

# G-inflation

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Based on work with:

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**G**-inflation = Inflation driven by  
the **G**alileon field

# The Galileon field

$\mathcal{L}_1 = \phi$       Field equations are 2nd order

$$\mathcal{L}_2 = (\nabla\phi)^2$$

$$\mathcal{L}_3 = (\nabla\phi)^2 \square\phi$$

$$\mathcal{L}_4 = (\nabla\phi)^2 \left[ 2(\square\phi)^2 \right.$$

$$\left. -2(\nabla_\mu\nabla_\nu\phi)^2 - \frac{R}{2}(\nabla\phi)^2 \right]$$

$$\mathcal{L}_5 = (\nabla\phi)^2 \left[ (\square\phi)^3 + \dots \right]$$

Galilean shift symmetry in flat space

$$\partial_\mu\phi \rightarrow \partial_\mu\phi + b_\mu$$

$$\mathcal{L}_n \sim \partial^{2(n-1)}\phi^n$$

Nicolis et al. '09;  
Deffayet et al. '09

# Our inflaton Lagrangian

$$\mathcal{L} = \frac{R}{2} + K(\phi, X) - F(\phi, X)\square\phi$$

where  $X := -\frac{1}{2}(\nabla\phi)^2$

Field equations are 2nd order

# Simple motivation

The Galileon field has been used to explain **current cosmic acceleration**.....

Chow, Khoury '09; Silva, Koyama '09;

**TK**, Tashiro, Suzuki '09; **TK** '10;

Gannouji, Sami '10;

De Felice, Tsujikawa '10; De Felice, Mukohyama, Tsujikawa '10; ...

# Simple motivation

Why don't we use the Galileon field to drive inflation in the early universe?

# Talk plan

- I. Introduction
- II. G-inflation
- III. Primordial perturbations
- IV. Summary

# G-inflation

The image features a clear blue sky with scattered white cumulus clouds. A vertical strip of textured, light-colored material, possibly paper or fabric, runs down the left side of the frame. The text 'G-inflation' is centered in the middle of the image in a white, serif font with a subtle drop shadow.

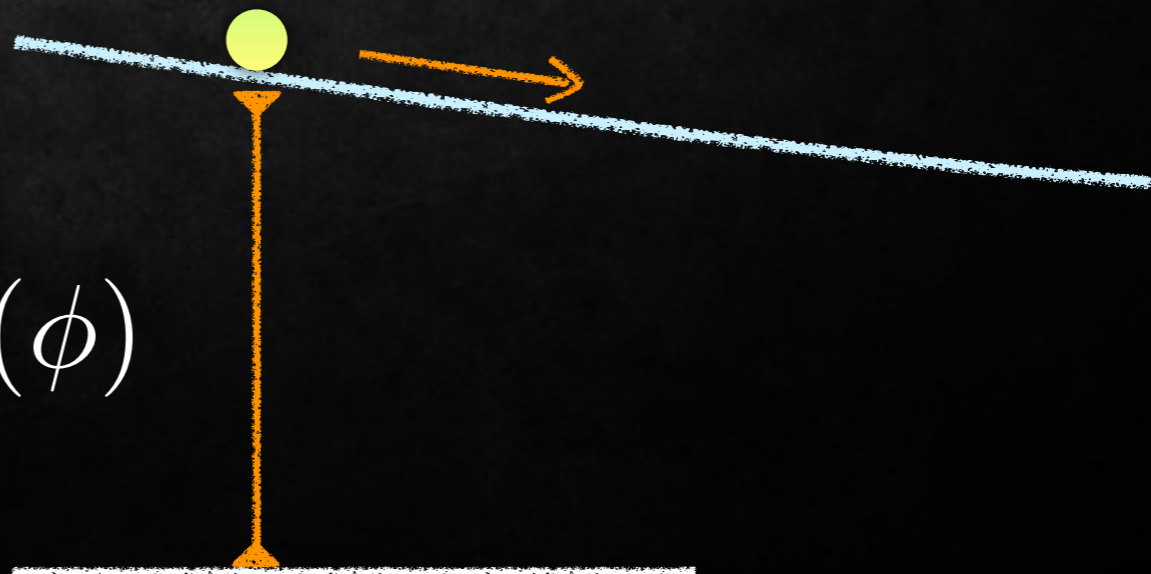


# Standard picture of inflation

One (or more) canonical scalar field(s) rolling slowly down a nearly flat potential

$$\mathcal{L} = X - V(\phi), \quad X = -\frac{1}{2}(\partial\phi)^2$$

$$3M_{\text{Pl}}^2 H^2 \simeq V(\phi)$$



# Kinematically driven inflation

$$\mathcal{L} = K(\phi, X)$$

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$$\mathcal{L} = K(\phi, X)$$

