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Oscillating Universe in Hořava-Lifshitz Gravity

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Motivation

◆Horava-Lifshitz gravity (HL gravity)　P.Horava (2009)

- $x^i \to bx^i, \mathrm{t} \to b^z t$: power-counting renormalizable ($z = 3$)
- Action includes higher curvature term → Singularity avoidance

detailed balance condition
- restriction of Action

→ impose(original)

→ relax (SVW)

◆Discussion in FLRW universe

✓ original : refined classification M. Minamitsuji (2009)

✓ SVW : Analysis with non clear assumption

Preceding studies

◆ SVW version

1. P. Wu and H. Yu (2009)

   unstable static solution ⟶ expand, oscillation…

   Emergent universe : singularity avoidance

2. E. J. Son and W. Kim (2010)

   inflation ⟶ exponential expand in present

discussion about phase transition including matter

◆ Equation of state in HL gravity : possibility of change e.o.s

e.o.s of radiation : $\rho_r \propto a^{-4} \longrightarrow a^{-6}$

⟶ Analyze vacuum solution in SVW version

Action in SVW HL gravity

- SVW : potential term includes all possible term $z = 3$

T. P. Sotiriou, M. Visser and S. Weinfurtner (2009)

$$\mathcal{V}_{\rm HL} = 2\Lambda + g_1 \mathcal{R} + \kappa^2 \left(g_2 \mathcal{R}^2 + g_3 \mathcal{R}^i{}_j \mathcal{R}^j{}_i \right) + \kappa^3 g_4 \epsilon^{ijk} \mathcal{R}_{i\ell} \nabla_j \mathcal{R}^\ell{}_k$$
$$+\kappa^4 \left(g_5 \mathcal{R}^3 + g_6 \mathcal{R}\, \mathcal{R}^i{}_j \mathcal{R}^j{}_i + g_7 \mathcal{R}^i{}_j \mathcal{R}^j{}_k \mathcal{R}^k{}_i + g_8 \mathcal{R} \Delta \mathcal{R} + g_9 \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk} \right),$$

g_i : dimensionless coupling constant

- restriction to g_i : stability of flat background spacetime

$g_1 < 0,\ g_9 > 0,\ \lambda > 1$ → $g_1 = -1$ (rescaling of time)

- $\kappa^2 = (1/M_{PL})^2 = 1$

Application to FLRW universe

◆Background : homogeneous and isotropic ($K \neq 0$)

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right) \, ,$$

Hamiltonian constraint

$$H^2 + \frac{2}{(3\lambda-1)}\frac{K}{a^2} = \frac{2}{3(3\lambda-1)}\left[\Lambda + \frac{g_{\rm r}}{a^4} + \frac{g_{\rm s}}{a^6}\right]$$

$$g_r \equiv 6(g_3 + 3g_2)K^2 \; , g_s \equiv 12(9g_5 + 3g_6 + g_7)K^3$$

If $K \neq 0, \quad g_s, g_r \neq 0$

◆original : $g_s = 0, g_r < 0$

◆SVW : $g_r, \; g_s$ are arbitrary

→ it is possible that energy density is negative.

How to Classify

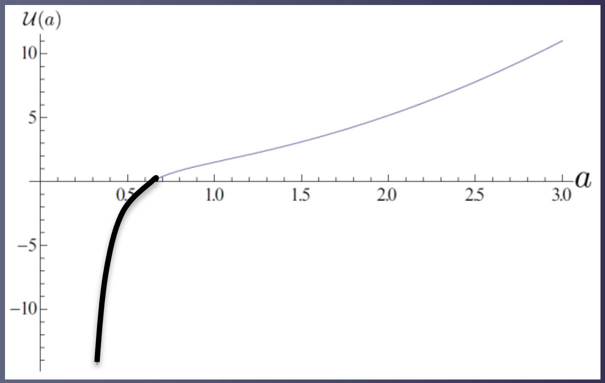
◆Hamiltonian constraint

$$\frac{1}{2}\dot{a}^2 + \mathcal{U}(a) = 0 \ , \ \mathcal{U}(a) = \frac{1}{3\lambda - 1}\left[K - \frac{\Lambda}{3}a^2 - \frac{g_{\rm r}}{3a^2} - \frac{g_{\rm s}}{3a^4}\right] .$$

