Non-Gaussianity in the CMB from nonlinear effects

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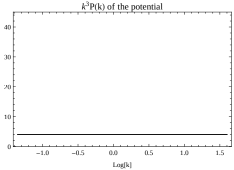
Standard lore of perturbation theory

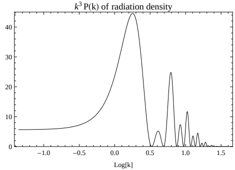
- *Initial conditions*: quantization of the free theory implies Gaussian initial conditions: $P(k)$
  $\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\rangle = \delta(\mathbf{k}+\mathbf{k}')P(k)$
- *Evolution*: linearisation of GR.

Transfer scheme of perturbations

- Linear equations, modes k are independent,
  $\Longrightarrow$ Gaussianity conserved.
- $P(k) \to \Theta(k,\eta) \to a_{\ell m} \to C_\ell$



$\longrightarrow$



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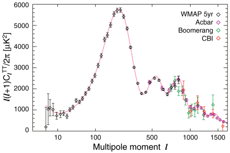


Non-Gaussianity

non-Gaussianity (NG)

- Initial conditions non-Gaussian?
  We want to test the models of inflation with other moments of the statistics
- Non-linear dynamics is intrinsic to GR

Statistics of the primordial gravitational potential $\Phi = \Phi^{(1)} + \frac{1}{2}\Phi^{(2)}$

- Gaussian part $\Phi^{(1)}$ and non-Gaussian part $\Phi^{(2)}$:
- $\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\rangle = \delta(\mathbf{k}+\mathbf{k}')P(k)$
- $\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) f_{\rm NL} F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
- $F(\dots) =$ type of non-Gaussianity
- $f_{\rm NL} =$ its amplitude.

The transfer to temperature fluctuations $\Theta_{\ell m}$

- In general $\Theta \equiv \mathcal{T}(\Phi)$
- Order 1 $\Theta^{(1)}_{\ell m} \equiv \mathcal{T}^{\ell m}_{\rm L}(\Phi^{(1)})$
- Order 2 $\Theta^{(2)}_{\ell m} \equiv \mathcal{T}^{\ell m}_{\rm L}(\Phi^{(2)}) + \mathcal{T}^{\ell m}_{\rm NL}(\Phi^{(1)}\Phi^{(1)})$

In Fourier space

- $\Theta^{(1)}_{\ell m}(\mathbf{k}) = \mathcal{T}^{\ell m}_{\rm L}(k)\Phi^{(1)}_{\mathbf{k}}$
- $\Theta^{(2)}_{\ell m}(\mathbf{k}) = \mathcal{T}^{\ell m}_{\rm L}(k)\Phi^{(2)}_{\mathbf{k}}$
  $+ \int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \delta^3(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\mathcal{T}^{\ell m}_{\rm NL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k})\Phi^{(1)}_{\mathbf{k}_1}\Phi^{(1)}_{\mathbf{k}_2}$

$f_{\rm NL}$ or $\mathcal{T}_{\rm NL}$?

$\langle \Theta_{\ell_1 m_1}\Theta_{\ell_2 m_2}\Theta_{\ell_3 m_3}\rangle \neq 0$ because of $\langle \Theta^{(1)}\Theta^{(1)}\Theta^{(2)}\rangle$.

Evolution of the distribution function of radiation

Boltzmann equation

$$L[f] = C[f]$$

- Liouville operator: Free-fall

$$L[f] = \frac{\mathrm{d}f}{\mathrm{d}s} = p^c \nabla_c f(x, p^a) + \frac{\partial f(x, p^a)}{\partial p^c} \frac{\mathrm{d}p^c}{\mathrm{d}s}$$

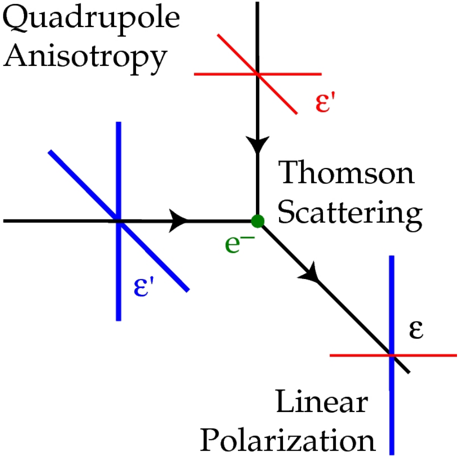
  Geodesic equation

$$p^b \nabla_b p_a = \frac{\mathrm{d}p_a}{\mathrm{d}s} + \omega_{bac} p^c p^b = 0$$

- Collision operator: Compton scattering on free electrons.

