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Trispectrum estimator in equilateral type non-Gaussian models

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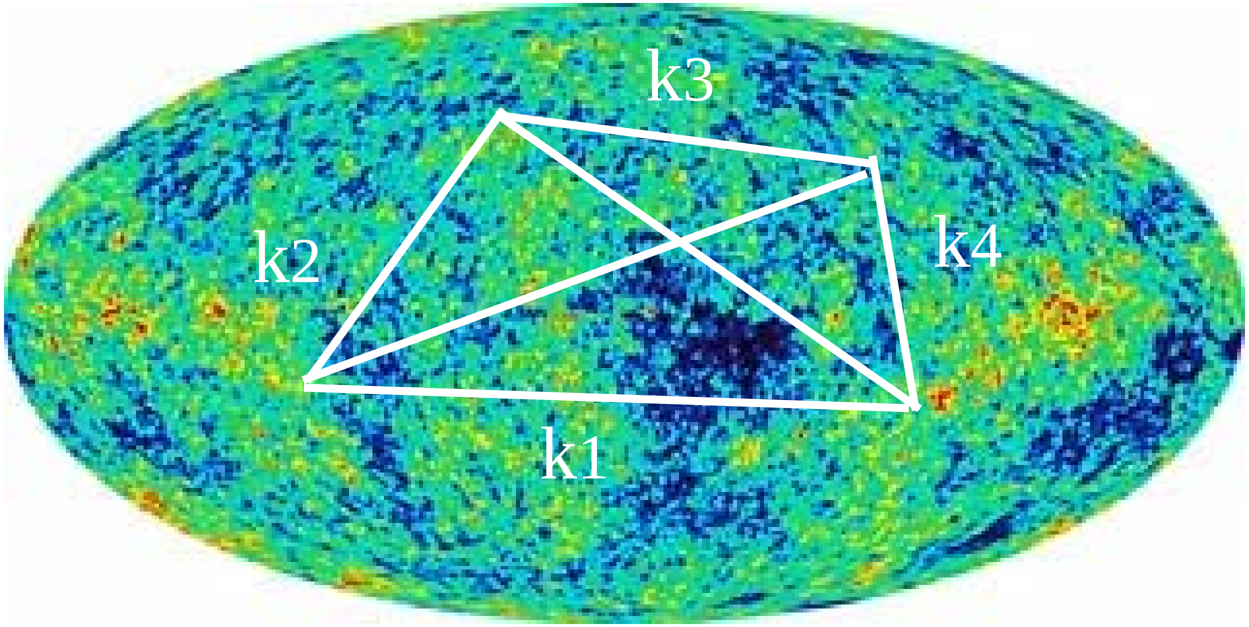
Primordial Trispectrum

- Definition

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4)\rangle_c = (2\pi)^3\delta(k_1+k_2+k_3+k_4)T'_\zeta(k_1,k_2,k_3,k_4)$$

gives also important information !!



Takahashi

Kobayashi

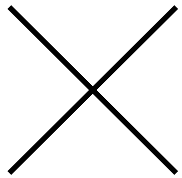
Izumi

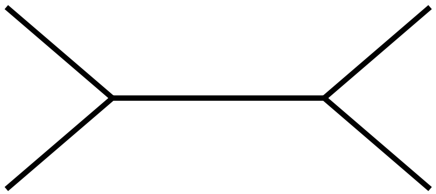
$$\langle\zeta^n\rangle \sim \frac{1}{c_s^{3n-5}} \quad \Rightarrow \quad f_{NL} \sim \frac{1}{c_s^2} \quad \tau_{NL} \sim \frac{1}{c_s^4}$$

Primordial Trispectrum in equilateral type model

Ex)
- single field DBI — Arroja, SM, Koyama, Tanaka `09
- multi-field DBI — SM, Arroja, Koyama `09

