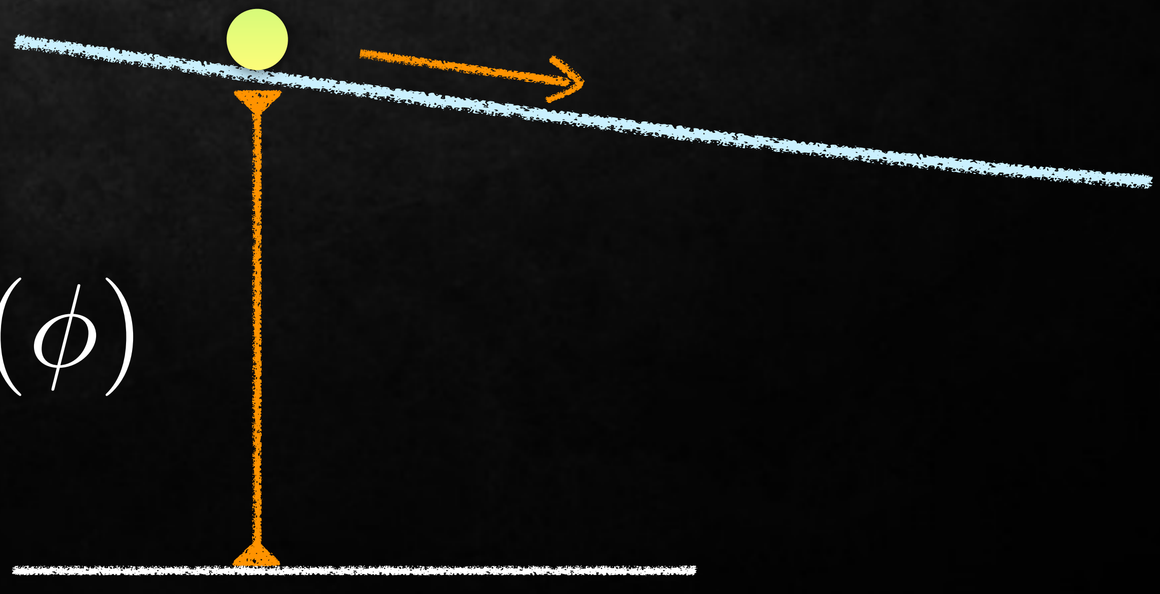
IV. Summary

G-inflation

Standard picture of inflation

One (or more) canonical scalar field(s) rolling slowly down a nearly flat potential

$$\mathcal{L} = X - V(\phi), \quad X = -\frac{1}{2}(\partial\phi)^2$$



$$3M_{\rm Pl}^2 H^2 \simeq V(\phi)$$

Kinematically driven inflation

$$\mathcal{L} = K(\phi, X)$$

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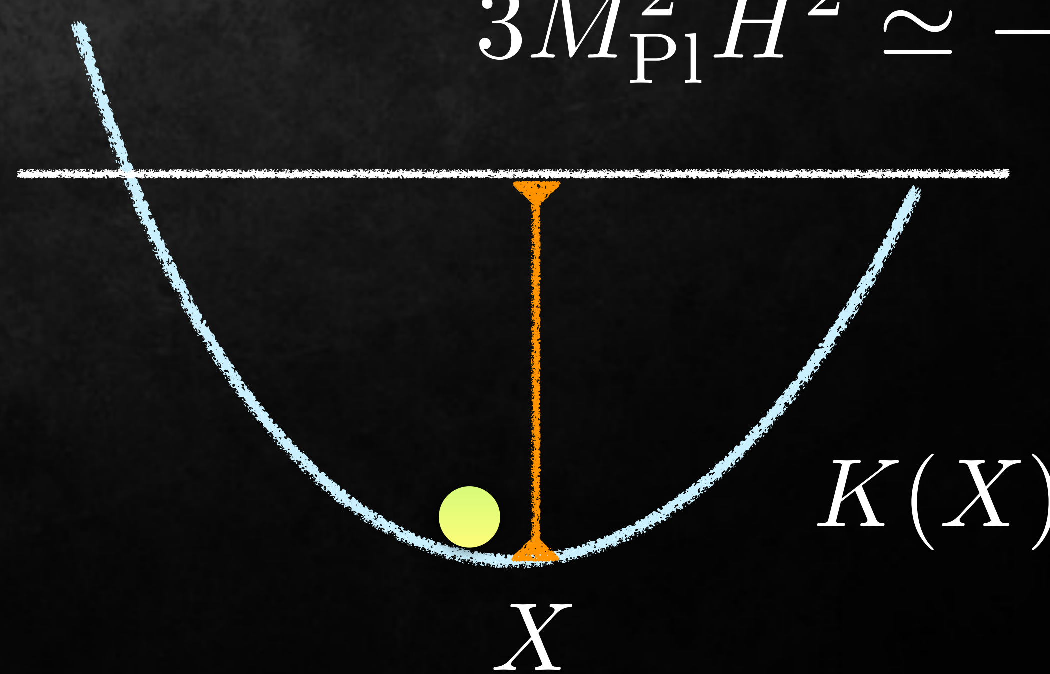
$$\mathcal{L} = K(\phi, X)$$

