

Non-Gaussianity from Preheating

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27 September 2010

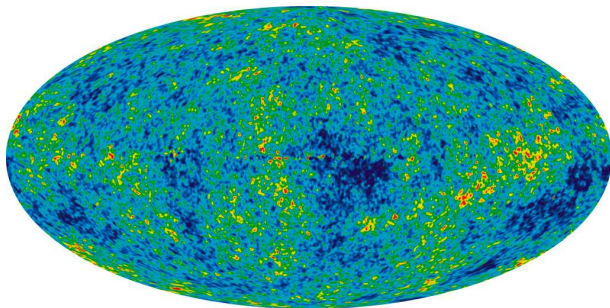
A.Chambers & AR, PRL100(2008)041302

A.Chambers & AR, JCAP08(2008)002

A.Chambers, S.Nurmi & AR, JCAP01(2010)012

$$\frac{\partial}{\partial a} \ln f_{a,\sigma^2}(\xi_i) = \frac{(\xi_i - a)}{\sigma^2} f_{a,\sigma^2}(\xi_i) - \frac{1}{2\sigma^2} \ln \left(\frac{1}{2\pi\sigma^2} \right)$$
$$\int \mathcal{T}(x) \frac{\partial}{\partial \theta} f(x, \theta) dx = \int \mathcal{T}(x) \frac{\partial}{\partial \theta} \ln f(x, \theta) f(x, \theta) dx$$

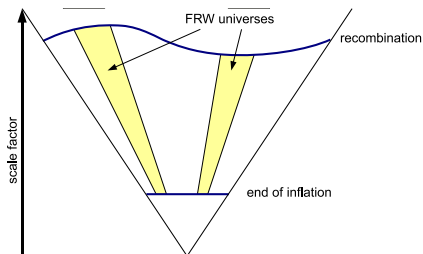
Perturbations from Non-Equilibrium Fields



- Light scalar fields \Rightarrow Nearly scale invariant perturbations
- Non-equilibrium field evolution \Rightarrow Non-Gaussian contribution
- Sensitive test of models – But how can we calculate it?

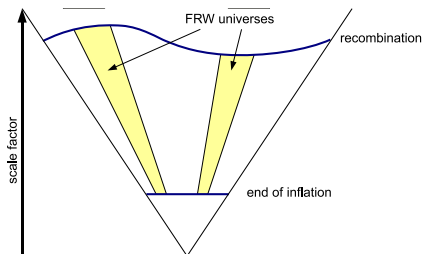
$$\frac{\partial}{\partial a} \ln f_{a,\sigma^2}(\xi_i) = \frac{(\xi_i - a)}{\sigma^2} f_{a,\sigma^2}(\xi_i) - \frac{1}{2\sigma^2} \int_{-\infty}^{\infty} \tau(x) \frac{\partial}{\partial \theta} f(x, \theta) dx - \frac{1}{2} \left(\frac{\partial}{\partial \theta} \right)^2 \ln L(\theta)$$

Separate Universes



- Each Hubble patch \sim separate FRW universe (Salopek&Bond 1990)
- Curvature perturbation $\zeta = \delta N = \delta \ln a|_{\rho=\rho_*}$
- Valid at distances $d \gg 1/H$

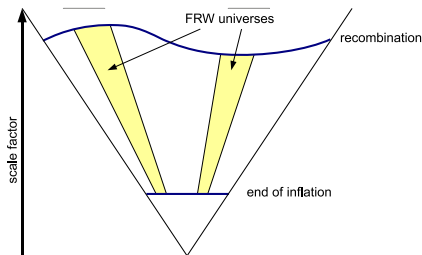
Separate Universes



- Curvature perturbation $\zeta = \delta N = \delta \ln a|_{\rho=\rho_*}$
- Conserved if $w = w(\rho)$:

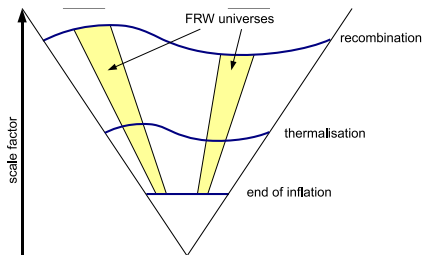
$$\frac{d \ln a}{d \rho} = - \frac{1}{3\rho(1 + w(\rho))}$$