In-in formalism





Includes complicated functions like

$$
\begin{aligned}
\tilde{\mathcal{F}}_2(k_1,k_2,k_3,k_4,k_5,k_6) \;=\; & -\frac{|k_1k_4|^{\frac{1}{2}}}{|k_2k_3k_5k_6|^{\frac{3}{2}}}\frac{1}{\mathcal{A}\mathcal{C}}\Bigg[1+\frac{k_5+k_6}{\mathcal{A}}+2\frac{k_5k_6}{\mathcal{A}^2} \\
& +\frac{1}{\mathcal{C}}\left(k_2+k_3+k_5+k_6+\frac{1}{\mathcal{A}}\left((k_2+k_3)(k_5+k_6)+2k_5k_6\right)+2\frac{k_5k_6(k_2+k_3)}{\mathcal{A}^2}\right) \\
& +\frac{2}{\mathcal{C}^2}\left(k_5k_6+(k_2+k_3)(k_5+k_6)+k_2k_3+\frac{1}{\mathcal{A}}\left(k_2k_3(k_5+k_6)+2k_5k_6(k_2+k_3)\right)\right. \\
& \left.+2\frac{k_2k_3k_5k_6}{\mathcal{A}^2}\right)+\frac{6}{\mathcal{C}^3}\left(k_2k_3(k_5+k_6)+k_5k_6(k_2+k_3)+2\frac{k_2k_3k_5k_6}{\mathcal{A}}\right) \\
& +24\frac{k_2k_3k_5k_6}{\mathcal{C}^4}\Bigg],
\end{aligned}
$$

practically, not so useful as a theoretical template !!

Bispectrum estimator

- Estimator for general case model

theory

observation

$$\mathcal{E} = \frac{1}{\mathcal{N}^2} \sum_{l_i m_i} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \frac{B_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}} a^{obs}_{l_1 m_1} a^{obs}_{l_2 m_2} a^{obs}_{l_3 m_3}$$

calculation time $N_{pix}^{5/2}$ in general

Komatsu, Spergel `01

- Estimator for local type model

$$B_\zeta(k_1, k_2, k_3) \propto [P_\zeta(k_1) P_\zeta(k_2) + \text{perms.}]$$

separable form

$$\mathcal{E} = \frac{1}{\mathcal{N}^2} \int d^3x A(x) \left(B(x)\right)^2$$

with

$$\begin{cases} A(x) = \sum_{l,m} \frac{2}{\pi} \int k^2 dk \Delta_l(k) j_l(kx) \frac{a^{obs}_{lm}}{c_l} Y_{lm}(\hat{x}) \\ B(x) = \sum_{l,m} \frac{2}{\pi} \int k^2 dk \frac{25}{9} P_\zeta(k) \Delta_l(k) j_l(kx) \frac{a^{obs}_{lm}}{c_l} Y_{lm}(\hat{x}) \end{cases}$$

→ calculation time $N_{pix}^{3/2}$

HEALPix

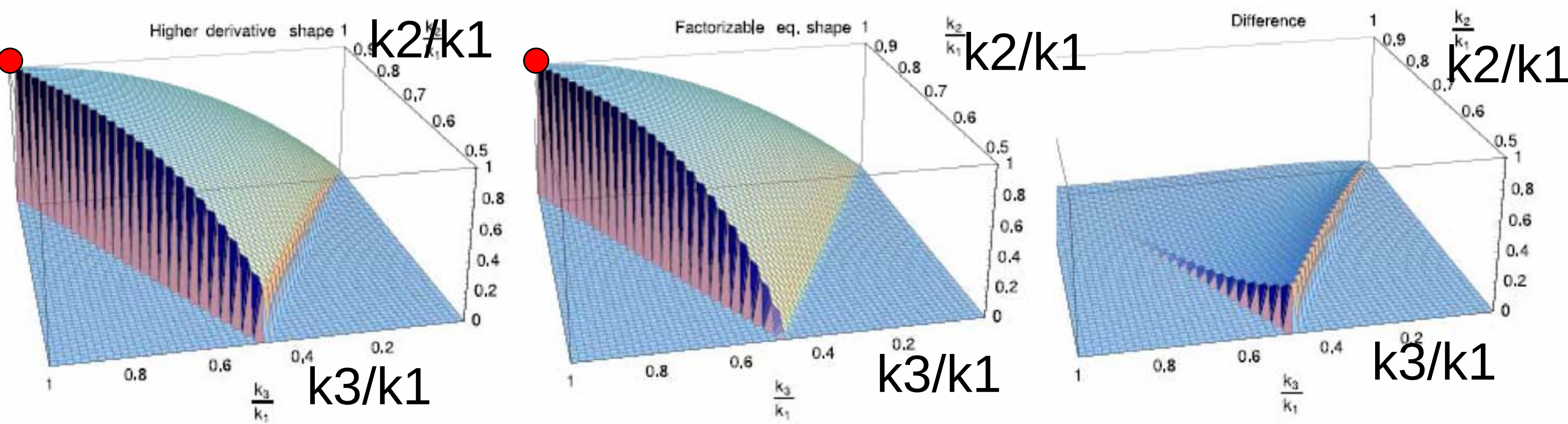
Bispectrum estimator in equilateral type model