$\longrightarrow$ $K = K(X)$

$$\frac{d}{dt}\left(a^3 K_X \dot{\phi}\right) = 0$$

$$3M_{\rm Pl}^2 H^2 \simeq -K$$



“k-inflation”

Armendariz-Picon et al. ’99;

“Ghost condensate”

Arkani-Hamed et al. ’04

G-inflation: background

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\Box\phi$$

$$\downarrow$$

$$\begin{aligned} 3H^2 &= \rho \\ -3H^2 - 2\dot{H} &= p \end{aligned}$$

+ Scalar field EOM is automatically satisfied

$$\begin{aligned} \rho &= 2XK_X - K + 3F_X H\dot{\phi}^3 - 2F_\phi X \\ p &= K - 2\left(F_\phi + F_X\ddot{\phi}\right)X \end{aligned}$$

de Sitter G-inflation

$$K = K(X), \;\; F = fX, \;\; f = \text{const}$$

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Look for exactly de Sitter solution:

$$\begin{aligned} H &= \text{const} \\ \dot{\phi} &= \text{const} \end{aligned}$$

satisfying:

$$\begin{aligned} 3H^2 &= -K \\ K_X &= -3fH\dot{\phi} \end{aligned}$$

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$$\begin{aligned} 3H^2 &= -K \\ K_X &= -3fH\dot{\phi} \end{aligned} \longrightarrow K = -\frac{1}{6f^2}\frac{(K_X)^2}{X}$$

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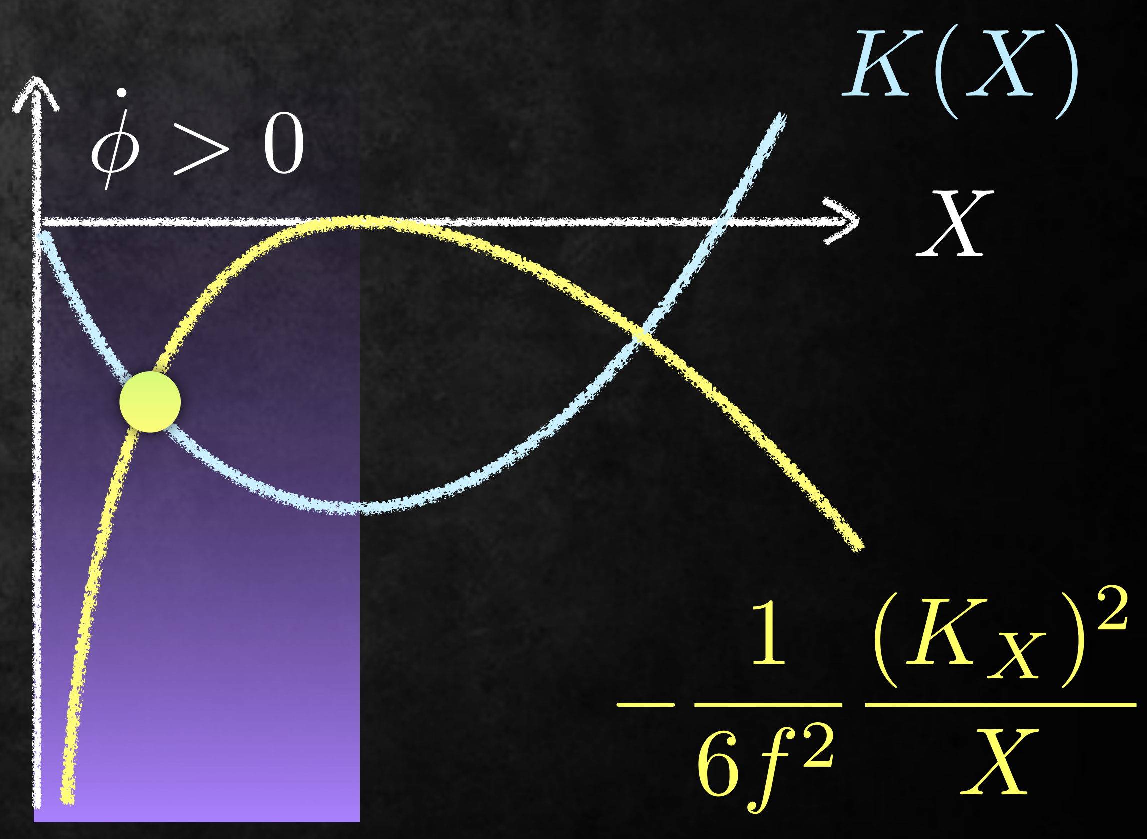