$$K = K(X)$$

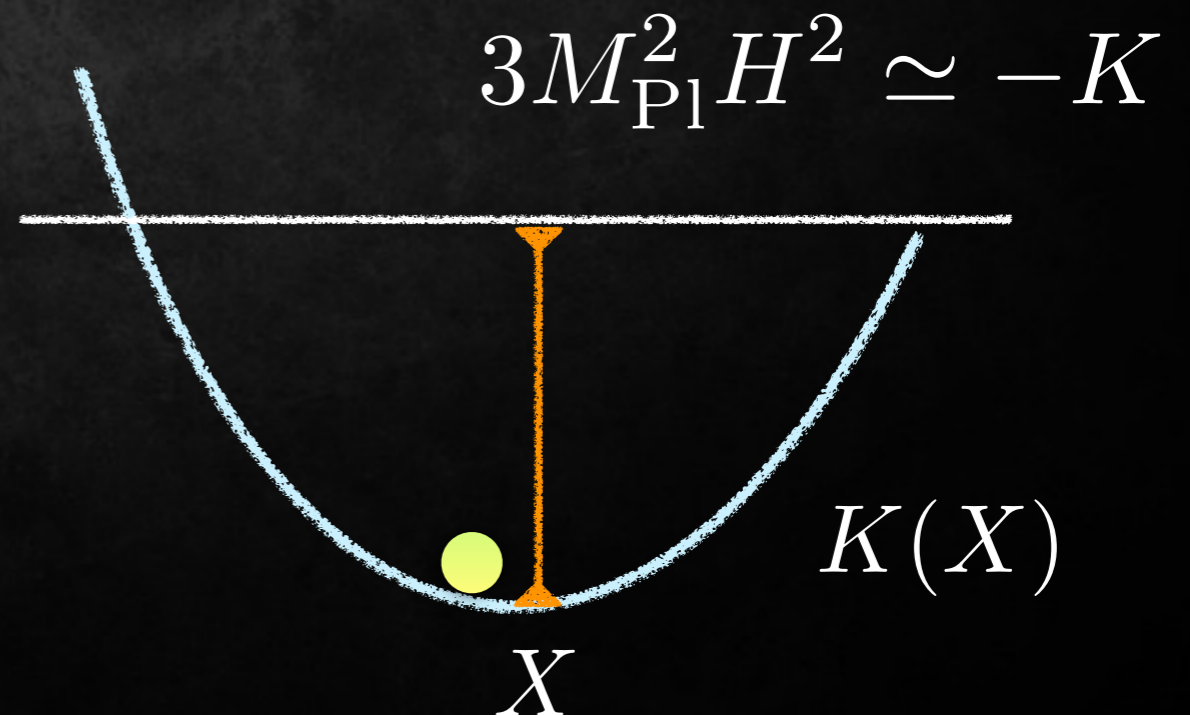
$$\frac{d}{dt} \left( a^3 K_X \dot{\phi} \right) = 0$$

“k-inflation”

Armendariz-Picon et al. '99;

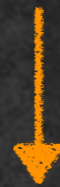
“Ghost condensate”

Arkani-Hamed et al. '04



# G-inflation: background

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\square\phi$$



$$\begin{aligned} 3H^2 &= \rho \\ -3H^2 - 2\dot{H} &= p \end{aligned} + \text{Scalar field EOM is automatically satisfied}$$

$$\rho = 2XK_X - K + 3F_X H \dot{\phi}^3 - 2F_\phi X$$

$$p = K - 2 \left( F_\phi + F_X \ddot{\phi} \right) X$$

# de Sitter G-inflation

$$K = K(X), \quad F = fX, \quad f = \text{const}$$

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Look for **exactly de Sitter** solution:

$$H = \text{const}$$

$$\dot{\phi} = \text{const}$$

satisfying:

$$3H^2 = -K$$

$$K_X = -3fH\dot{\phi}$$

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$$\begin{aligned} 3H^2 &= -K \\ K_X &= -3fH\dot{\phi} \end{aligned} \longrightarrow K = -\frac{1}{6f^2} \frac{(K_X)^2}{X}$$

