A study of high-order Non-Gaussianity

with applications to clusters and voids

Sirichai Chongchitnan

based on 1007.1230 (ApJ)

with Joseph Silk

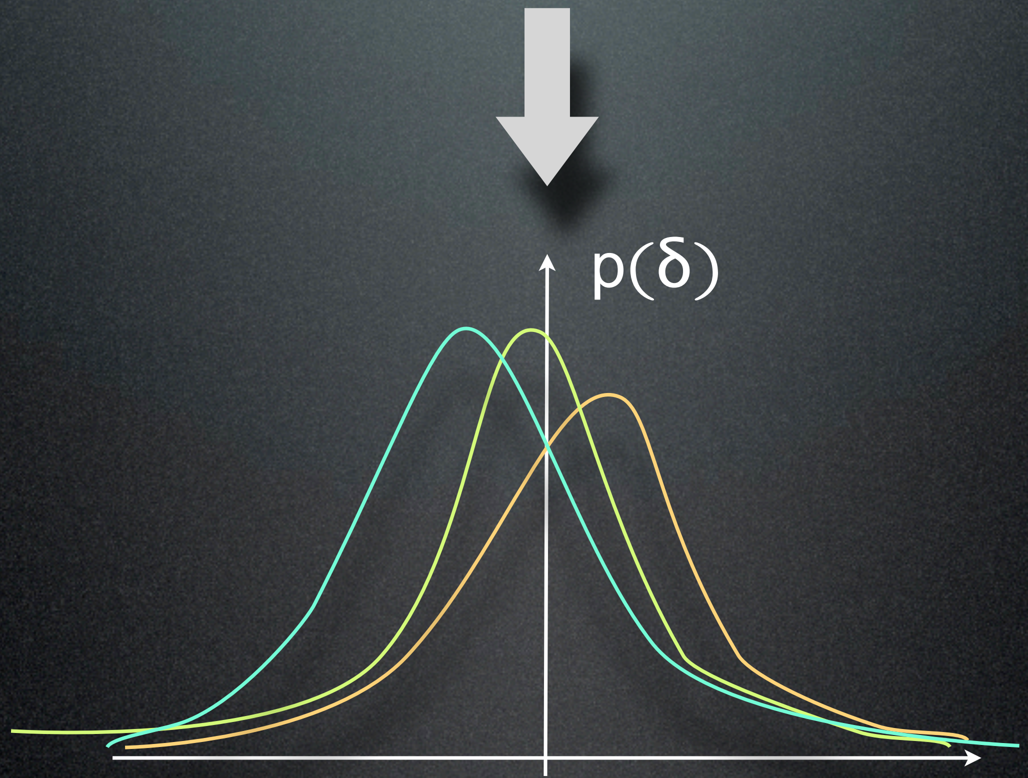
UNIVERSITY OF OXFORD

f_{NL}, g_{NL}

↓

Non-Gaussian pdf

$$\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\rm NL}\left(\phi^2(\mathbf{x}) - \langle\phi^2\rangle\right) + g_{\rm NL}\phi^3(\mathbf{x}) + \ldots .$$



Edgeworth Expansion

Non-Gaussian pdf $=$ Gaussian$\times$(1+ deviation)

Edgeworth Expansion

Non-Gaussian pdf = Gaussian×(1+ deviation)

Deviation = cumulants S_3 S_4 S_5 ....

$$S_n(R) \equiv \frac{\langle \delta_R^n \rangle_c}{\sigma_R^{2n-2}},$$

Edgeworth Expansion

linear

$$p(x) = N(x)\left[1 + \frac{\sigma_R S_3}{6}(x^3 - 3x)\right],$$

Edgeworth Expansion

linear

$$p(x) = N(x)\left[1 + \frac{\sigma_R S_3}{6}(x^3 - 3x)\right],$$

quadratic

$$p(x) = N(x)\left[1 + \frac{\sigma_R S_3}{6} H_3(x) + \sigma_R^2 \left(\frac{S_4}{24} H_4(x) + \frac{S_3^2}{72} H_6(x)\right)\right].$$

Edgeworth Expansion

linear

$$p(x) = N(x)\left[1 + \frac{\sigma_R S_3}{6}(x^3 - 3x)\right],$$

quadratic

$$p(x) = N(x)\left[1 + \frac{\sigma_R S_3}{6} H_3(x) + \sigma_R^2 \left(\frac{S_4}{24} H_4(x) + \frac{S_3^2}{72} H_6(x)\right)\right].$$

general

$$p(\delta_R) = N(\delta_R)\left[1 + \sum_{s=1}^{\infty} \sigma_R^s E_s\left(\frac{\delta_R}{\sigma_R}\right)\right],$$