Quasi-dS G-inflation

$$K = K(X), \quad F = f(\phi)X$$

Required to get $n_s - 1 \neq 0$

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Quasi-dS G-inflation

$$K = K(X), \quad F = f(\phi) X$$

Quasi-de Sitter solution:

Required to get $n_s - 1 \neq 0$

$$H = H(t), \quad \dot{\phi} = \dot{\phi}(t)$$

← Small rate of change

satisfying:

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

$$H^2 \simeq -K(X)$$

$$K_X \simeq -3 f(\phi) H \dot{\phi}$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

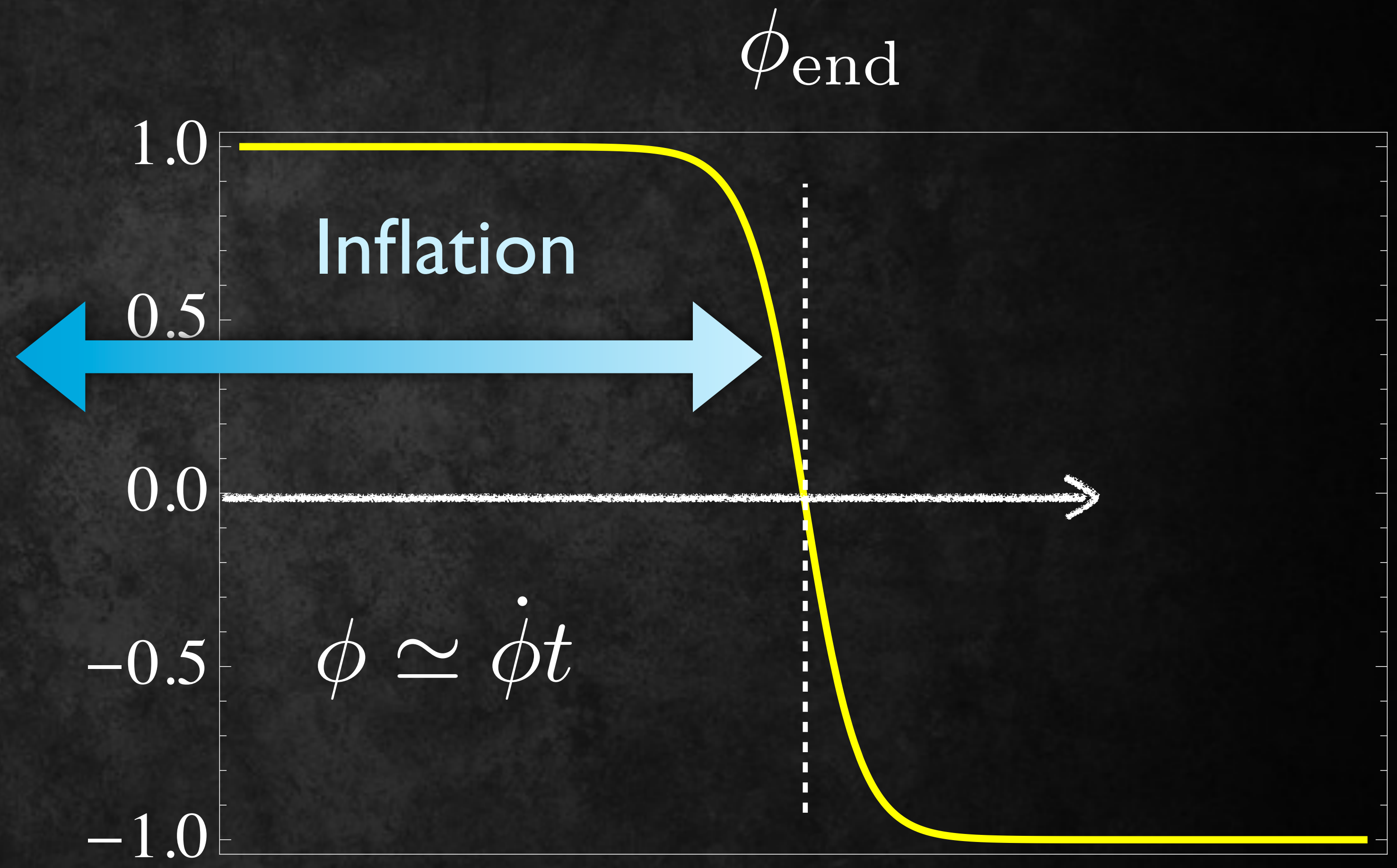
Graceful exit & Reheating

Basic idea

$$K = -A(\phi)X + \cdots$$

Example:

$$A = \tanh\left[\lambda(\phi_{\rm end} - \phi)\right]$$



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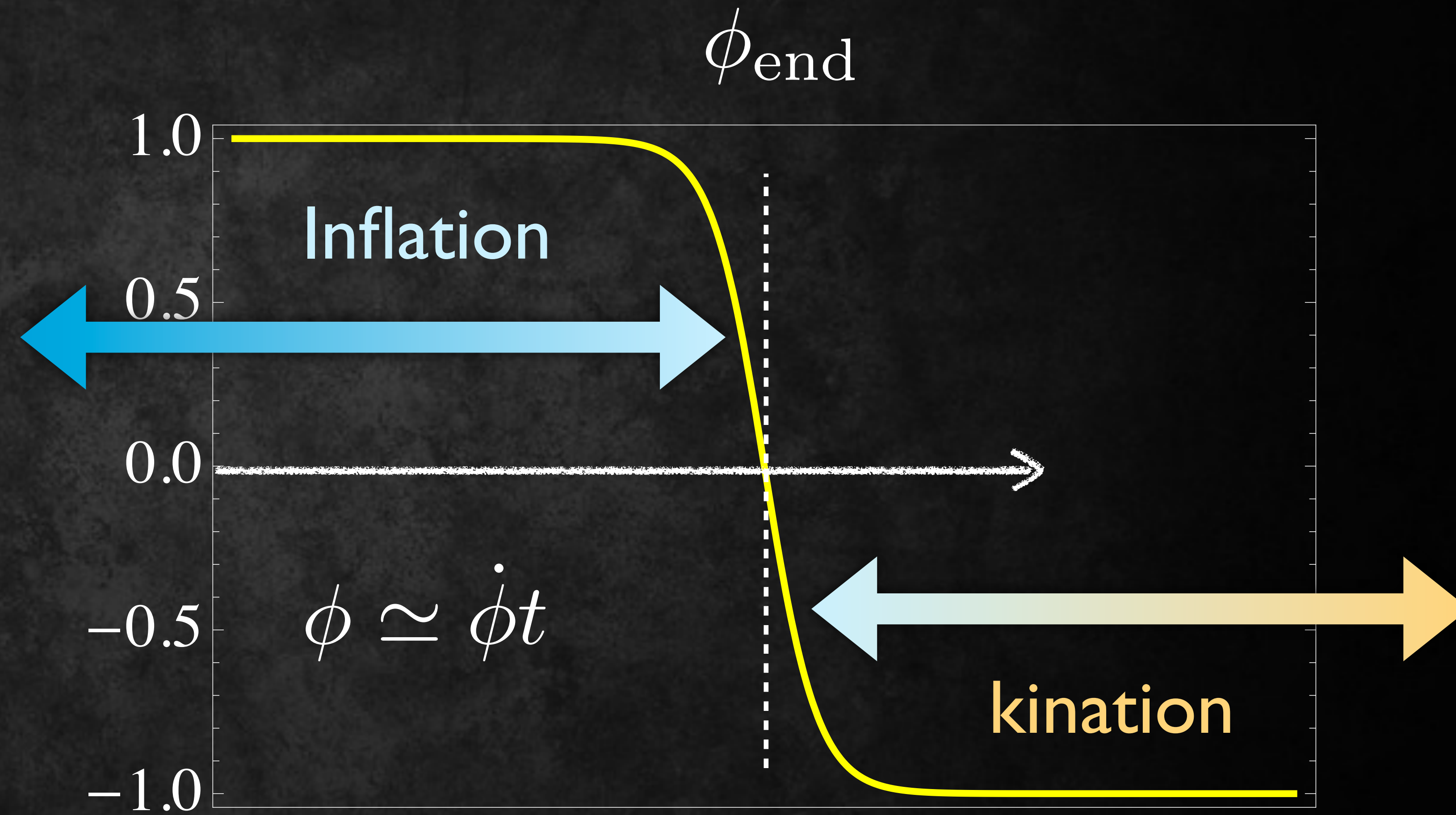