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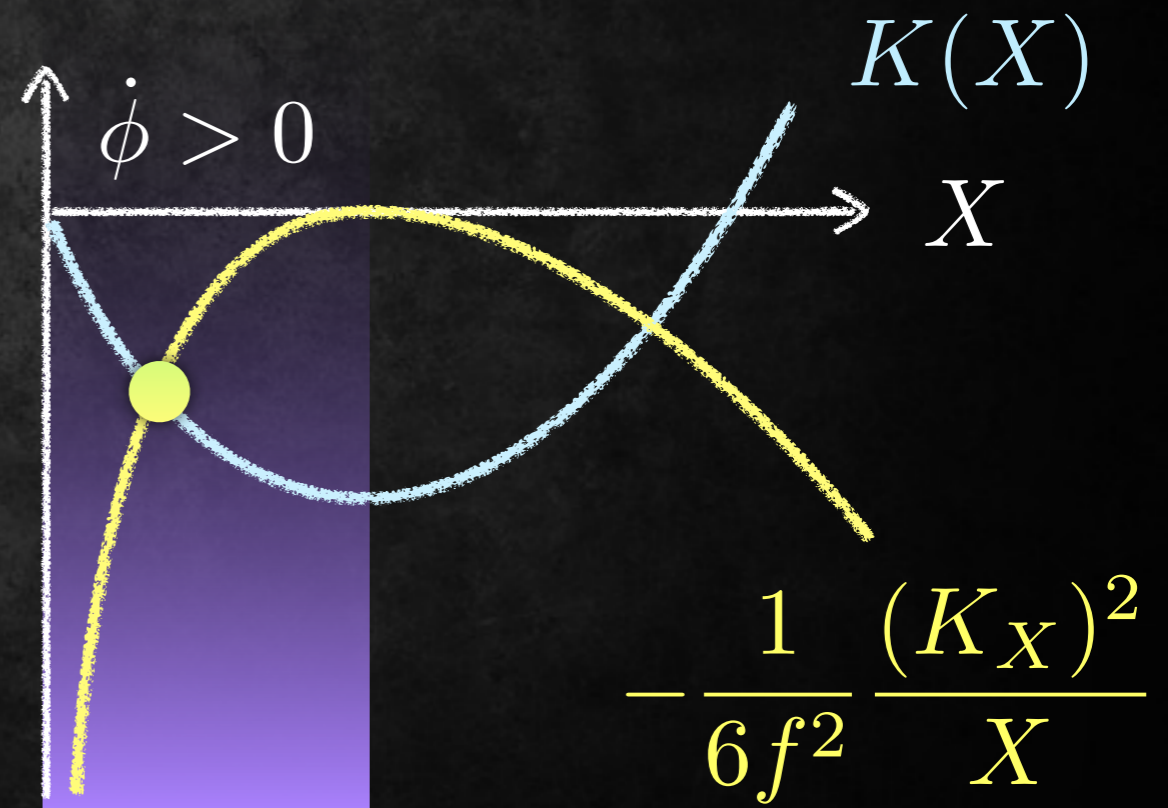
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# Quasi-dS G-inflation

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Quasi-de Sitter solution:

Required to get  $n_s - 1 \neq 0$

$$H = H(t), \quad \dot{\phi} = \dot{\phi}(t)$$



Small rate of change

satisfying:

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

$$H^2 \simeq -K(X)$$

$$K_X \simeq -3f(\phi)H\dot{\phi}$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

