Quantum Mechanics of Leptogenesis

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Quantum Mechanics of Leptogenesis?

- Why do we need a full QFT treatment of leptogenesis?
- How can a QFT treatment of leptogenesis be formulated?
  - full QFT treatment applicable to any production and freezeout processes in a thermal bath!!!
- example for full QFT computation of an asymmetry generation in realistic seesaw model
- not yet a full theory of "historical" leptogenesis as some important ingredients (expansion, certain "washout" processes, SM widths, spectator processes) are simplified/neglected. . .
- . . . but first principles computation without semiclassical assumptions (reference to particle numbers/asymptotic states, molecular chaos, kinetic equilibrium, close to equilibrium,. . .) and related problems (RIS substraction, definition of asymptotic states, coherence, flavour issues)

$\Rightarrow$ allows comparison between QFT and Boltzmann!

Thermal Leptogenesis

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \partial\!\!\!/ \nu_R - \bar{l}_L \tilde{\Phi} \lambda \nu_R - \frac{1}{2} \bar{\nu^c_R} M \nu_R + h.c.$$

- explains small neutrino masses via see-saw mechanism
- fulfils Sakharov conditions
  - complex phases violate CP
  - singlet fermions are out of equilibrium
  - B-violating sphaleron processes can transfer asymmetry to baryonic sector

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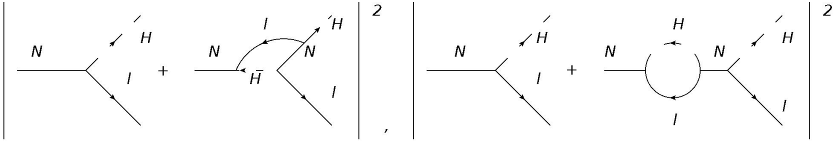
$\Rightarrow$ baryon asymmetry can be generated

Quantum Genesis

- usually studied in terms of semiclassical Boltzmann equations (classical particle numbers , collision terms from vacuum S-matrix)
- creation (of matter) from interference is a quantum effect



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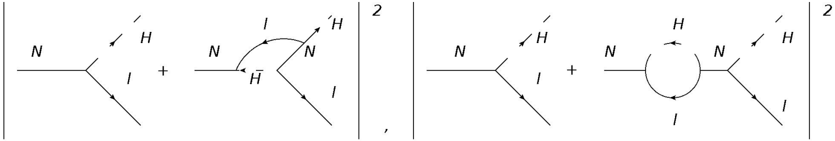
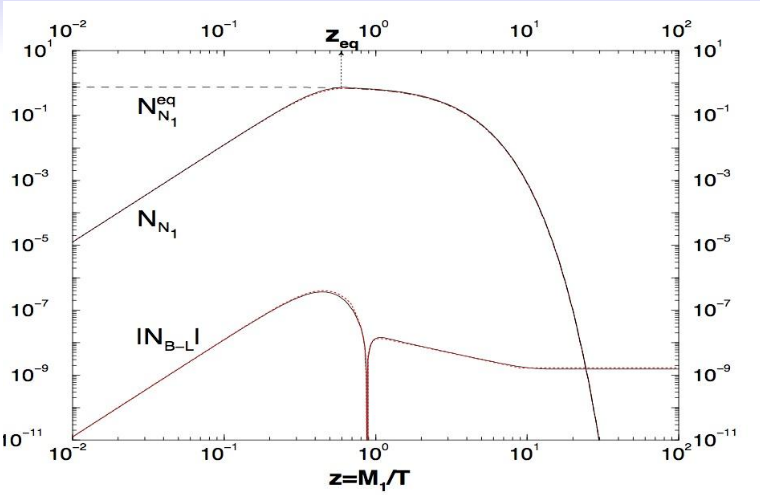


conceptual problems in the semi-classical description

- non-Markovian / memory effects
- no asymptotic states / particle number in omnipresent plasma
- off-shell effects
- flavour effects: coherent oscillations, quantum zeno effect...
- modified spectrum (quasiparticles, collective excitations...)



Buchmüller/Plümacher 2002

Methods

- Boltzmann equations (BE)
- quantum Boltzmann equations (QBE)
- (effective) kinetic equations for reduced density matrices
- Kadanoff-Baym equations (KBE)

Is a Quantum Treatment possible?