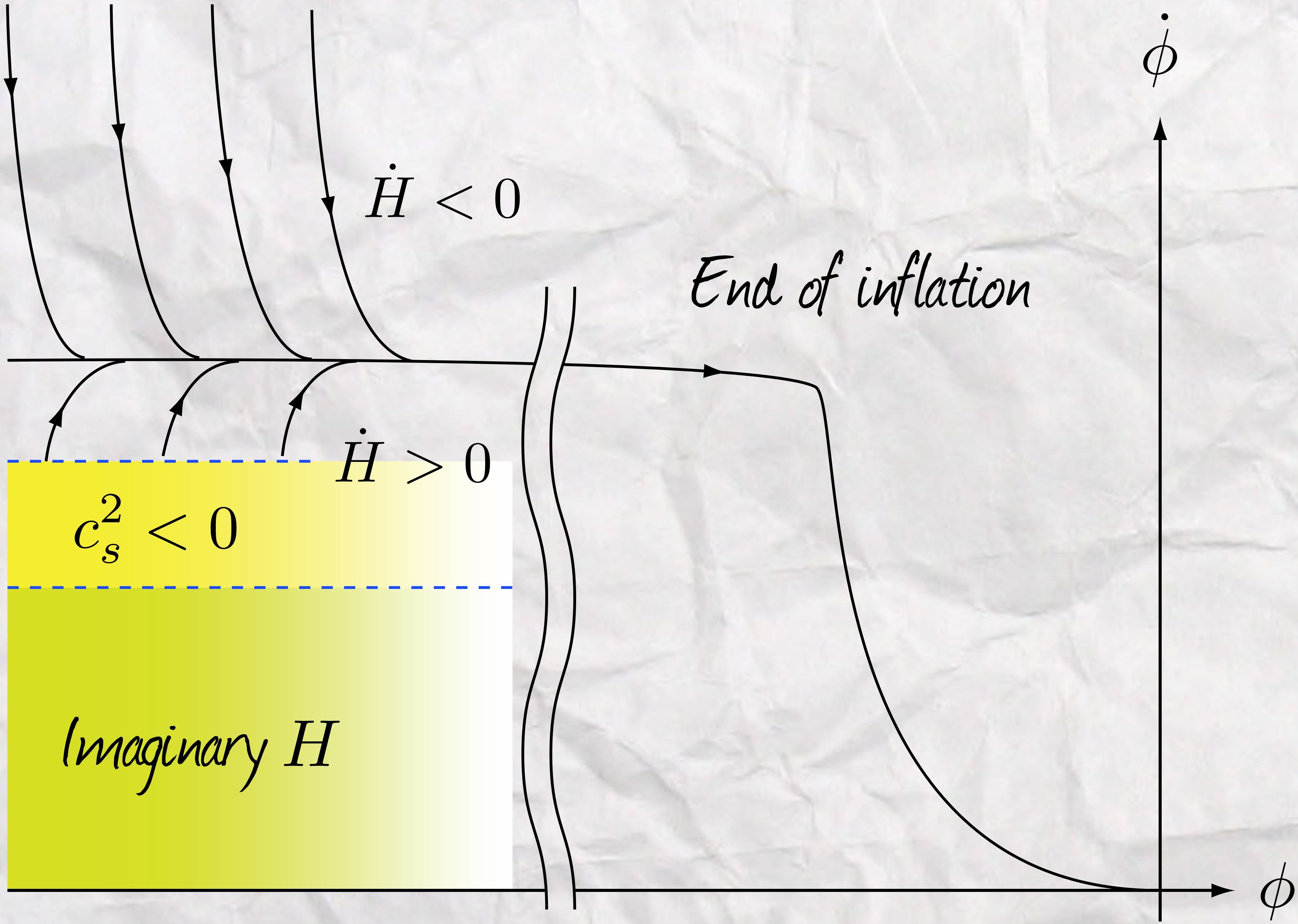
$$\rho \simeq p \simeq X \propto a^{-6}$$

Reheating through gravitational particle production

Ford '87

~ massless, canonical field (normal sign)