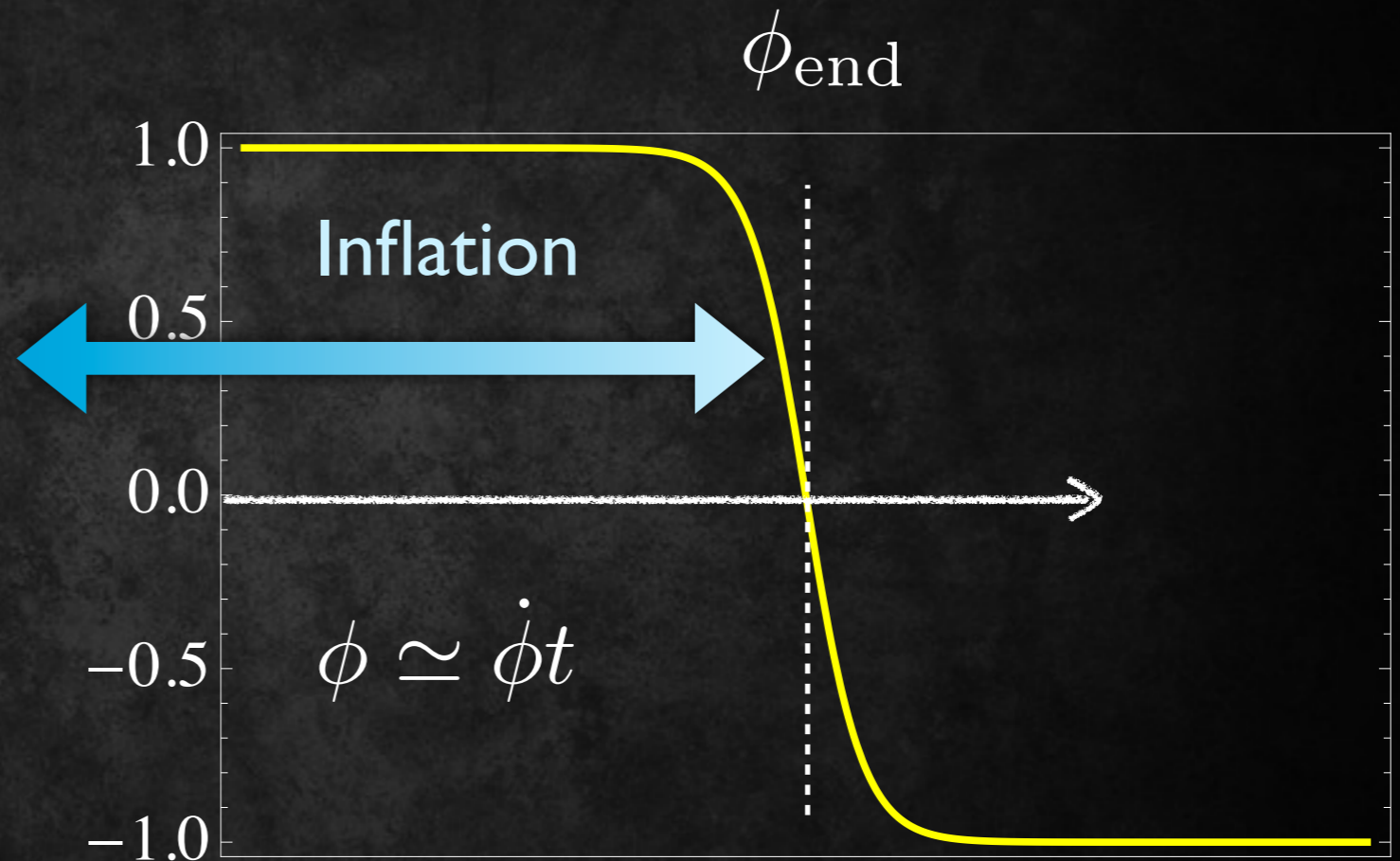
# Graceful exit & Reheating

## Basic idea

$$K = -A(\phi)X + \dots$$

Example:

$$A = \tanh[\lambda(\phi_{\text{end}} - \phi)]$$



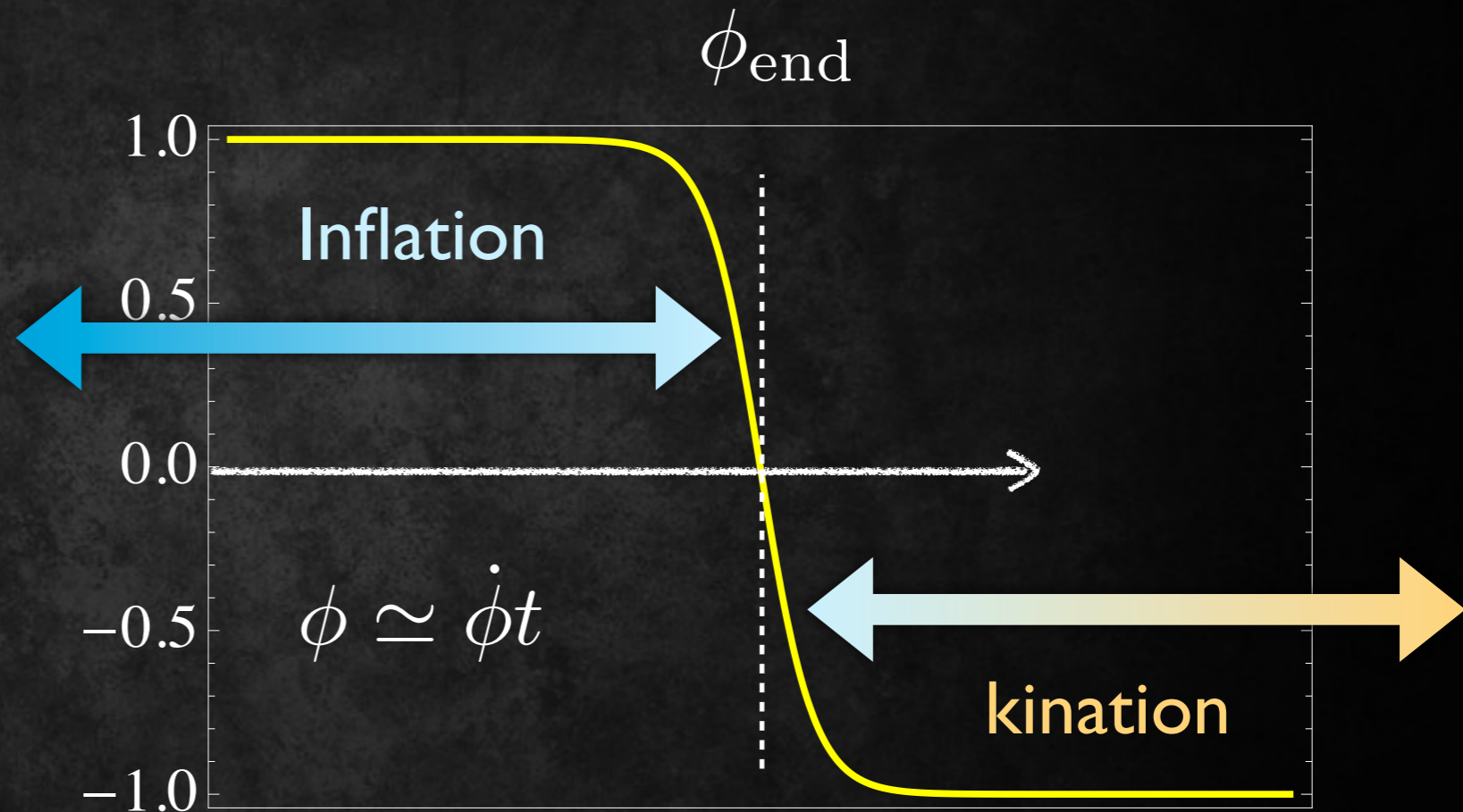
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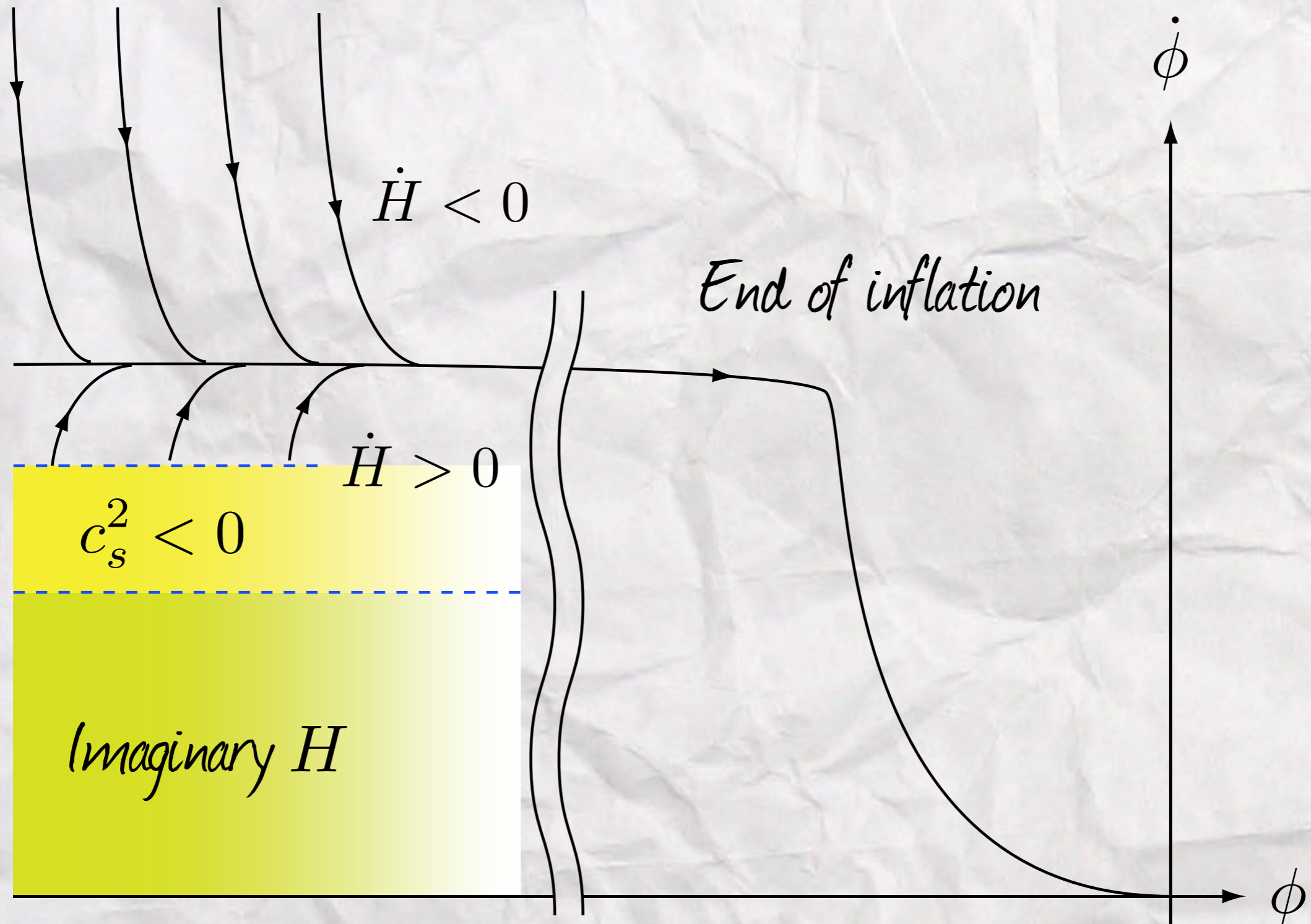
$$\rho \simeq p \simeq X \propto a^{-6}$$