- Separable equilateral shape function

Babichi, Creminelli, Zaldarriaga `04

$$F(k_1,k_2,k_3) = (2\pi)^4 \left(\frac{9}{10} f_{\rm NL}^{equil}\right) \left(-\frac{1}{k_1^3 k_2^3} - \frac{1}{k_1^3 k_3^3} - \frac{1}{k_2^3 k_3^3} - \frac{2}{k_1^2 k_2^2 k_3^2} + \frac{1}{k_1 k_2^2 k_3^3} + 5 \ \text{perms.}\right)$$

This template fits the bispectrum in DBI inflation very well !!



DBI　　Separable form　　Difference

matching the amplitudes at $k_1 = k_2 = k_3$ $\longrightarrow$ $f_{\rm NL}^{equil} = -\frac{35}{108}\frac{1}{c_s^2}$

Separable Trispectrum

Trispectrum in DBI inflation is not separable, but there is a quasi-separable and symmetric trispectrum.

- Quasi-separable trispectrum

$$T_\zeta(k_1,k_2,k_3,k_4) = \frac{g_{\rm NL}^{equil}}{k_1k_2k_3k_4(\frac{k_1+k_2+k_3+k_4}{4})^5}\mathcal{P}_\zeta(k)^3$$

- appears for k-inflation with $P_{,XXXX} \neq 0$
- not constrained by the bispectrum measurements

separable by introducing the integral

$$\frac{1}{M^n} = \frac{1}{\Gamma(n)}\int_0^\infty t^{(n-1)}e^{-Mt}$$

Chen et al `09

validity to use this shape as a template is nontrivial

Shape function

- Shape function motivated by CMB trispectrum

$$S_{\mathcal{T}}(k_1, k_2, k_3, k_4; k_{12}) = (k_1 k_2 k_3 k_4)^2 k_{12} \mathcal{T}_\zeta(k_1, k_2, k_3, k_4; k_{12})$$

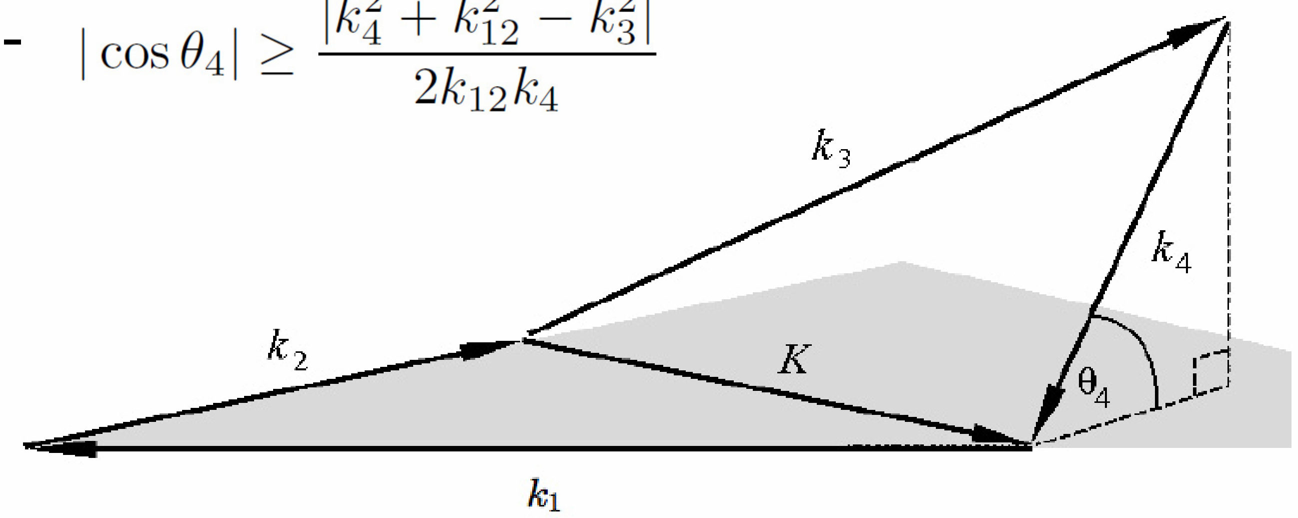
reduced trispectrum

configurations are specified by $(k_1, k_2, k_3, k_4, k_{12}, \theta_4)$

- Physical region should satisfy

-Triangle conditions for (k_1, k_2, k_{12}) and (k_3, k_4, k_{12})