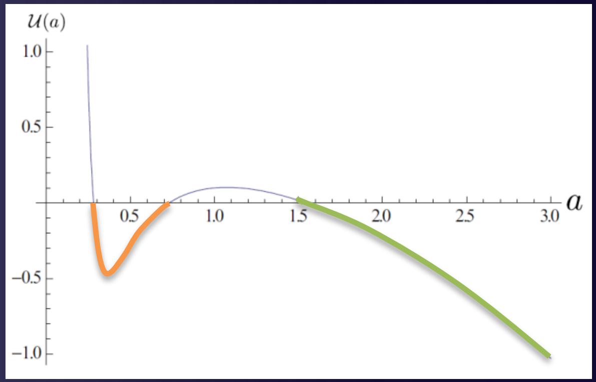
Moving zero−energy particle in the potential $\mathcal{U}(a)$

→ scale factor can move in only $\mathcal{U}(a) \leq 0$

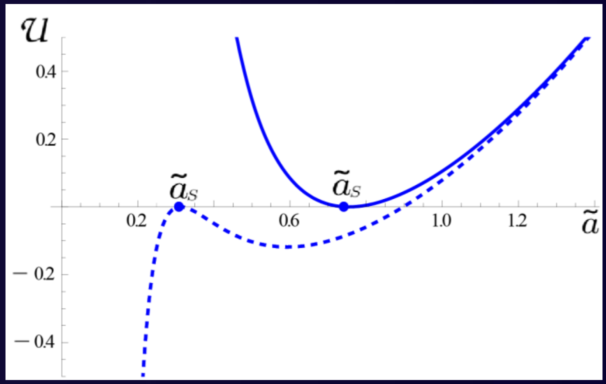
例：



big bang → big crunch



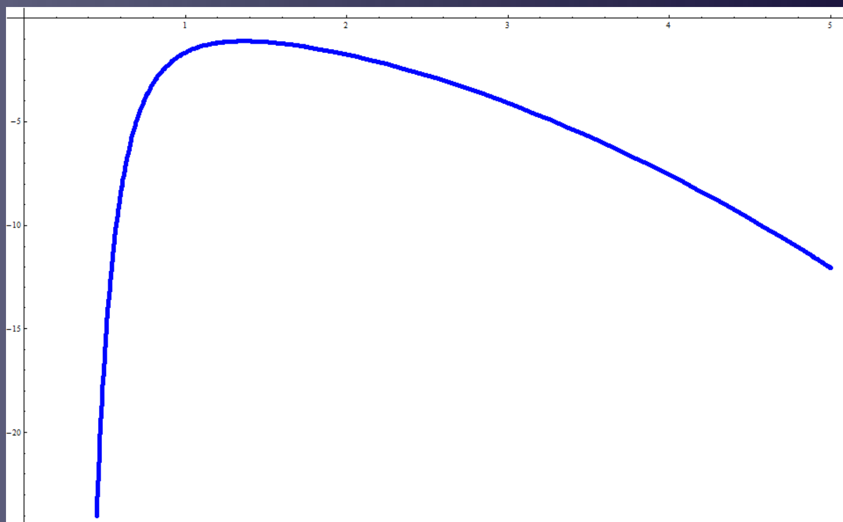
oscillation or bounce



stable static
unstable static