[Petrov 1975]

‘Skewness’ and f_{NL}

$$\sigma^4 S_3(R) = f_{\rm NL} \int_0^\infty \frac{dk_1}{k_1} A(k_1)\mathcal{P}(k_1) \int_0^\infty \frac{dk_2}{k_2} A(k_2)\mathcal{P}(k_2) \int_{-1}^{1} d\mu \, A(k_3) \left[1 + 2\frac{P_\phi(k_3)}{P_\phi(k_2)}\right],$$

$$S_3 \simeq \frac{3.14 \times 10^{-3} \times f_{\rm NL}}{\sigma_R^{0.835}},$$

‘Kurtosis’ and g_{NL}

$$\langle \delta_R^4 \rangle_c = \frac{3}{4\pi} g_{\rm NL} \left(\prod_{i=1}^{3} \int_0^\infty \frac{dk_i}{k_i} \, A(k_i) \mathcal{P}_\phi(k_i) \right) \int_{-1}^{1} d\mu_1 \int_{-1}^{1} d\mu_2 \int_0^{2\pi} d\phi \, A(k_4) \left[1 + \frac{P_\phi(k_4)}{P_\phi(k_1)} + \frac{P_\phi(k_4)}{P_\phi(k_2)} + \frac{P_\phi(k_4)}{P_\phi(k_3)} \right],$$

$$S_4 \simeq \frac{1.14 \times 10^{-5} \times g_{\rm NL}}{\sigma_R^{1.27}},$$

[see Desjacques+Seljak]

Q: If I know f_{NL}, g_{NL} , can I reconstruct the pdf ?

Q: If I know f_{NL}, g_{NL} , can I reconstruct the pdf ?

No.