Reheating through gravitational  
particle production

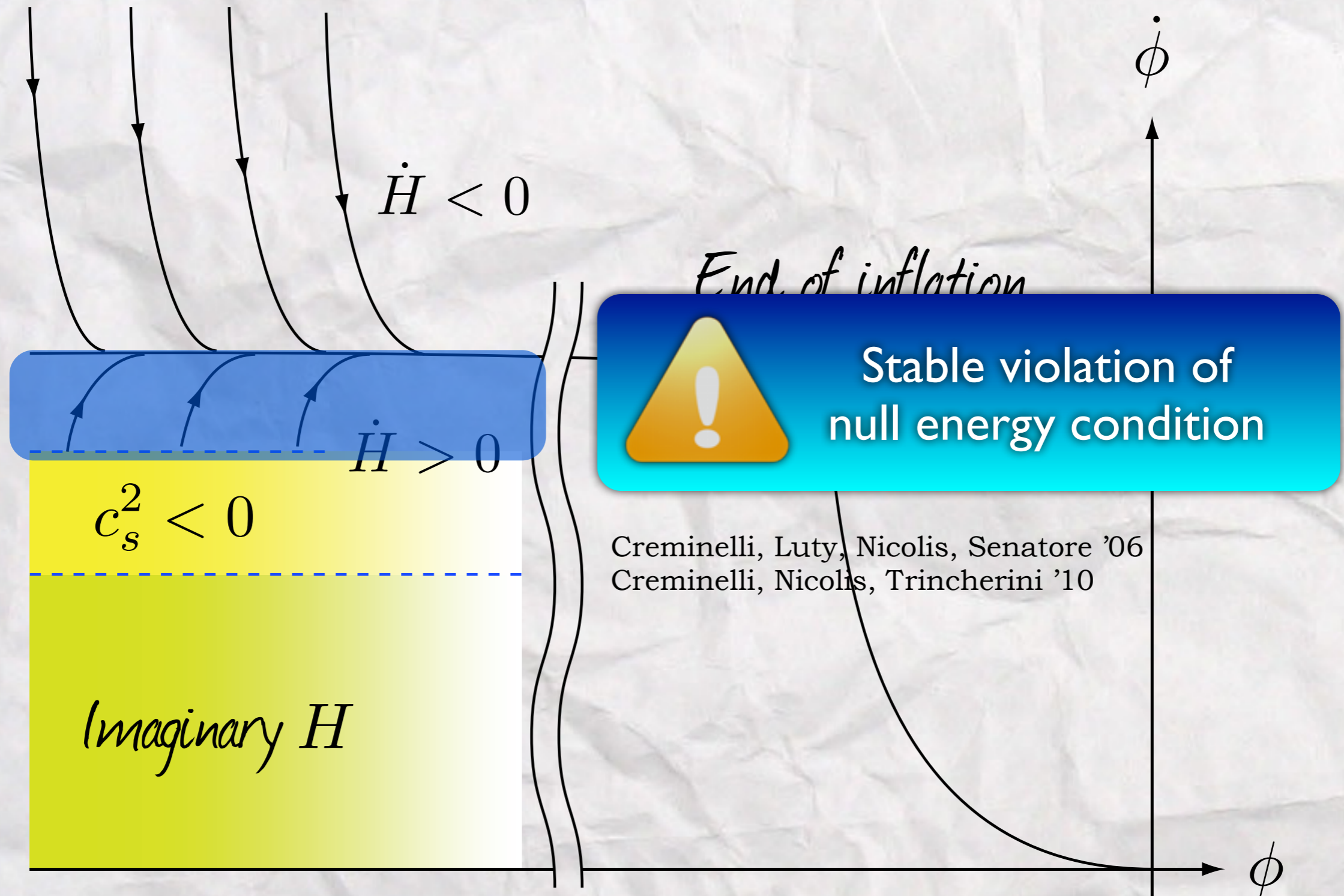
*~ massless, canonical field  
(normal sign)*

Ford '87

# Phase diagram



# Phase diagram





# Primordial perturbations



# Cosmological perturbations

$$ds^2 = -(1 + 2\alpha)dt^2 + 2a^2\beta_{,i}dtdx^i + a^2(1 + 2\mathcal{R})\delta_{ij}dx^i dx^j$$

$$\phi = \phi(t)$$

Unitary gauge:  $\delta\phi = 0$

1. Expand the action to 2nd order
2. Eliminate  $\alpha$  and  $\beta$  using constraint eqs
3. Quadratic action for  $\mathcal{R}$

# Cosmological perturbations

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$$\delta T_i^0 = -F_X \dot{\phi}^3 \alpha_{,i}$$