Light Scalar Field χ



- $m_\chi < H \Rightarrow$ Gaussian scale-invariant perturbations
- Different "universes" have different initial value χ_0
- Affects expansion \Rightarrow Scale factor depends on χ_0

Light Scalar Field χ



- Thermalisation erases memory of $\chi_0 \Rightarrow w = w(\rho)$
- Curvature perturbation determined at thermalisation:

$$\zeta_{\text{rec}} = \zeta_{\text{therm}} = \delta \ln a|_{\rho=\rho_{\text{therm}}}$$

Calculating the Curvature Perturbation

- Solve Friedmann eq. for each separate universe $\Rightarrow a(t), \rho(t)$
 - Non-linear, includes gravity, valid at $d \gg H^{-1}$
- Pick $\rho_* < \rho_{\text{therm}}$, and calculate $\zeta = \delta \ln a|_{\rho=\rho_*}$
 - Two fields: $\zeta = \zeta(\phi_0, \chi_0)$
 - $\delta\phi_0 \Leftrightarrow$ Shift in time:
Usual inflationary perturbations
 - $\delta\chi_0$: New contribution
- Need to calculate $N(\chi_0) = \ln a(\chi_0)$
 - Perturbative approach: Taylor expand

$$\zeta = \zeta_\phi + \frac{\partial N}{\partial \chi_0} \delta\chi_0 + \frac{1}{2} \frac{\partial^2 N}{\partial \chi_0^2} \delta\chi_0^2 + \dots$$

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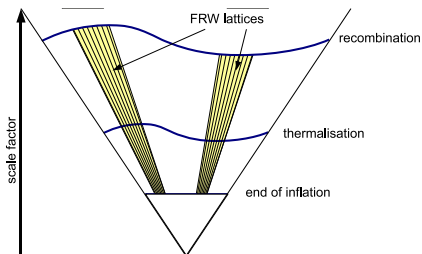
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Usual inflationary perturbations
 - $\delta\chi_0$: New contribution
 - Need to calculate $N(\chi_0) = \ln a(\chi_0)$
 - Instead: Solve field and Friedmann eqs numerically
 - Solve for many different initial values χ
- \Rightarrow Whole non-linear function $N(\chi_0)$ (Bassett&Tanaka 2003, Suyama&Yokoyama 2006)

$$\frac{\partial}{\partial a} \ln f_{a,\sigma^2}(\xi_i) = \frac{(\xi_i - a)}{\sigma^2} f_{a,\sigma^2}(\xi_i) - \frac{1}{2\sigma^2} \ln f_{a,\sigma^2}(\xi_i)$$

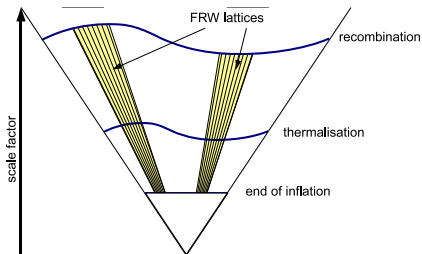
$$\int \mathcal{T}(x) \frac{\partial}{\partial \theta} f(x, \theta) dx = \mathcal{N} \left(\mathcal{T}(x) \frac{\partial}{\partial \theta} \ln f(x, \theta) \right)$$

Lattice Calculation of Curvature Perturbations



- Describe each "universe" as a lattice (Chambers&AR 2007)
- Inhomogeneous fields, FRW metric
- Lattice size $L \sim 1/H$

Lattice Calculation of Curvature Perturbations



- Solve field evolution on lattice, coupled to Friedmann eq with average ρ
- Find curvature perturbation as $\zeta(\chi_0) = \delta \ln a(\chi_0)|_{\rho=\rho_*}$, where χ_0 is the lattice average