Why do we also need to describe polarization?

Because if radiation has a quadrupole,
Compton scattering generates polarisation.



Description of polarisation by the Stokes parameters

Tensorial distribution function

If $n^i = (0,0,1)$: $f_{ab} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & I+Q & U+\mathrm{i}V & 0 \\ 0 & U-\mathrm{i}V & I-Q & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Covariant expression

$$f_{\mu\nu}(x,p^a) \equiv \frac{1}{2} I(x,p^a) S_{\mu\nu} + P_{\mu\nu}(x,p^a) + \frac{\mathrm{i}}{2} V(x,p^a) e^{\rho}_{o} \epsilon_{\rho\mu\nu\sigma} n^{\sigma}$$

Multipolar expansion

Multipoles for scalar functions (I and V)

$$I(x, p^o, n^a) = \sum_{\ell=0}^{\infty} I_{\underline{a_\ell}}(x, p^o) n^{\underline{a_\ell}}$$

And for polarisation, E and B modes...

$$P_{ab}(x, p^a) = \sum_{\ell=2}^{\infty} \left[E_{ab\underline{c_{\ell-2}}}(x, p^o) n^{\underline{c_{\ell-2}}} - n_c \epsilon^{cd}{}_{(a} B_{b)d\underline{c_{\ell-2}}}(x, p^o) n^{\underline{c_{\ell-2}}} \right]^{\mathrm{TT}}$$

Steps to follow

1. Perturb the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + g^{(1)}_{\mu\nu} + \frac{1}{2} g^{(2)}_{\mu\nu}$
2. Perturb the tetrad $e_a^\mu = \bar{e}_a^\mu + e_a^{(1)\mu} + \frac{1}{2} e_a^{(2)\mu}$
3. Perturb the connections $\omega_{abc} = \bar{\omega}_{abc} + \omega^{(1)}_{abc} + \frac{1}{2}\omega^{(2)}_{abc}$
4. Find the perturbed geodesic equations
5. Compute the perturbed Liouville operator
6. Compute the Thomson scattering for each electron
7. Sum over the electrons distribution to obtain the Collision tensor in full generalities
8. Expand it in perturbations
9. Take the multipoles $I_{\underline{a_\ell}}$ $E_{\underline{a_\ell}}$ and $B_{\underline{a_\ell}}$ of the Boltzmann equation
10. Solve it or integrate it numerically

Evolution of brightness $\mathcal{I} = T^4$ along geodesics

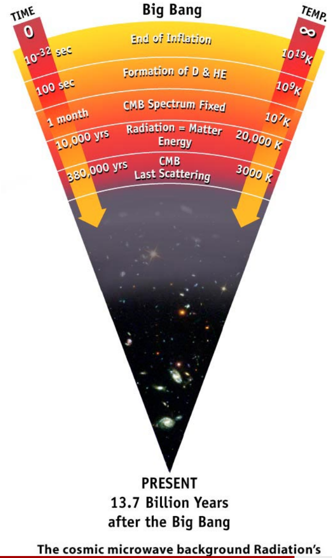
$$\frac{\mathrm{d}\left[e^{-\bar{\tau}}\mathcal{I}E^{-4}\right]}{\mathrm{d}\eta} = \bar{g}(\eta)E^{-4}\left[e^{\Phi}\mathcal{C}[\mathcal{I}] + \bar{\tau}'\mathcal{I}\right]$$