Phase diagram



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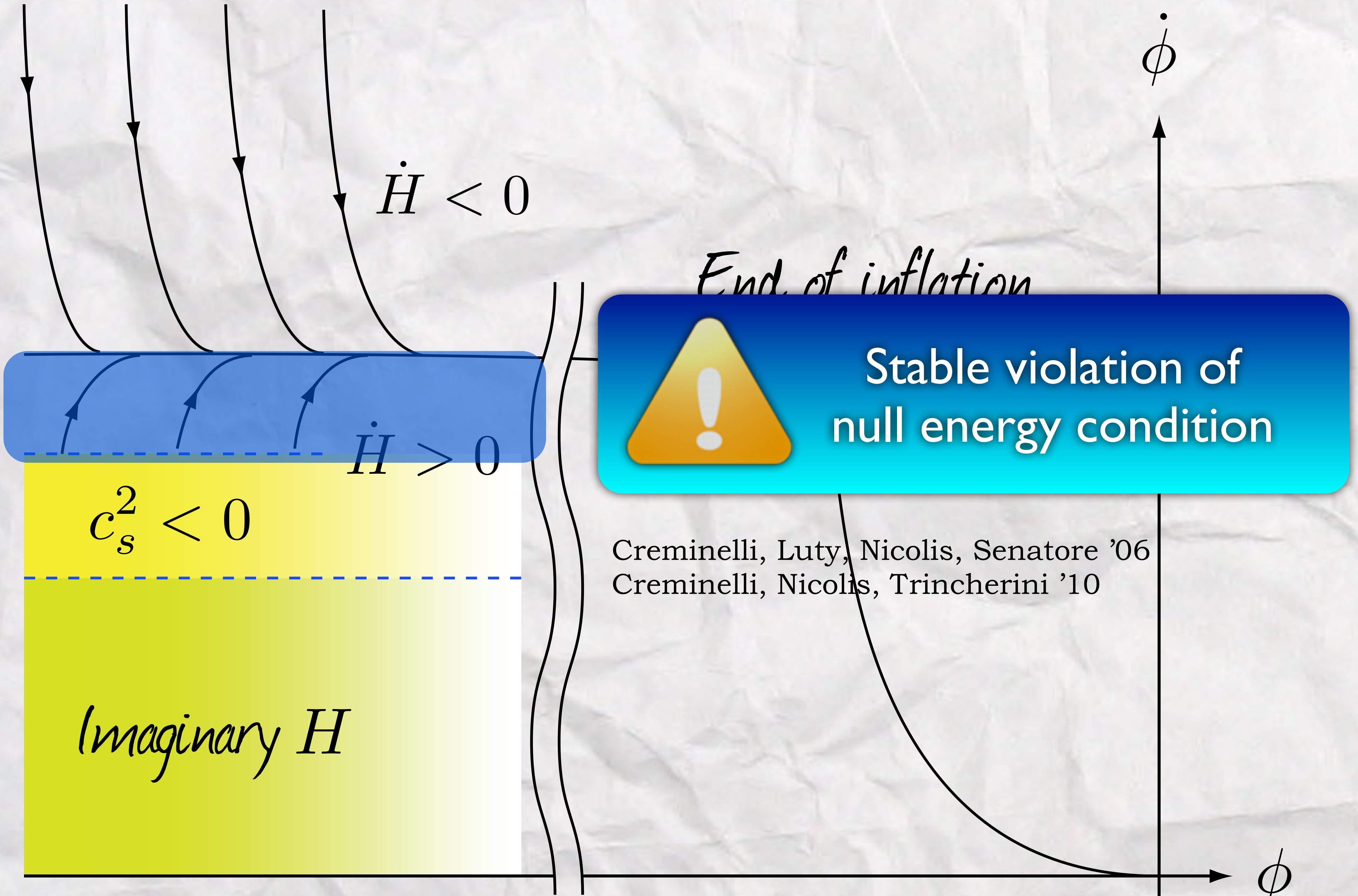

Primordial perturbations

Cosmological perturbations

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\beta_{,i}dtdx^i + a^2(1+2\mathcal{R})\delta_{ij}dx^i dx^j$$

$$\phi = \phi(t)$$

Unitary gauge: $\delta\phi = 0$

1. Expand the action to 2nd order
2. Eliminate α and β using constraint eqs
3. Quadratic action for $\mathcal{R}$

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Unitary gauge: $\delta\phi = 0$

1. Expand the action to 2nd order
2. Eliminate α and β using constraint eqs
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$$\delta T_i{}^0 = -F_X\dot{\phi}^3\alpha_{,i}$$