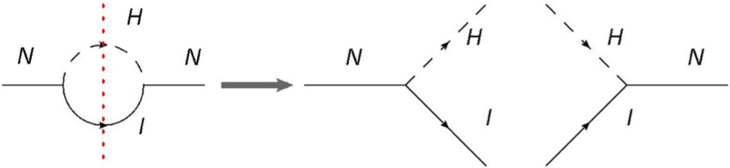
- spacial homogeneity
- weak coupling $\Rightarrow$ perturbative
- background plasma is in equilibrium
- backreaction can be neglected

'Weak coupling to a thermal bath'

Boltzmann vs Kadanoff-Baym Equations

- initial value problem for density matrix $\rho(t)\ldots$
- $\ldots$or for correlation functions $\langle\ldots\rangle = \mathrm{tr}(\rho\ldots)$
- KBE contain full quantum mechanics

$$\text{particle numbers} \Leftrightarrow \text{correlation functions}$$
$$\text{collision term} \Leftrightarrow \text{self energies}$$



Statistical and Spectral Propagators

$$\begin{aligned}\Delta^+(x_1,x_2) &= \frac{1}{2}\langle\{\Phi(x_1),\Phi(x_2)\}\rangle_c \\ \Delta^-(x_1,x_2) &= i\langle[\Phi(x_1),\Phi(x_2)]\rangle_c\end{aligned}$$

$$\begin{aligned}S^+_{\alpha\beta}(x_1,x_2) &= \frac{1}{2}\langle[\Psi(x_1)_\alpha,\bar{\Psi}_\beta(x_2)]\rangle_c \\ S^-_{\alpha\beta}(x_1,x_2) &= i\langle\{\Psi(x_1)_\alpha,\bar{\Psi}_\beta(x_2)\}\rangle_c\end{aligned}$$

$$\begin{aligned}G^+_{\alpha\beta}(x_1,x_2) &= \frac{1}{2}\langle[N_\alpha(x_1),N_\beta(x_2)]\rangle_c \\ G^-_{\alpha\beta}(x_1,x_2) &= i\langle\{N_\alpha(x_1),N_\beta(x_2)\}\rangle_c\end{aligned}$$

Equilibrium: KMS-relations, e.g. $\Delta^+_{\mathbf{q}}(\omega) = -i\left(\frac{1}{2} + f^{eq}_\phi(\omega)\right)\Delta^-_{\mathbf{q}}(\omega)$

$\Rightarrow$ equilibrium propagators not independent, Bose/Fermi statistics

Kadanoff Baym Equations

$$C(i\partial\!\!\!/_1 - m)G^-(x_1,x_2) = -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt' \Sigma^-(x_1,x')G^-(x',x_2)$$

$$C(i\partial\!\!\!/_1 - m)G^+(x_1,x_2) = \int d^3\mathbf{x}' \int_{t_i}^{t_1} dt' \Sigma^-(x_1,x')G^+(x',x_2) + \int d^3\mathbf{x}' \int_{t_i}^{t_2} dt' \Sigma^+(x_1,x')G^-(x',x_2)$$

Weak Coupling to a thermal Bath

- dressed spectral propagators in LO time translation invariant
- KBE are equivalent to a stochastic Langevin equation
- KBE can be solved analytically up to a *memory integral*

Lepton Asymmetry

- observables can be computed from correlation functions

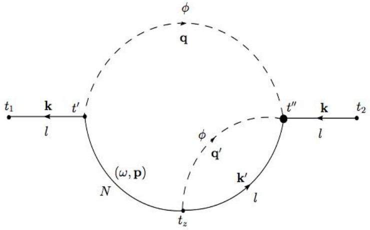
Lepton Number Matrix

$$L_{\mathbf{k}ij}(t_1, t_2) = -\mathrm{tr}[\gamma^0 S^+_{\mathbf{k}ij}(t_1, t_2)].$$

- asymmtry is given by $L_{\mathbf{k}ii}(t, t)$
- equation of motion for $L_{\mathbf{k}ii}(t, t)$ can be derived from KBE for statistical lepton propagator S^+
- knowledge of the nonequilibrium propagators allows to compute Feynman diagrams
- additional complication due to explicit time dependence
- Since leptogenesis comes from a LO-NLO interference, we need the dressed NLO nonequilibrium lepton propagators!