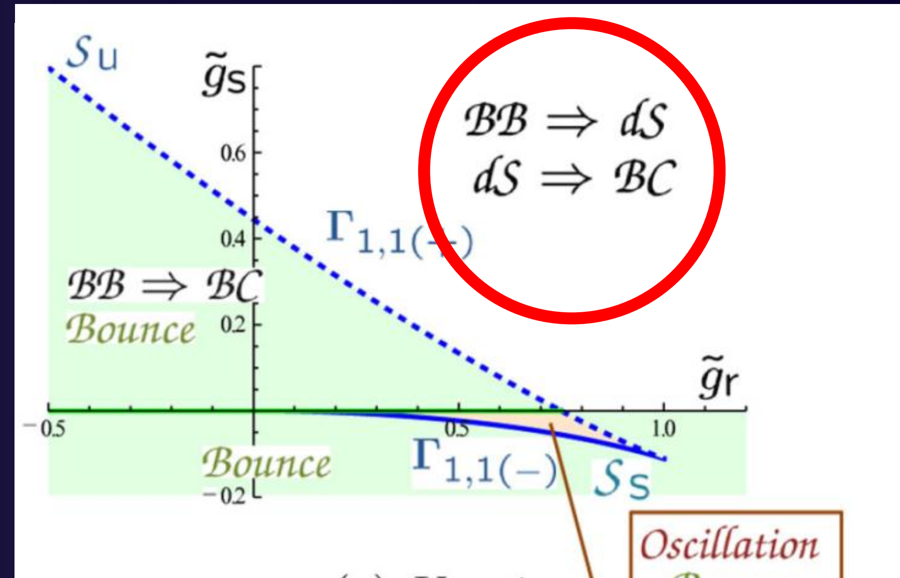
Classification $(\Lambda > 0, K = 1)$

1. Big bang → de Sitter



$\tilde{g}_s = 5, \tilde{g}_r = 5$

with singularity



(a) $K = 1$

$\mathcal{BB}$: big bang $\mathcal{BC}$: big crunch

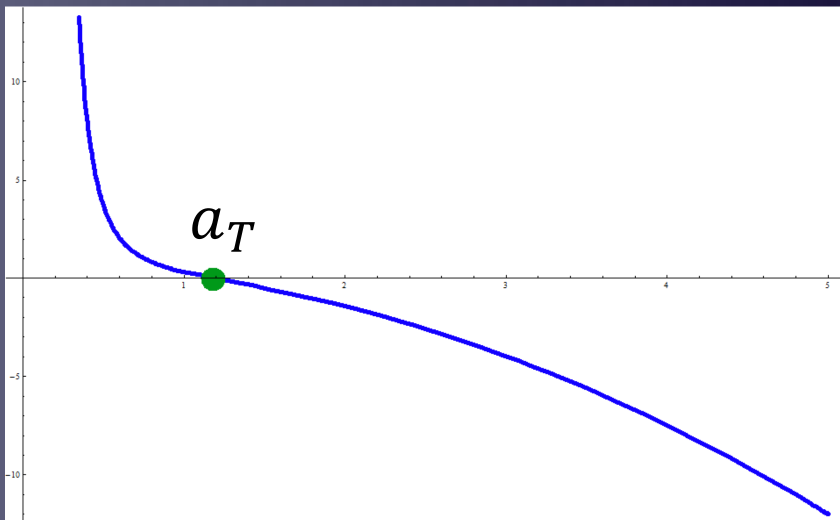
$d\mathcal{S}$: de Sitter $\mathcal{M}$: Milne