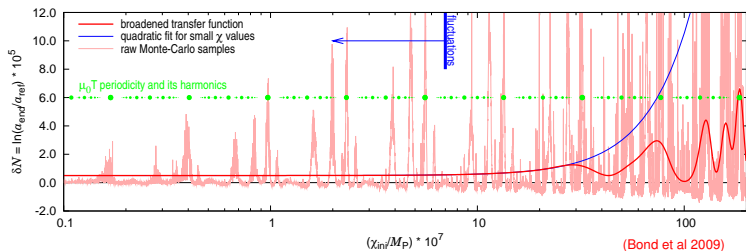
Initial Conditions

- Sub-horizon modes: Quantum vacuum
 - Gaussian classical fluctuations with (Khlebnikov&Tkachev 1996)

$$\overline{|\chi_k|^2} = \frac{1}{V} \frac{1}{2\omega_k}, \quad \overline{|\dot{\chi}_k|^2} = \frac{1}{V} \frac{\omega_k}{2}$$

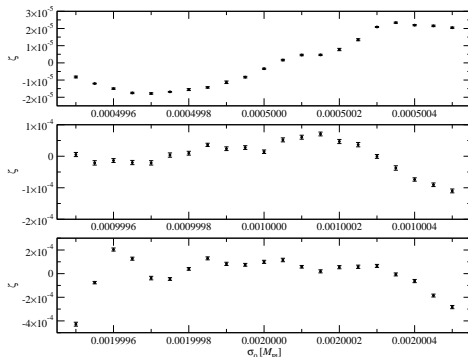
- Linear dynamics: quantum = classical
 - Non-linear dynamics: quantum \approx classical
- Mean value \leftrightarrow Super-horizon modes
 - Initial field value χ_0
 - Input parameter – we are calculating $\zeta(\chi_0)$!

Massless Preheating



- Chaotic behaviour: Highly non-Gaussian (Chambers&AR 2007)
- Not well approximated by a quadratic f_{NL}

Curvaton Resonance



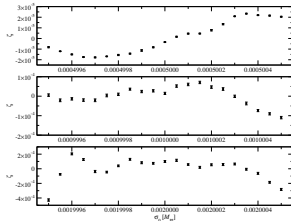
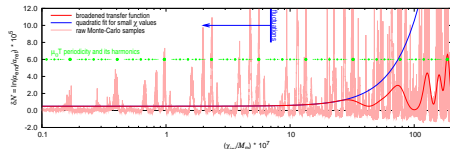
- Curvaton decays through parametric resonance (Enqvist et al 2009)
- Again, not well described by f_{NL} (Chambers, Nurmi&AR 2009)

$$\frac{\partial}{\partial a} \ln f_{a,\sigma^2}(\xi_i) = \frac{(\xi_i - a)}{\sigma^2} f_{a,\sigma^2}(\xi_i) - \frac{1}{2\sigma^2} \left[\frac{\partial^2}{\partial \xi_i^2} \ln f_{a,\sigma^2}(\xi_i) \right]$$

$$\int \mathcal{T}(x) \frac{\partial}{\partial \theta} f(x, \theta) dx = \mathcal{M} \left(\mathcal{T}(\xi) \frac{\partial}{\partial \xi} \ln f_{a,\sigma^2}(\xi) \right)$$

Conclusions

- Non-linear calculation of curvature perturbation due to non-equilibrium physics
 - Separate universes + lattice field theory
 - Works with any (bosonic) field dynamics
- Massless preheating, Curvaton resonance:
 - Possibly observable effects $\Delta\zeta \sim 10^{-5}$
 - Highly non-linear
 - How should we look for these signals?



$$\frac{\partial}{\partial a} \ln f_{a,\sigma^2}(\xi_i) = \frac{(\xi_i - a)}{\sigma^2} f_{a,\sigma^2}(\xi_i) - \frac{1}{2\sigma^2} \ln f_{a,\sigma^2}(\xi_i)$$

$$\int \mathcal{T}(x) \frac{\partial}{\partial \theta} f(x, \theta) dx = \mathcal{M} \left(\mathcal{T}(\xi) \frac{\partial}{\partial \theta} f(\xi, \theta) \right) \int f_{a,\sigma^2}(\xi_i)$$