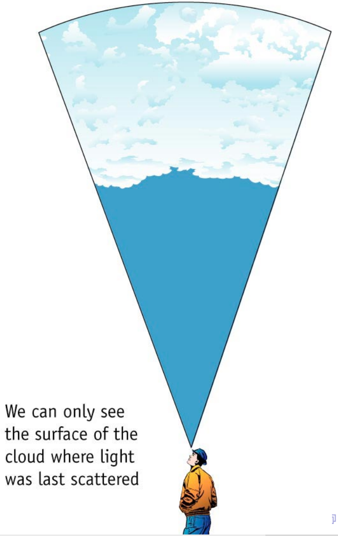
$$\frac{\mathrm{d}}{\mathrm{d}\eta} = \frac{\partial}{\partial\eta} + \frac{\mathrm{d}x^l}{\mathrm{d}\eta}\frac{\partial}{\partial x^l} + \frac{\mathrm{d}n^i}{\mathrm{d}\eta}\frac{\partial}{\partial n^i}\,, \qquad \frac{\mathrm{d}\ln E}{\mathrm{d}\eta} \simeq -\frac{\mathrm{d}\Phi}{\mathrm{d}\eta} + \Phi' + \Psi'$$

Classification of effects:

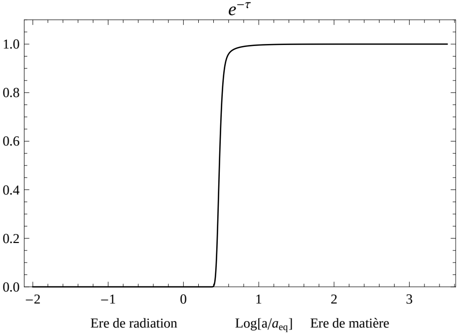
- 1) $\mathrm{d}E/\mathrm{d}\eta$: Evolution of the energy of photons:
  Einstein effect (potential Φ) and *integrated effects*
- 2) $\bar{g}(\eta)\ldots$ Collisions on the last scattering surface (LSS):
  Intrinsic temperature, and *Doppler effect*.
- 3) *Lensing effect* $\delta\left(\frac{\mathrm{d}n^i}{\mathrm{d}\eta}\right)$ **Order 2**
- 4) *Time-delay* $\delta\left(\frac{\mathrm{d}x^l}{\mathrm{d}\eta}\right)$ **Order 2**