- $|\cos\theta_4| \geq \dfrac{|k_4^2 + k_{12}^2 - k_3^2|}{2k_{12}k_4}$



Shape correlator

Regan, Shellard, Furgusson `10

- Overlap integral

$$F(S_{\mathcal{T}}, S_{\mathcal{T}'}) = \int d\mathcal{V}_k \int d(\cos\theta_4) S_{\mathcal{T}}(k_1, k_2, k_3, k_4, k_{12}, \theta_4) S_{\mathcal{T}'}(k_1, k_2, k_3, k_4, k_{12}, \theta_4) w$$

-integration is over physical region

-weight function motivated by the estimator in l space

$$w(k_1, k_2, k_3, k_4, k_{12}) = \frac{1}{k_{12}(k_1 + k_2 + k_{12})(k_3 + k_4 + k_{12})}$$

- Shape correlator

$$\bar{\mathcal{C}}(S_{\mathcal{T}}, S_{\mathcal{T}'}) = \frac{F(S_{\mathcal{T}}, S_{\mathcal{T}'})}{\sqrt{F(S_{\mathcal{T}}, S_{\mathcal{T}})F(S_{\mathcal{T}'}, S_{\mathcal{T}'})}}$$

⇒ We can check the validity of the estimator

Parametrisation of allowed region

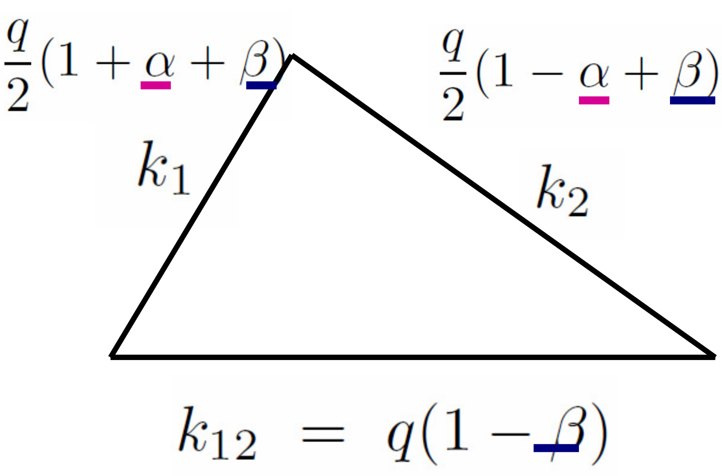
From the scaling behaviour, we can separate out the overall scale q, from the behaviour on a cross-sectional slice

$$S_{\mathcal{T}}(k_1,k_2,k_3,k_4,k_{12},\theta_4) = f(q)\bar{S}_{\mathcal{T}}\left(\frac{k_1}{q},\frac{k_2}{q},\frac{k_3}{q},\frac{k_4}{q},\frac{k_{12}}{q},\theta_4\right)$$

only this part is important

here, we set $q \equiv \frac{1}{2}(k_1 + k_2 + k_{12})$

- Intro. variables (α, β) for triangle (k_1, k_2, k_{12})



α : asymmetry between k_1 and k_2

$$-(1-\beta) \le \alpha \le 1-\beta$$

β : ratio between $k_1 + k_2$ and k_{12}

$$0 \le \beta \le 1$$

Parametrisation of allowed region (cont.)

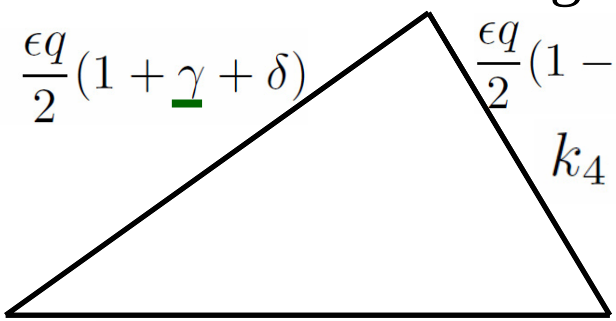
- Intro. variables $(\gamma, \delta, \epsilon)$ for triangle (k_3, k_4, k_{12})