Moment problem has no solution

f_{NL}, g_{NL}

Non-Gaussian pdf

f_{NL}, g_{NL}

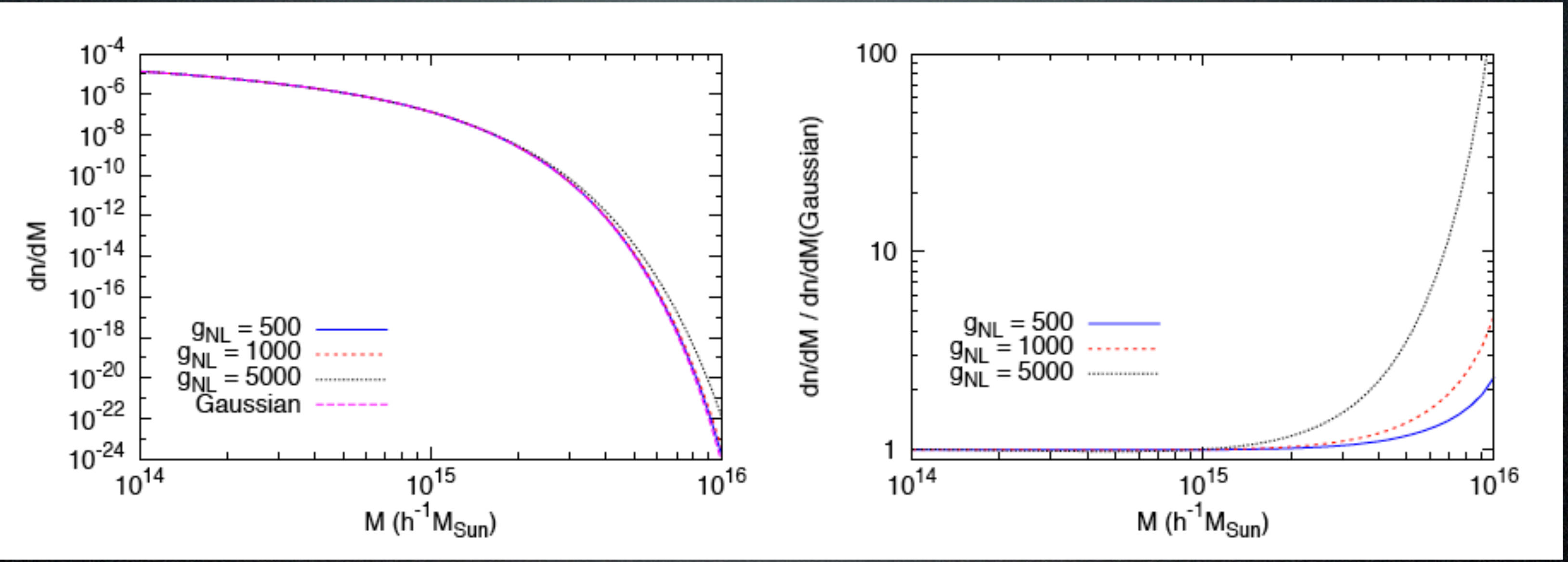
↓

- Assume S_5, S_6 etc = 0
- Use a truncated Edgeworth expansion - but downgrade statistical meaning of f_{NL}, g_{NL}

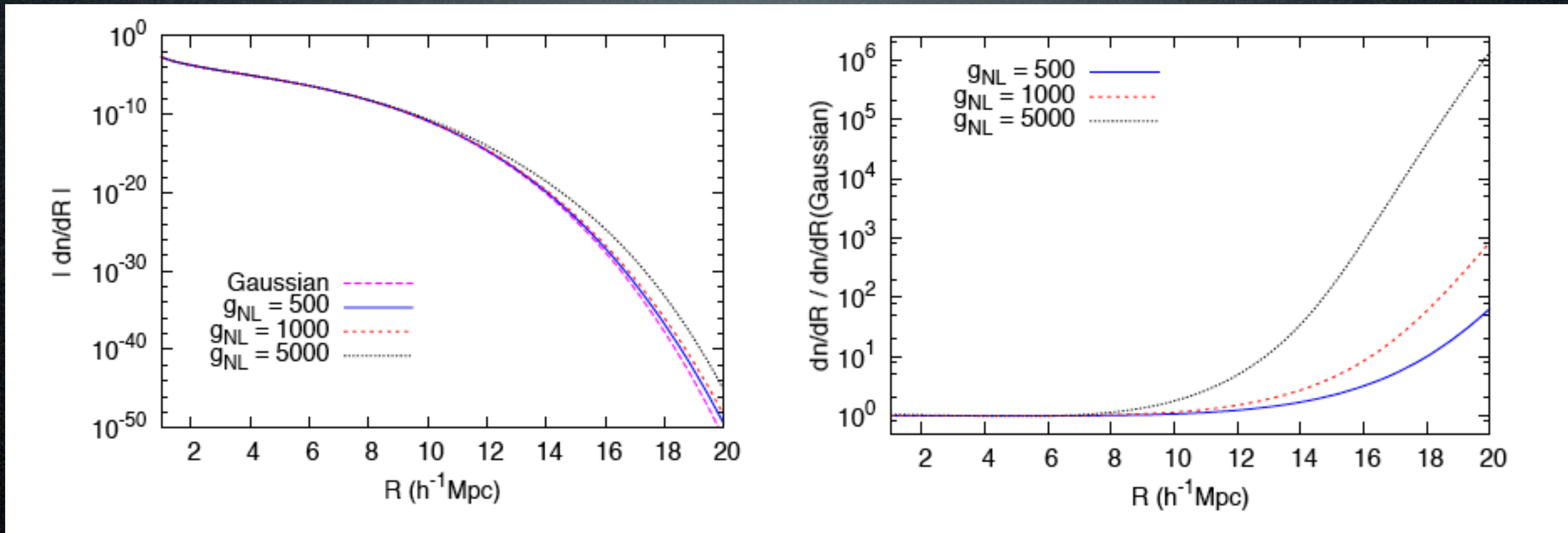
↓

Non-Gaussian pdf

g_{NL} and clusters



g_{NL} and voids



Lower bound on g_{NL}

$$\sigma^2 S_4 \geq -2.$$

see our proof, or Feller's *Probability*

Lower bound on g_{NL}

$$\sigma^2 S_4 \geq -2.$$

see our proof, or Feller's *Probability*

$$g_{\rm NL}^{\rm LSS} \gtrsim -1.2 \times 10^5,$$

$$g_{\rm NL}^{LSS} > -3.5 \times 10^5$$ [Desjacques & Seljak]

$$g_{\rm NL}^{CMB} > -5.6 \times 10^5$$ [Vielva & Sanz]