Uniform  $\phi$  hypersurfaces  
 $\neq$  comoving hypersurfaces

# Quadratic action

$$S^{(2)} = \frac{1}{2} \int d\tau d^3x z^2 \left[ \mathcal{G}(\mathcal{R}')^2 - \mathcal{F}(\vec{\nabla}\mathcal{R})^2 \right]$$

where

$$z = \frac{a\dot{\phi}}{H - F_X \dot{\phi}^3 / 2}$$

$$\begin{aligned} \mathcal{F} = & K_X + 2F_X \left( \ddot{\phi} + 2H\dot{\phi} \right) - 2F_X^2 X^2 \\ & + 2F_{XX} X \ddot{\phi} - 2(F_\phi - X F_{\phi X}) \end{aligned}$$

$$\begin{aligned} \mathcal{G} = & K_X + 2X K_{XX} + 6F_X H \dot{\phi} + 6F_X^2 X^2 \\ & - 2(F_\phi + X F_{\phi X}) + 6F_{XX} H X \dot{\phi} \end{aligned}$$

# Quadratic action

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where

No ghost and gradient instabilities if

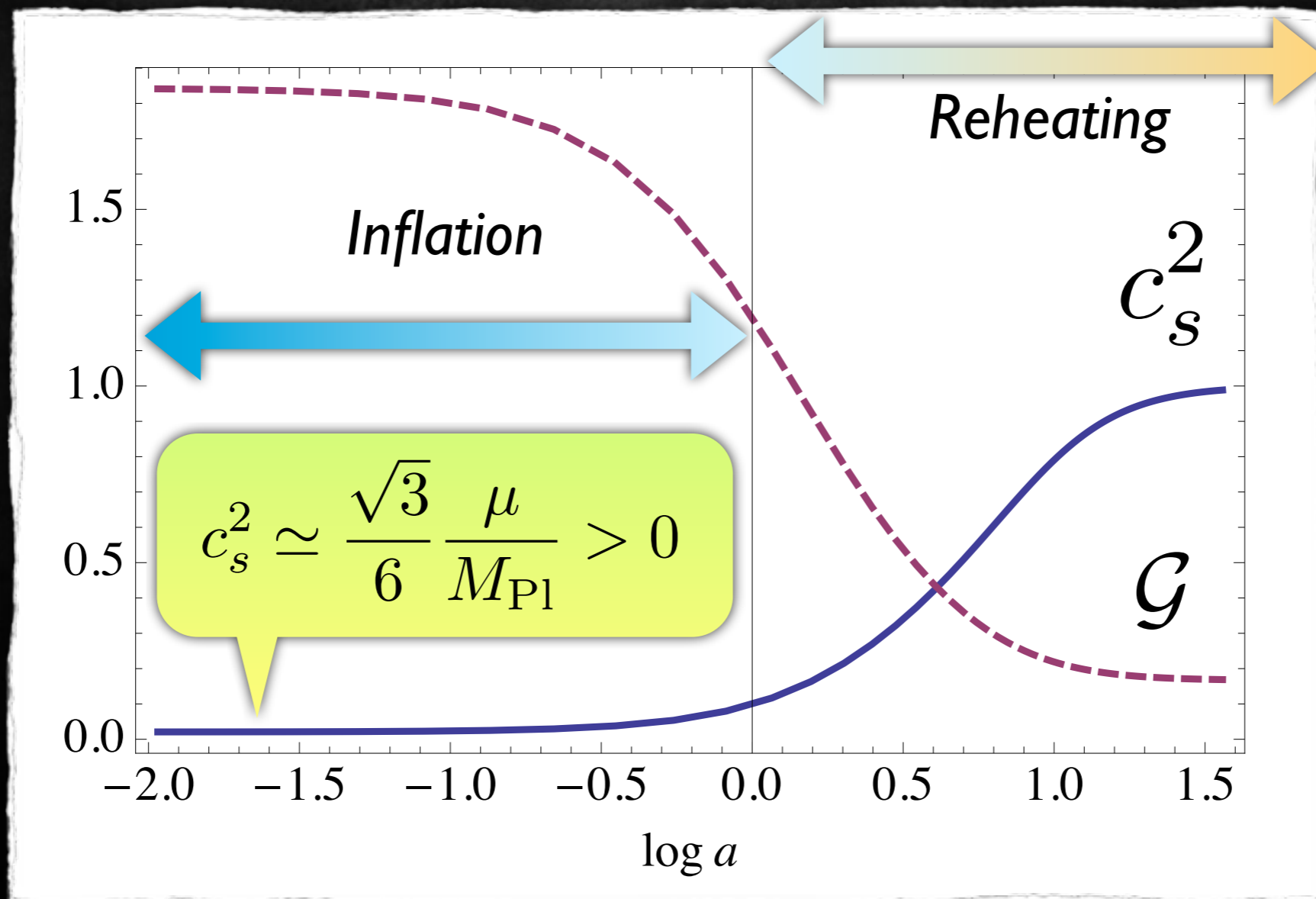
$$z \quad \mathcal{G} > 0, \quad c_s^2 = \mathcal{F}/\mathcal{G} > 0$$