Geometry of the background space-time



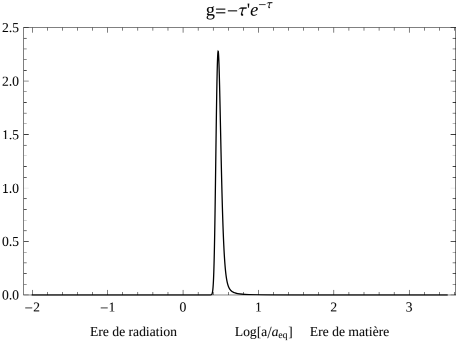


Integrated effects efficient since recombination

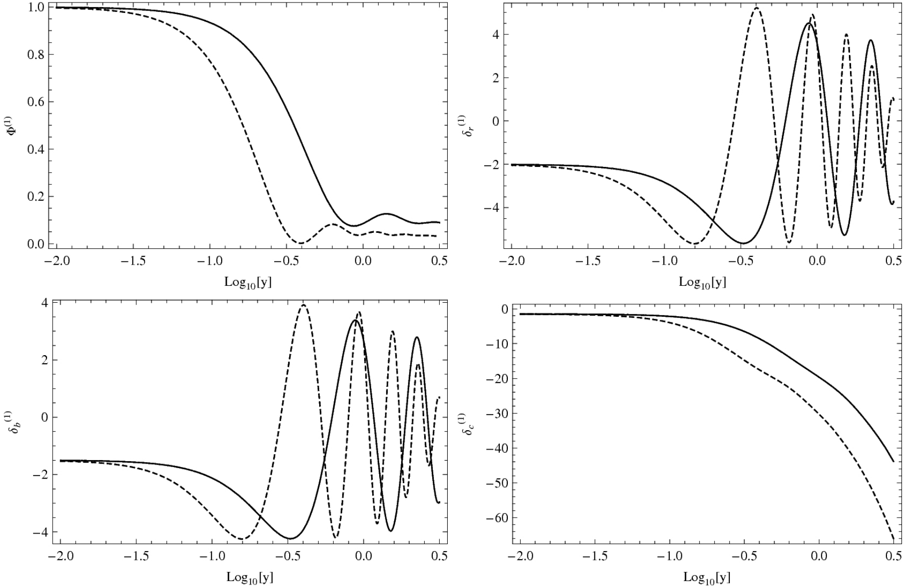


In the width of the LSS

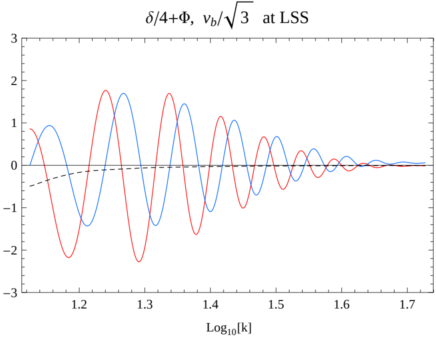
Intrinsic Θ, Φ, Doppler

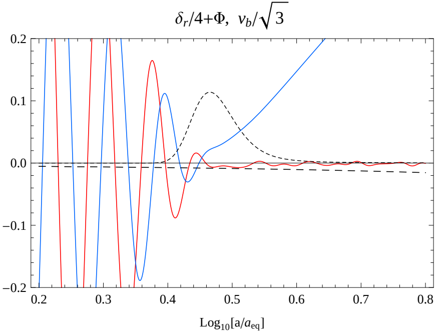


Linear evolution of perturbations (order 1)



Perturbations on the LSS





- Red: Intrinsic Θ and Einstein effect
- Blue: Doppler effect

Linear response of radiation

$$\left[(1+R)\frac{\delta_r^{(1)\prime}}{4}\right]' + \mathrm{visc} + \frac{k^2}{3}\frac{\delta_r^{(1)}}{4} \simeq -\frac{k^2}{3}(1+R)\Phi^{(1)} + \left[(1+R)\Phi^{(1)\prime}\right]'$$

with $R = 3\bar{\rho}_b/(4\bar{\rho}_r)$.

Because of viscosity, at small scale:

$\Theta^{(1)} \simeq \frac{\delta_r^{(1)}}{4} + \Phi^{(1)} \to -R\Phi^{(1)}$

Dynamics of primary effects

Non-linear evolution (order 2)

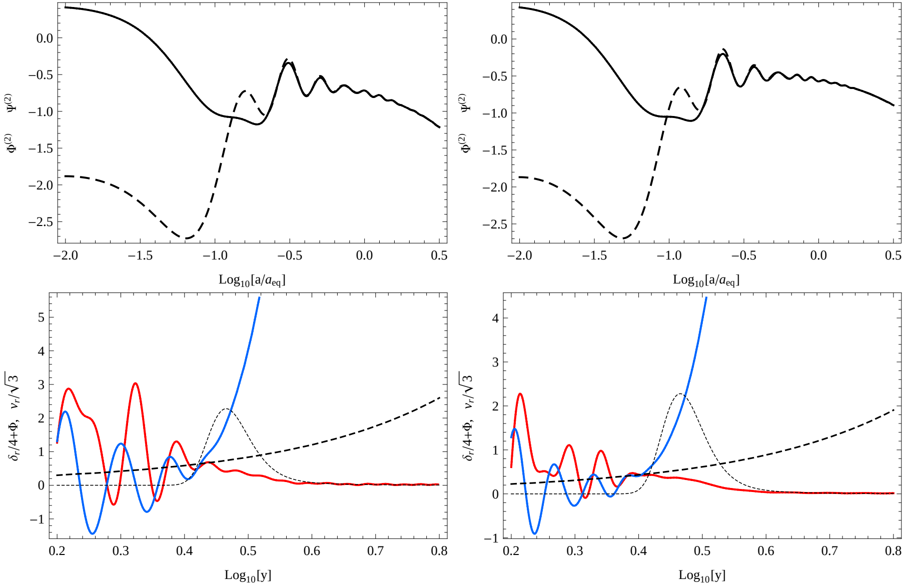
$$\left[(1+R)\frac{\delta_r^{(2)\prime}}{4}\right]' + \text{visc} + \frac{k^2}{3}\frac{\delta_r^{(2)}}{4} \simeq$$
$$-\frac{k^2}{3}(1+R)\Phi^{(2)} + \left[(1+R)\Phi^{(2)\prime}\right]' + \text{quadr}$$

Behaviour on small scales (viscosity)

$$\Theta^{(2)\text{intr}} \simeq \frac{\delta_r^{(2)}}{4} + \Phi^{(2)} \rightarrow -R\Phi^{(2)}$$