Work in progress: Bias

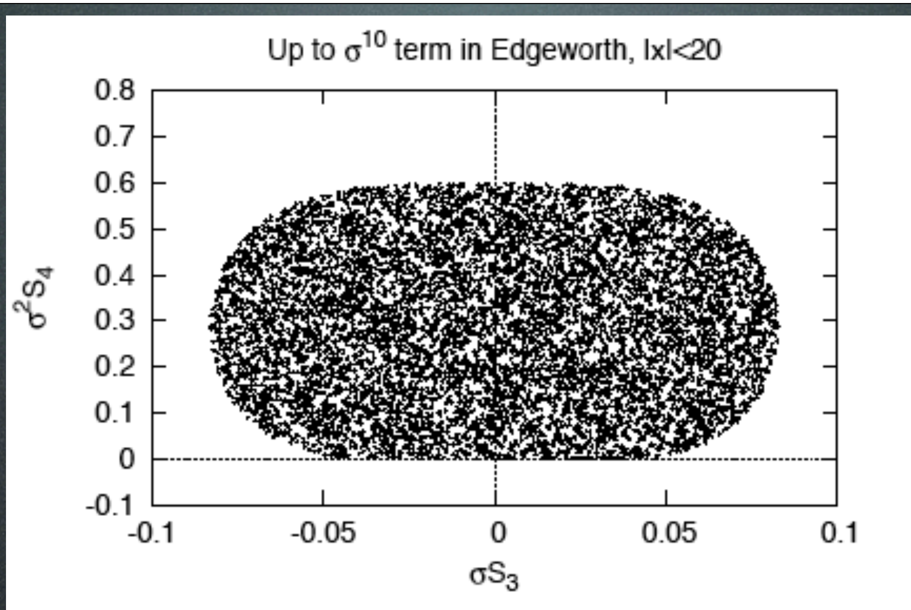
$$p(x_R, x'_R) = N(x_R, x'_R)\left[1 + \sum_{s=1}^{\infty} \sum_{\{P_m\}} \sum_{\{p_i, q_i, \pi_i\}} F\left(x_R, x'_R\right)\right],$$

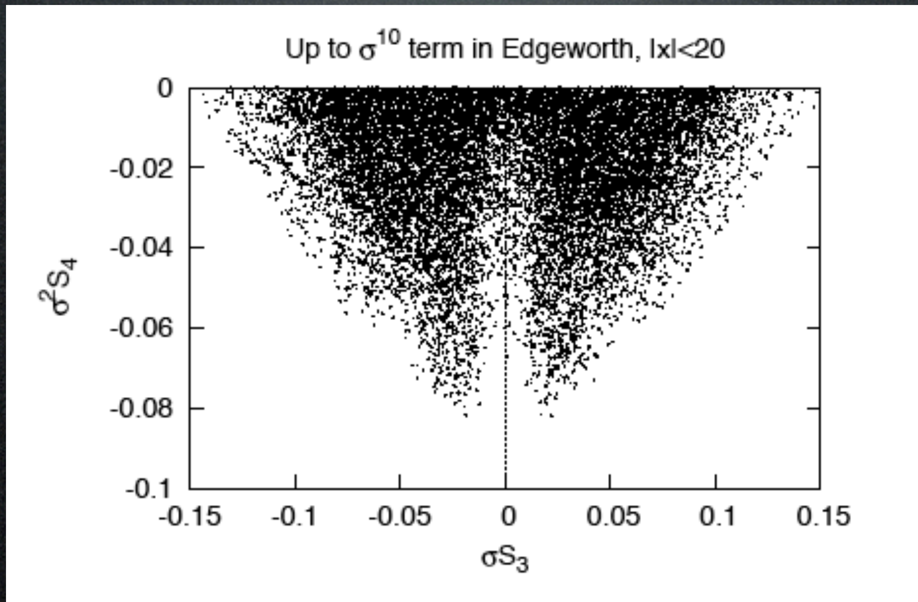
$$\xi_{\rm pk} = \frac{P_2}{P_1^2} - 1.$$

$$b^2 = \frac{\xi_{\rm pk}}{\xi(r)},$$

Conclusions

- Clarified $f_{NL}, g_{NL} \longrightarrow$ pdf link via Edgeworth series.
- Easy-to-use formulae linking f_{NL}, g_{NL} to cumulants.
- Effects of g_{NL} on cluster and void abundances.
- Look out for our follow-up papers.


Up to σ^10 term in Edgeworth, IxI<20
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
σ^2S_4
-0.1
-0.05
0
0.05
0.1
σS_3


Up to σ^10 term in Edgeworth, IxI<20
0
-0.02
-0.04
-0.06
-0.08
-0.1
σ^2S_4
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
σS_3