$$\begin{aligned} \mathcal{F} = & K_X + 2F_X \left( \ddot{\phi} + 2H\dot{\phi} \right) - 2F_X^2 X^2 \\ & + 2F_{XX} X \ddot{\phi} - 2(F_\phi - X F_{\phi X}) \end{aligned}$$

$$\begin{aligned} \mathcal{G} = & K_X + 2X K_{XX} + 6F_X H \dot{\phi} + 6F_X^2 X^2 \\ & - 2(F_\phi + X F_{\phi X}) + 6F_{XX} H X \dot{\phi} \end{aligned}$$

# Stable example

$$K = -A(\phi)X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$



# Primordial spectrum

Consider G-inflation with:

$$K = K(X), \quad F = f(\phi)X$$

New variables:

$$\begin{aligned} dy &= c_s d\tau \\ \tilde{z} &= (\mathcal{F}\mathcal{G})^{1/4} z \\ u &= \tilde{z}\mathcal{R} \end{aligned}$$

“Sasaki-Mukhanov equation”

$$\frac{d^2 u}{dy^2} + \left( k^2 - \frac{\tilde{z}_{,yy}}{\tilde{z}} \right) u = 0$$

$$\frac{\tilde{z}_{,yy}}{\tilde{z}} \simeq \frac{1}{(-y)^2} [2 + 3\epsilon\mathcal{C}(X)]$$

$$\mathcal{C}(X) = \frac{K}{K_X} \frac{Q_X}{Q}$$

$$Q(X) = \frac{(K - XK_X)^2}{18Xc_s^2\sqrt{\mathcal{F}\mathcal{G}}}$$

# Primordial spectrum

Normalized mode:  $u = \frac{\sqrt{\pi}}{2} \sqrt{-y} H_{3/2+\epsilon\mathcal{C}}^{(1)}(-ky)$



$$\mathcal{P}_{\mathcal{R}} = \frac{Q}{4\pi^2} \Big|_{c_s k=1/(-\tau)}, \quad n_s - 1 = -2\epsilon\mathcal{C}$$

$\propto f, \phi$

$\mathcal{R}$  can be generated even from exact de Sitter

where

$$Q(X) = \frac{(K - XK_X)^2}{18M_{\text{Pl}}^4 X c_s^2 \sqrt{\mathcal{F}\mathcal{G}}}$$

\* Tensor mode dynamics: unchanged

# Tensor-to-scalar ratio

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = fX \quad \longrightarrow \quad H^2 \sim \frac{\mu M^3}{M_{\text{Pl}}^2}$$

$$r \simeq \frac{16\sqrt{6}}{3} \left( \frac{\sqrt{3}\mu}{M_{\text{Pl}}} \right)^{3/2}$$

Conventional consistency relation is violated

$$r \neq -8c_s n_T$$

$$\uparrow \propto \epsilon \propto f, \phi$$

$$M = 0.00435 \times M_{\text{Pl}}, \quad \mu = 0.032 \times M_{\text{Pl}}$$

$$\longrightarrow \quad \mathcal{P}_{\mathcal{R}} = 2.4 \times 10^{-9}, \quad r = 0.17$$

$r$  can be large!



# Summary

The image features a vintage-style photograph of a bright blue sky filled with fluffy white clouds. The clouds are scattered across the lower half of the frame, with some appearing more prominent and detailed than others. The overall aesthetic is that of an old film or photograph, with a slightly grainy texture and some minor imperfections or dust spots visible. The word "Summary" is centered in the middle of the image in a white, serif font with a subtle drop shadow, making it stand out against the blue background.

# Summary

- **G-inflation**: A general class of single field inflation

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\square\phi$$

- $n_s - 1 \simeq 0$
- Large  $r$
- ~~Consistency relation~~

*G-inflation would make  
gravitational wave people happy!*

A scenic landscape photograph featuring a bright blue sky filled with large, fluffy white clouds. In the foreground, a snow-covered field is visible, bordered by a wooden fence with wire mesh. The background shows a silhouette of a residential area with houses and trees under a clear sky. The text "Thank you!" is overlaid in a large, bold, orange font across the center of the image.

Thank you!

# Example

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$



$$\mu \ll M_{\text{Pl}}$$

$$\frac{H^2}{M_{\text{Pl}}^2} \simeq \frac{1}{6} \frac{M^3}{M_{\text{Pl}}^3} \frac{\mu}{M_{\text{Pl}}}$$

$$X \simeq \left( 1 - \frac{\sqrt{3}\mu}{M_{\text{Pl}}} \right) \mu M^3$$

# Numerical Example

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = \frac{e^{\alpha\phi}}{M^3}X$$