Potential created by the collapse of cold dark matter

$$\frac{1}{2}\Phi^{(2)}(\mathbf{k},\eta) \simeq -\frac{1}{6}K(\mathbf{k}_1,\mathbf{k}_2)\left(\frac{k_1 k_2 \eta}{k}\right)^2 \Phi(k_1)\Phi(k_2), \qquad (1)$$


$\Psi^{(2)}$
$\Phi^{(2)}$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−2.0 −1.5 −1.0 −0.5 0.0 0.5
$\mathrm{Log}_{10}[a/a_{\mathrm{eq}}]$
$\delta_r/4+\Phi,\ v_r/\sqrt{3}$
5 4 3 2 1 0 −1
0.2 0.3 0.4 0.5 0.6 0.7 0.8
$\mathrm{Log}_{10}[y]$

Estimator of primordial non-Gaussianity

- We build an estimator for $f_{\rm NL}$ using all $\langle \Theta_{\ell_1 m_1} \Theta_{\ell_2 m_2} \Theta_{\ell_3 m_3} \rangle$ up to $\ell_{\rm max}$ without taking into account the non-linear dynamics.
- If $f_{\rm NL} = 0$, what measures this estimator? Answer : $f_{\rm NL}^{\rm eq}$

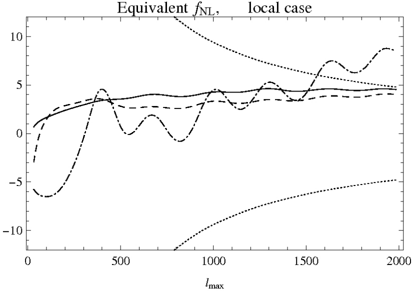
Observational constraints on $f_{\rm NL}$ (WMAP-5)

- Local NG: $-9 < f_{\rm NL} < 111$
- Equilateral NG: $-150 < f_{\rm NL} < 253$

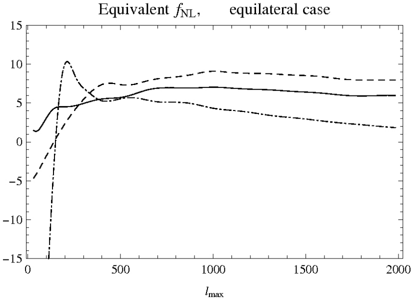
Planck is going to increase $\ell_{\rm max}$, thus reducing the cosmic variance limitation, thus increasing the precision.

Bispectrum generated by primary effects

Local NG



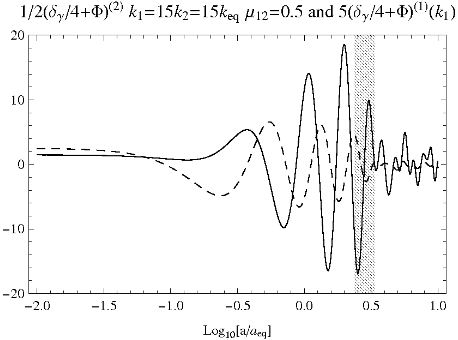
Equilateral NG

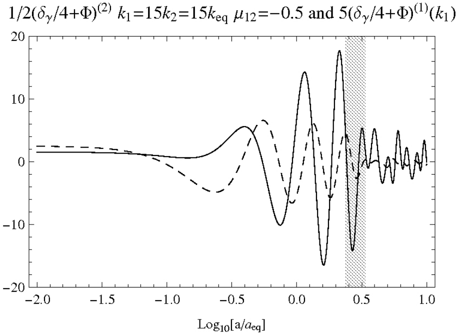


Conclusion

- Secondary effect add up (same sign)
- Non-linear evolution has to be taken into account in future constraints on non-Gaussianity.

Thanks a lot for your attention.


$1/2(\delta_\gamma/4+\Phi)^{(2)}$ k_1=15k_2=15k_{eq} μ_{12}=0.5 and $5(\delta_\gamma/4+\Phi)^{(1)}(k_1)$
20
10
0
−10
−20
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
$\mathrm{Log}_{10}[a/a_{eq}]$


$1/2(\delta_\gamma/4+\Phi)^{(2)}$ k_1=15k_2=15k_{eq} μ_{12}=−0.5 and $5(\delta_\gamma/4+\Phi)^{(1)}(k_1)$
20
10
0
−10
−20
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
$\mathrm{Log}_{10}[a/a_{eq}]$