Uniform ϕ hypersurfaces
≠ comoving hypersurfaces

Deffayet, Pujolas, Sawicki, Vikman 1008.0048; TK, Yamaguchi, Yokoyama 1008.0603

Quadratic action

$$S^{(2)} = \frac{1}{2}\int d\tau d^3x\, z^2 \left[\mathcal{G}(\mathcal{R}')^2 - \mathcal{F}(\vec{\nabla}\mathcal{R})^2\right]$$

where

$$\begin{aligned}
z &= \frac{a\dot{\phi}}{H - F_X\dot{\phi}^3/2} \\
\mathcal{F} &= K_X + 2F_X\left(\ddot{\phi} + 2H\dot{\phi}\right) - 2F_X^2X^2 \\
&\quad +2F_{XX}X\ddot{\phi} - 2\left(F_\phi - XF_{\phi X}\right) \\
\mathcal{G} &= K_X + 2XK_{XX} + 6F_XH\dot{\phi} + 6F_X^2X^2 \\
&\quad -2\left(F_\phi + XF_{\phi X}\right) + 6F_{XX}HX\dot{\phi}
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where

No ghost and gradient instabilities if

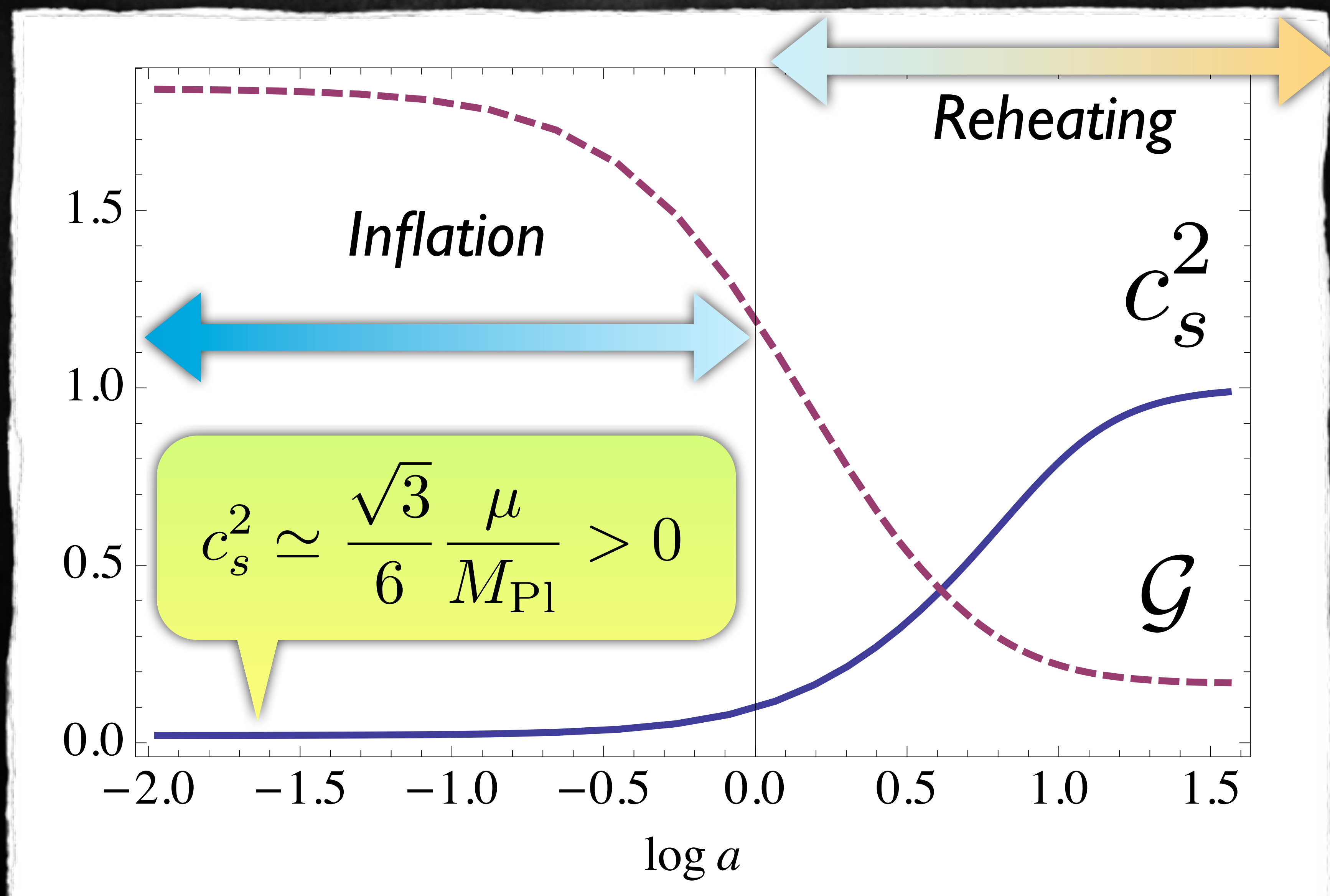
$$\mathcal{G} > 0, \quad c_s^2 = \mathcal{F}/\mathcal{G} > 0$$