$\epsilon = (k_3 + k_4 + k_{12})/(k_1 + k_2 + k_{12})$ ratio of the size of the two triangles

⇨ not lose the generality to consider $1 \leq \epsilon < \infty$

$$k_3 = \frac{\epsilon q}{2}(1 + \gamma + \delta)$$

$$\frac{\epsilon q}{2}(1 - \gamma + \delta)$$



k_4

$$k_{12} = \epsilon q(1 - \delta)$$

γ : asymmetry between k_3 and k_4

$$-\frac{1-\beta}{\epsilon} \leq \gamma \leq \frac{1-\beta}{\epsilon}$$

δ : given by $1 - \beta = \epsilon(1 - \delta)$

- necessary facts for the overlap integral

$$S_\mathcal{T}(k_1, k_2, k_3, k_4, k_{12}, \theta_4) = f(q)\bar{S}_\mathcal{T}(\alpha, \beta, \gamma, \epsilon, \theta_4),$$

$$w(k_1, k_2, k_3, k_4, k_{12}) = \frac{1}{4\epsilon(1-\beta)},$$

$$d\mathcal{V}_k = dk_1 dk_2 dk_3 dk_4 dK = \epsilon q^4 dq\, d\alpha\, d\beta\, d\gamma\, d\epsilon\,.$$

Results

	Overlap-full	$\epsilon = 1$	equilateral	theoretical prediction for $g_{\rm NL}^{equil}$	$f_{\rm NL}^{equil}$
equilateral shape	1	1	1	$(3X^3 P_{,4X})/(16 c_s P_{,X}^4)$	$f_{\rm NL}^{equil}$
single DBI	0.87	0.90	0.92	$17/c_s^4$	$-0.36/c_s^2$
multi DBI	0.33	0.60	0.85	$2.2/(c_s^4 T_{\mathcal{R}S}^2)$	$-0.36/(c_s^2 T_{\mathcal{R}S}^2)$

$$F(S_{\mathcal{T}}^{DBI(\sigma)}, S_{\mathcal{T}}^{DBI(\sigma)}) = F(S_{\mathcal{T}}^{equil}, S_{\mathcal{T}}^{equil}),$$
$$F(S_{\mathcal{T}}^{DBI(\sigma)}, S_{\mathcal{T}}^{equil}) > 0.$$

when $g_{\rm NL}^{equil} = 1$ and $\bar{\mathcal{C}}(S_{\mathcal{T}}^{DBI(s)}, S_{\mathcal{T}}^{equil}) = 1$

$$F(S_{\mathcal{T}}^{DBI(s)}, S_{\mathcal{T}}^{DBI(s)}) = F(S_{\mathcal{T}}^{equil}, S_{\mathcal{T}}^{equil}),$$
$$F(S_{\mathcal{T}}^{DBI(s)}, S_{\mathcal{T}}^{equil}) > 0.$$

when $g_{\rm NL}^{equil} = 1$ and $\bar{\mathcal{C}}(S_{\mathcal{T}}^{DBI(s)}, S_{\mathcal{T}}^{equil}) = 1$

Matching at a point underestimate the amplitudes by factor 2 ~ 4 !!

Cf. Bispectrum

$$f_{\rm NL}^{equil} = -\frac{35}{108}\frac{1}{c_s^2} \sim -0.33\frac{1}{c_s^2}$$

Matching at a point

Conclusion and discussion

- Estimator for equilateral type primordial non-Gaussianity
  - Separable shape $$T_\zeta(k_1,k_2,k_3,k_4) = \frac{g_{\rm NL}^{equil}}{k_1k_2k_3k_4(\frac{k_1+k_2+k_3+k_4}{4})^5}\mathcal{P}_\zeta(k)^3$$
  - Shape correlation $$\bar{\mathcal{C}}(S_\mathcal{T}, S_{\mathcal{T}'}) = \frac{F(S_\mathcal{T}, S_{\mathcal{T}'})}{\sqrt{F(S_\mathcal{T}, S_\mathcal{T})F(S_{\mathcal{T}'}, S_{\mathcal{T}'})}}$$

  More accurate than considering only the equilateral configuration

- Predictions
  - special class of k-inflation $$g_{\rm NL}^{equil} = \frac{3X^3P_{,4X}}{16c_sP_{,X}^4}$$
  - single field DBI inflation $$g_{NL}^{equil} = \frac{17}{c_s^4}$$ ⟵ 87 % correlation
  - multi-field DBI inflation $$g_{NL}^{equil} = \frac{2.2}{c_s^4T_{\mathcal{R}S}^2}$$ ⟵ 33 % correlation

- Detectability

  Creminelli, Senatore, Zaldarriaga \`07

  $$\Delta g_{\rm NL} \sim \frac{1}{\mathcal{P}_\zeta N_{pix}^{1/2}} \sim \frac{\Delta f_{\rm NL}}{\mathcal{P}_\zeta^{1/2}}$$

- Application to other models?