$\mathcal{S}_s$: stable static $\mathcal{S}_u$: unstable static

Big bang to expanding universe

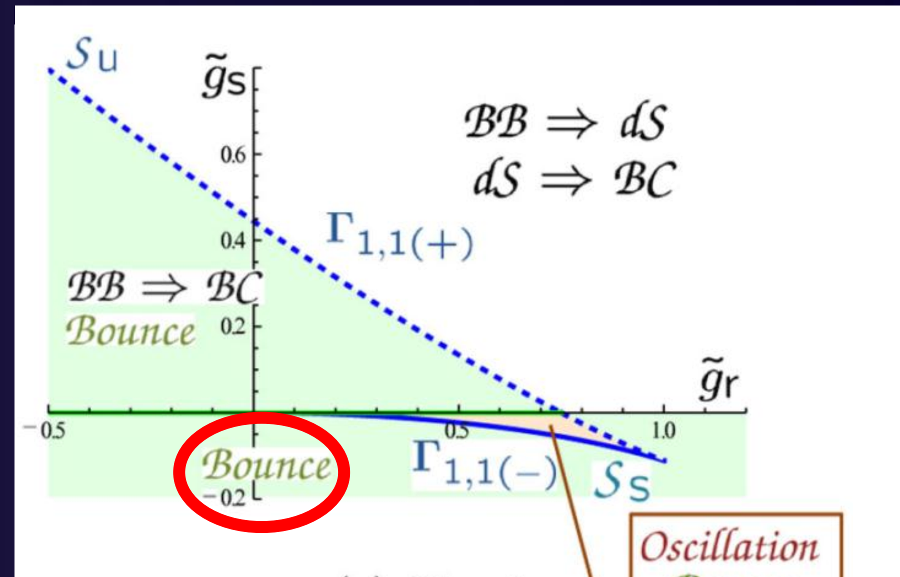
Classification ($\Lambda > 0, K = 1$)

2. bouncing universe



$$\tilde{g}_s = -1, \tilde{g}_r = -1$$

singularity avoidance



(a) $K = 1$

$\mathcal{BB}$: big bang $\mathcal{BC}$: big crunch

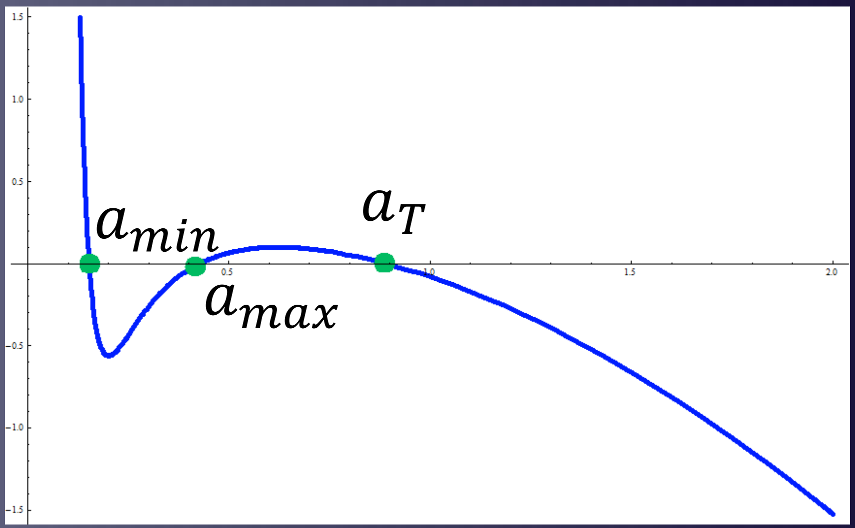
$d\mathcal{S}$: de Sitter $\mathcal{M}$: Milne

$\mathcal{S}_{\text{s}}$: stable static $\mathcal{S}_{\text{u}}$: unstable static