z

$$\begin{aligned}\mathcal{F} &= K_X + 2F_X\left(\ddot{\phi} + 2H\dot{\phi}\right) - 2F_X^2 X^2 \\ &\quad +2F_{XX}X\ddot{\phi} - 2\left(F_\phi - XF_{\phi X}\right) \\ \mathcal{G} &= K_X + 2XK_{XX} + 6F_X H\dot{\phi} + 6F_X^2 X^2 \\ &\quad -2\left(F_\phi + XF_{\phi X}\right) + 6F_{XX}HX\dot{\phi}\end{aligned}$$

Stable example

$$K = -A(\phi)X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$



Primordial spectrum

Consider G-inflation with:

$$K = K(X), \quad F = f(\phi)X$$

New variables:

$$\begin{aligned} dy &= c_s d\tau \\ \tilde{z} &= (\mathcal{F}\mathcal{G})^{1/4} z \\ u &= \tilde{z}\mathcal{R} \end{aligned}$$

"Sasaki-Mukhanov equation"

$$\frac{d^2u}{dy^2} + \left(k^2 - \frac{\tilde{z}_{,yy}}{\tilde{z}}\right) u = 0$$

$$\frac{\tilde{z}_{,yy}}{\tilde{z}} \simeq \frac{1}{(-y)^2}\left[2 + 3\epsilon\mathcal{C}(X)\right]$$

$$\mathcal{C}(X) = \frac{K}{K_X}\frac{Q_X}{Q}$$

$$Q(X) = \frac{(K - XK_X)^2}{18Xc_s^2\sqrt{\mathcal{F}\mathcal{G}}}$$

Primordial spectrum

Normalized mode: $$u = \frac{\sqrt{\pi}}{2}\sqrt{-y}H^{(1)}_{3/2+\epsilon\mathcal{C}}(-ky)$$

$$\mathcal{P}_{\mathcal{R}} = \frac{Q}{4\pi^2}\bigg|_{c_s k = 1/(-\tau)}, \quad n_s - 1 = -2\epsilon\mathcal{C}$$

$\propto f_{,\phi}$

where $$Q(X) = \frac{(K - XK_X)^2}{18M_{\rm Pl}^4 X c_s^2\sqrt{\mathcal{F}\mathcal{G}}}$$

$\mathcal{R}$ can be generated even from exact de Sitter

* Tensor mode dynamics: unchanged

Tensor-to-scalar ratio

$$K = -X + \frac{X^2}{2M^3\mu}, \;\; F = fX \quad \longrightarrow \quad H^2 \sim \frac{\mu M^3}{M_{\rm Pl}^2}$$

$$r \simeq \frac{16\sqrt{6}}{3}\left(\frac{\sqrt{3}\mu}{M_{\rm Pl}}\right)^{3/2}$$

Conventional consistency relation is violated

$$r \neq -8c_s n_T$$

$$\propto \epsilon \propto f_{,\phi}$$

$$M = 0.00435 \times M_{\rm Pl}, \;\; \mu = 0.032 \times M_{\rm Pl}$$

$$\longrightarrow \quad \mathcal{P}_{\mathcal{R}} = 2.4 \times 10^{-9}, \;\; r = 0.17$$

r can be large!

Summary

Summary

- G-inflation: A general class of single field inflation

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\Box\phi$$

  - $n_s - 1 \simeq 0$
  - Large r
  - ~~Consistency relation~~

G-inflation would make gravitational wave people happy!


Thank you!

Example

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$

$\longrightarrow$ $\mu \ll M_{\rm Pl}$

$$\frac{H^2}{M_{\rm Pl}^2} \simeq \frac{1}{6}\frac{M^3}{M_{\rm Pl}^3}\frac{\mu}{M_{\rm Pl}}$$

$$X \simeq \left(1 - \frac{\sqrt{3}\mu}{M_{\rm Pl}}\right)\mu M^3$$

Numerical Example

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = \frac{e^{\alpha\phi}}{M^3}X$$