Lepton Asymmetry

- for $L_{\mathbf{k}ii}(t,t)$, only the CP violating part of the self energy is relevant
- hierarchical masses: the diagram that generates the asymmetry is



- source of the deviation from equilibrium is the nonequilibrium Majorana propagator

Spectral Majorana Propagator

$$G^{-}_{\mathbf{q}}(t_1 - t_2) = i \int \frac{d\omega}{2\pi} e^{-i\omega(t_1-t_2)} \rho_{\mathbf{q}}(\omega)$$

$$\rho_{\mathbf{q}}(\omega) = \left(\frac{i}{\not{q} - m - C\Sigma^R_{\mathbf{q}}(\omega) + i\not{u}\epsilon} - \frac{i}{\not{q} - m - C\Sigma^A_{\mathbf{q}}(\omega) - i\not{u}\epsilon} \right) C^{-1}$$

- (quasi)poles of ρ give spectrum of resonances
- determined by retarded self energy Σ^R
- $\Sigma^R = \Sigma^R|_{T=0} + \delta\Sigma^R(T)$ has a vacuum part and a correction due to the medium
- rich phenomenology (flavour structure, collective excitations...) can be encoded therein

Statistical Majorana Propagator

$$
\begin{aligned}
G^{+}_{\mathbf{q}}(t_1, t_2) &= -G^{-}_{\mathbf{q}}(t_1) C\gamma^0 G^{+}_{\mathbf{q}}(0,0)\gamma^0 C^{-1} G^{-}_{\mathbf{q}}(-t_2) \\
&+ \int_0^{t_1} dt' G^{-}_{\mathbf{q}}(t_1 - t') \int_0^{t_2} dt'' C^{-1} \Sigma^{+}_{\mathbf{q}}(t' - t'') G^{-}_{\mathbf{q}}(t'' - t_2)
\end{aligned}
$$

- no restriction on the size of initial deviation from equilibrium!
- no a priori parameterisation of the propagators by distribution functions!

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\end{aligned}
$$

- no restriction on the size of initial deviation from equilibrium!
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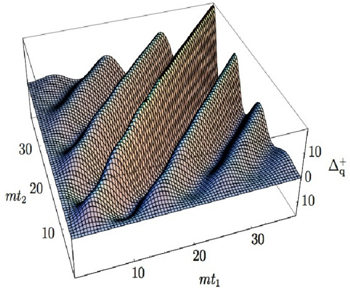
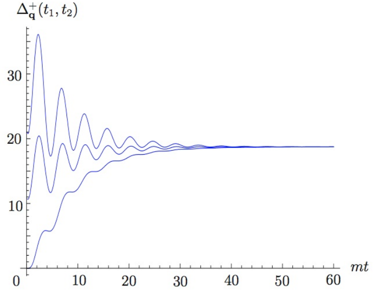
In the narrow width limit for vanishing initial particle number:

$$
\begin{aligned}
G^{+}_{\mathbf{q}}(t; y) = &- \left(i\gamma_0 \sin(\omega_{\mathbf{q}} y) - \frac{M - \mathbf{q}\gamma}{\omega_{\mathbf{q}}} \cos(\omega_{\mathbf{q}} y) \right) \\
&\times \left(\frac{1}{2} \tanh(\beta\omega_{\mathbf{q}}/2) e^{-\Gamma_{\mathbf{q}}|y|/2} + f_N^{eq}(\omega_{\mathbf{q}}) e^{-\Gamma_q t} \right) C^{-1}
\end{aligned}
$$

with $y = t_1 - t_2$, $t = (t_1 + t_2)/2$ and $\Gamma \propto \mathrm{disc}\Sigma$

The Statistical Propagator





- depends on two time arguments
- equilibrates independent of initial conditions after characteristic time $\tau \sim {}^{1}/_{\Gamma}$
- oscillates with plasma frequency

$$L_{\mathbf{k}ii}(t,t) \supset -\epsilon_{ii}\, 8\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{|\mathbf{k}||\mathbf{k}'|\omega} f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega)$$
$$\times \frac{\frac{1}{2}\Gamma}{((\omega - k - q)^2 + \frac{\Gamma^2}{4})((\omega - |\mathbf{k}'| - |\mathbf{q}'|)^2 + \frac{\Gamma^2}{4})}$$
$$\times \Big(\cos[(|\mathbf{k}| + |\mathbf{q}| - |\mathbf{k}'| - |\mathbf{q}'|)t] + e^{-\Gamma t}$$
$$-(\cos[(\omega - |\mathbf{k}| - |\mathbf{q}|)t] + \cos[(\omega - |\mathbf{k}'| - |\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big)\,,$$