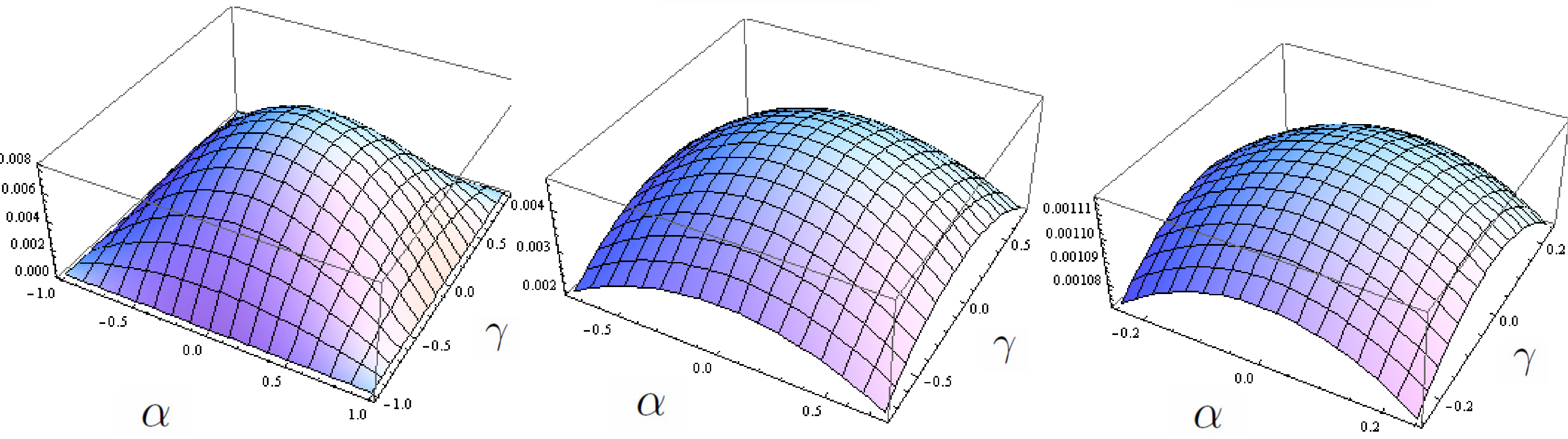
  Ghost inflation, Lifshitz field, DBI Galileon, ….

Thank you very much !

Equilateral shape

$$S_T^{equil}(\alpha, \beta, \gamma, 1)$$

$\beta = 0$ $\beta = 0.25$ $\beta = 0.75$



This shape is obtained by a special class of k-inflation

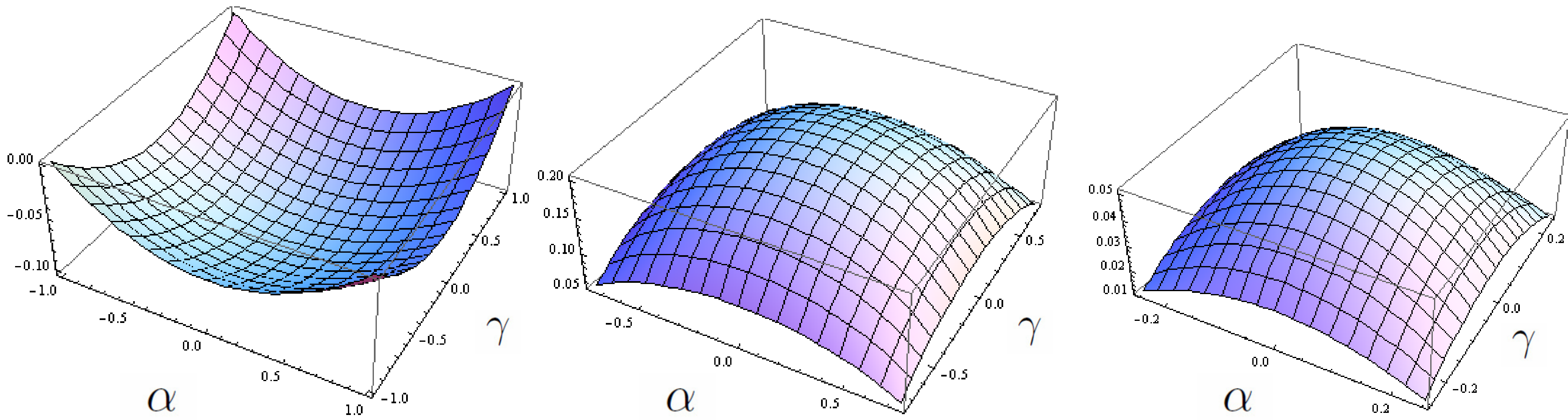
shown to be natural from effective theory of inflation