Solution with minimum value a_T : bouncing universe

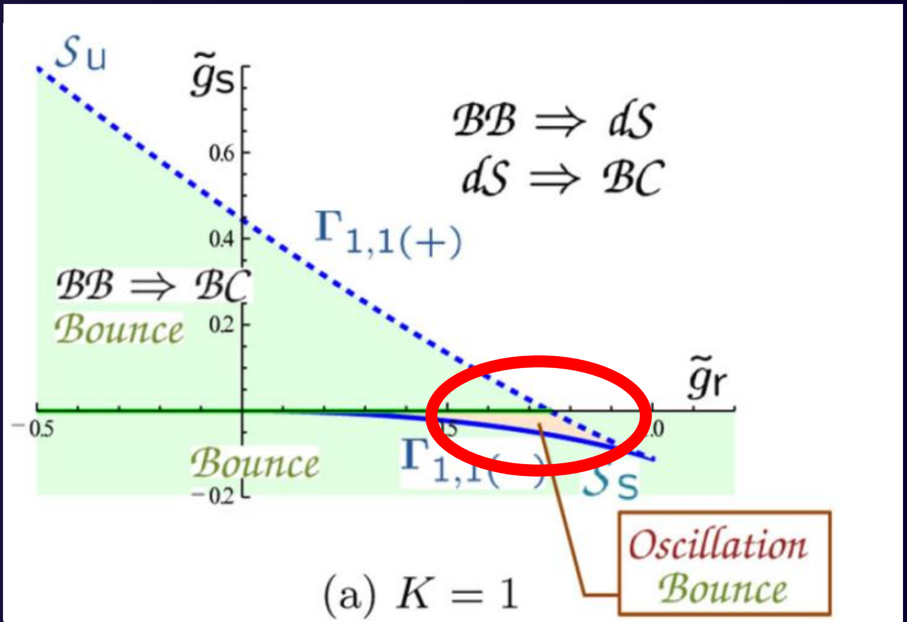
Classification $(\Lambda > 0, K = 1)$

3. oscillation and bounce



$\tilde{g}_s = -0.01,\ \tilde{g}_r = 0.5$

singularity avoidance



$\mathcal{BB}$: big bang $\mathcal{BC}$: big crunch

$d\mathcal{S}$: de Sitter $\mathcal{M}$: Milne

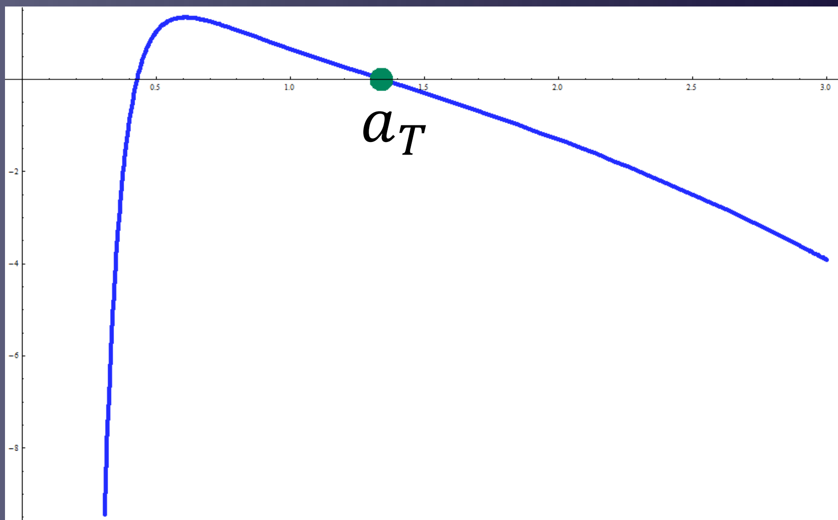
$\mathcal{S}_s$: stable static $\mathcal{S}_u$: unstable static