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$$\times \Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t] + e^{-\Gamma t}$$
$$-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t] + \cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big)\,,$$

with $\mathbf{p} = \mathbf{q}+\mathbf{k} = \mathbf{q}'+\mathbf{k}'$

$$\int_{\mathbf{p}} \ldots = \int \frac{d^3p}{(2\pi)^3 2\omega_{\mathbf{p}}} \ldots$$

$$f_{l\phi}(k,q) = f_l(k) f_\phi(q) + (1-f_l(k))(1+f_\phi(q))$$
$$= 1 - f_l(k) + f_\phi(q)$$

Comparison to Boltzmann Result

$$L_{\mathbf{k}ii}(t,t) \supset -\epsilon_{ii}\,\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega}f_{l\phi}(k,q)f_N^{eq}(\omega)f_{l\phi}(k',q')$$

$$\times\frac{\frac{1}{4}\Gamma}{((\omega-k-q)^2+\frac{\Gamma^2}{4})((\omega-|\mathbf{k}'|-|\mathbf{q}'|)^2+\frac{\Gamma^2}{4})}$$

$$\times\Big(\cos[(|\mathbf{k}|+|\mathbf{q}|-|\mathbf{k}'|-|\mathbf{q}'|)t]+e^{-\Gamma t}$$

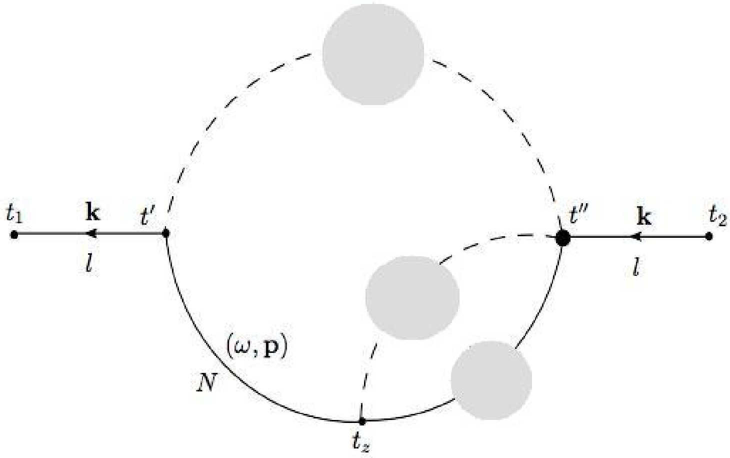
$$-(\cos[(\omega-|\mathbf{k}|-|\mathbf{q}|)t]+\cos[(\omega-|\mathbf{k}'|-|\mathbf{q}'|)t])e^{-\frac{\Gamma t}{2}}\Big)\,,$$

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' f_{l\phi}(k,q)f_N^{eq}(\omega)$$

$$\times\frac{1}{\Gamma}(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p)$$

$$\times\left(1-e^{-\Gamma t}\right)$$

Inclusion of SM widths



Inclusion of SM widths

$$\tilde{L}_{\mathbf{k}ii}(t,t) \supset -\epsilon_{ii}\frac{16\pi}{|\mathbf{k}|}\int_{\mathbf{q},\mathbf{q}'}\frac{k\cdot k'}{|\mathbf{k}'|\omega} f_{l\phi}(k,q) f_N^{eq}(\omega) f_{l\phi}(k',q')$$
$$\times \frac{1}{\Gamma}\frac{\frac{1}{4}\Gamma_{l\phi}\Gamma_{\phi}}{((\omega-k-q)^2+\frac{1}{4}\Gamma_{l\phi}^2)((\omega-k'-q')^2+\frac{1}{4}\Gamma_{\phi}^2)}$$
$$\left(1-e^{-\Gamma t}\right)$$

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$$\times \left(1-e^{-\Gamma t}\right)$$

Conclusion

- leptogenesis is a nonequilibrium quantum process
- semiclassical methods suffer from severe conceptional problems
- KBE allow full quantum treatment
- solutions can deviate significantly from BE due to memory, coherent and off-shell effects
- however, for narrow widths, hierarchical masses and fast SM interactions the BE appear to be recovered
- physical picture: fast interactions restore locality by reducing the coherence length
- BUT: in more complex scenarios of leptogenesis (flavoured, resonant. . .) there may be nontrivial effects