Senatore and Zaldarriaga `10

Shape function for single field DBI inflation

$S_{\mathcal{T}}^{DBI(\sigma)}(\alpha, \beta, \gamma, 1)$

$\beta = 0$ | $\beta = 0.25$ | $\beta = 0.75$



- Shape correlation $\bar{\mathcal{C}}(S_{\mathcal{T}}^{equil}, S_{\mathcal{T}}^{DBI(\sigma)})$

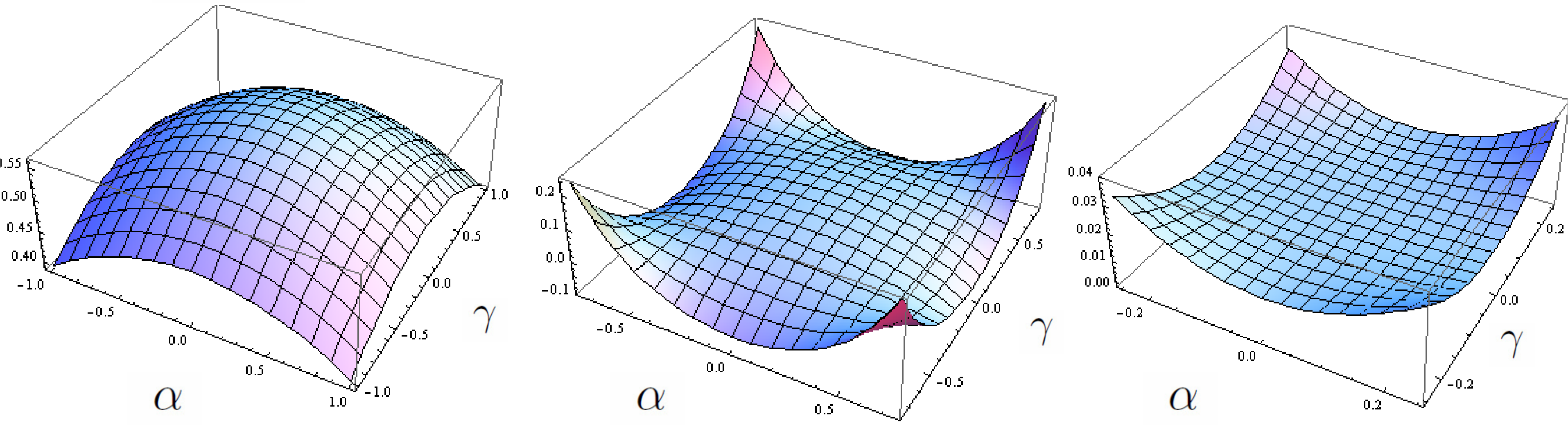
0.90 (configs. with $\epsilon \sim 1$) → 0.87 (full configs.)

Sufficiently large overlap

Shape function for multi-field DBI inflation

$S_{\mathcal{T}}^{DBI(s)}(\alpha, \beta, \gamma, 1)$

$\beta = 0$ $\beta = 0.25$ $\beta = 0.75$



- Shape correlation $\bar{\mathcal{C}}(S_{\mathcal{T}}^{equil}, S_{\mathcal{T}}^{DBI(s)})$

0.60 (configs. with $\epsilon \sim 1$) ⇒ 0.33 (full configs.)

Trispectrum can distinguish single DBI from multi DBI

SM, Arroja, Koyama `09