Oscillating solution($a_{min} \leq a \leq a_{max}$)+ bouncing solution

$g_s < 0$: sufficient condition for singularity avoidance

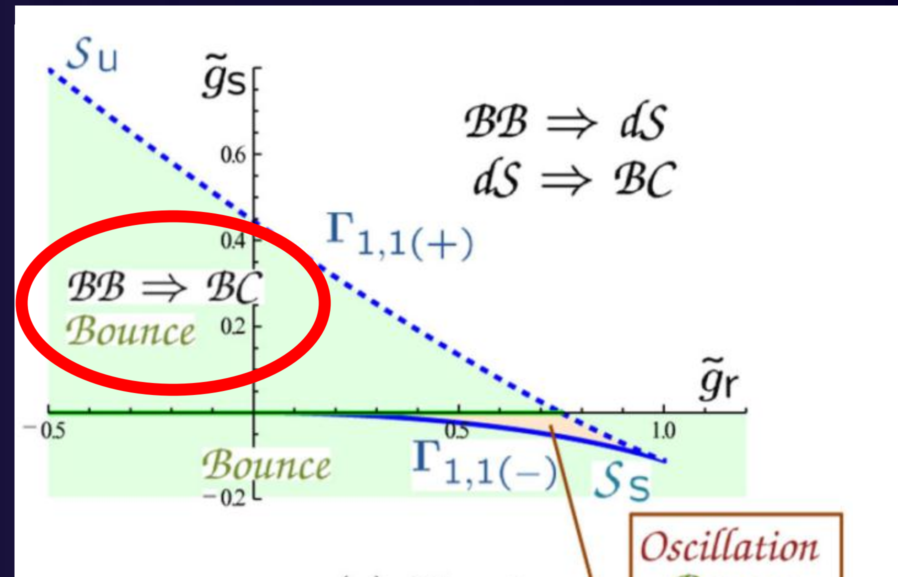
Classification $(\Lambda > 0, K = 1)$

4. bounce or big crunch



$\tilde{g}_s = -5,\ \tilde{g}_r = 1$

singularity avoidance is possible



(a) $K = 1$

$\mathcal{BB}$: big bang　　$\mathcal{BC}$: big crunch

$d\mathcal{S}$: de Sitter　　$\mathcal{M}$: Milne

$\mathcal{S}_\mathrm{s}$: stable static　　$\mathcal{S}_\mathrm{u}$: unstable static

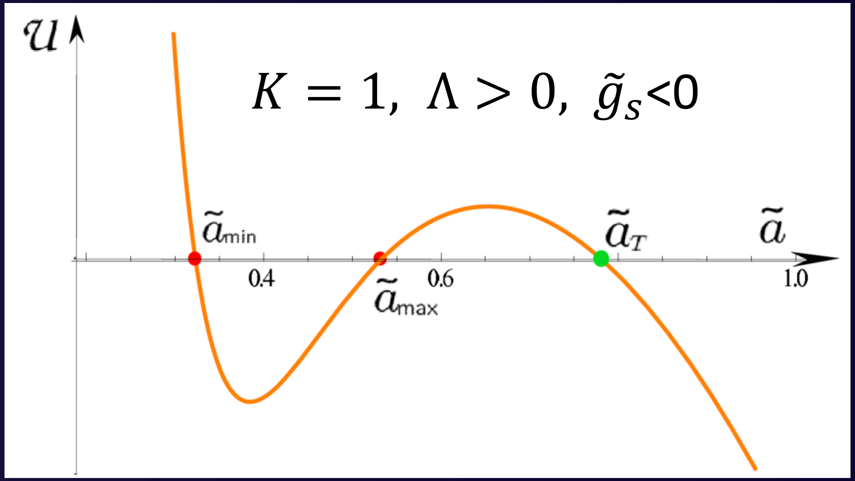
If initial value takes $a_T \leq a$: bouncing universe

Toward a cyclic universe

◆ How to escape to macroscopic universe.

1. $K = 1, \Lambda > 0$: oscillation
2. quantum tunneling to a_T
3. de Sitter phase

⟶ macroscopic universe



◆ How to obtain cyclic universe.

1. Scalar field is responsible for inflation.
   - Before tunneling : behave as cosmological constant
   - After tunneling : slow-roll inflation ⟶ reheating ($\mathcal{U}(\infty)$>0)
2. upper bound and lower bound : cyclic universe

reheating negative "stiff matter"

Summary and future works

- Classification vacuum FLRW universe in SVW HL gravity
  - Property of solution : largily depand in g_i

    Sufficient condition for singularity avoidance : $g_s < 0$

    (oscillation, bounce)

- With matter : condition for singularity avoidance ?

  $$g_s \to g_s + \underline{g_{stiff}}, \quad g_r \to g_r + \underline{g_{rad}}, \quad g_d \to \underline{g_{dust}}$$

  energy density of matter (≥ 0)

- future work
  - Discussion about stability of non-singular solution
    - Inhomogeneity
    - anisotropy : Bianchi type IX (in preparation)