Trispectrum from ghost inflation

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Keisuke Izumi (IPMU)

in collaboration with Shinji Mukohyama (IPMU)

Introduction

Inflation provides a solution to the flatness problem and horizon problem and explain production of density perturbations.
Almost gaussian and almost scale-invariant perturbations predicted by inflation fits observational data very well.

Which inflation occur in our universe??

There are some ways to distinguish these models by observations,

- observation of gravitational wave
- observation of deviation from gaussian or scale-invariance

Here, we focus on non-gaussianity.
Bispectrum and trispectrum are parameters which show the deviation from gaussianity.

To restrict models from observation, we should calculate bispectrum and trispectrum in each model and compare them to observational data.

We calculate trispectrum from the ghost inflation.

Contents

Gravity at long distance

Galaxy rotation curves are flattened.
Expansion of our universe is accelerated.

If we believe general relativity,

we must introduce new forms of matter (Dark Matter) and energy (Dark energy).

However, these might stem from modification of gravity sector at long distance.
Modifications of gravity sector can be classified into two categories.

Change theory

Massive gravity, DGP, ・・・・

Change state

Ghost condensation

Higgs phase of gravity

(Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004)

The ghost condensation is the simplest Higgs phase for gravity and modifies gravity in the infrared.

Higgs mechanism

Spontaneously breaks gauge symmetry.

(Theory itself has gauge symmetry.)

⇒ Gives mass to gauge boson.

Ghost condensation = Higgs phase of gravity

Spontaneously breaks diffeomorphism invariance.

(Theory itself has diffeomorphism invariance.)

⇒ Gives mass to graviton.

↳ Gravity is modified at long distance.

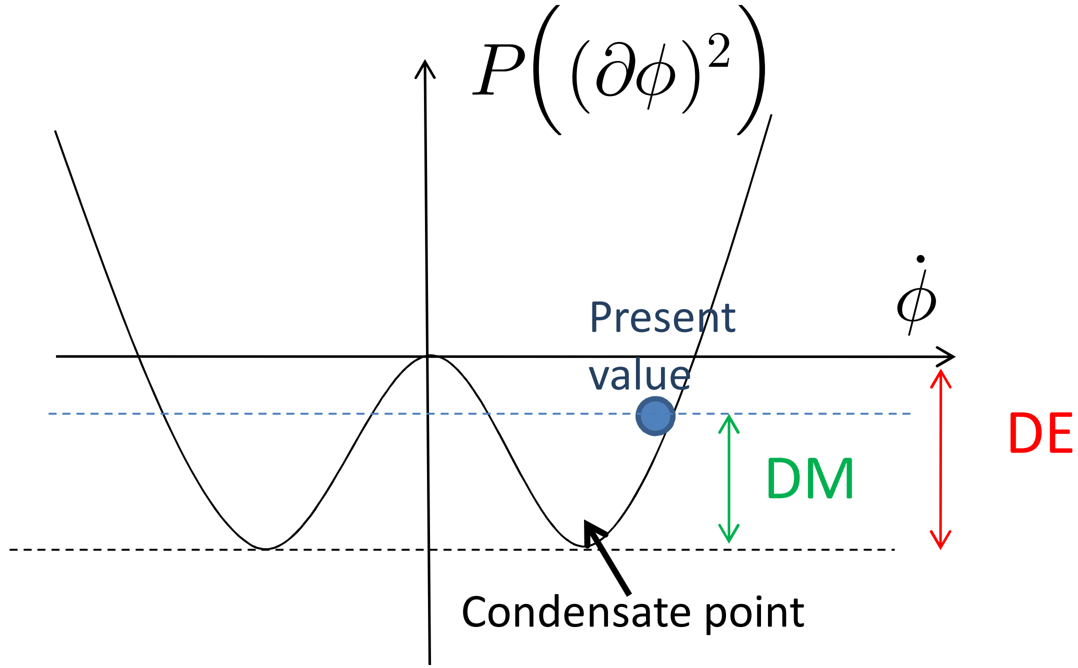
Ghost condensation

(Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004)

In ghost condensation, diffeomorphism invariance is broken by timelike vacuum expectation value of scalar field.
This can be realized by introducing scalar field called ghost field whose Lagrangian is $L = P\Big((\partial\phi)^2\Big)$

where function P has minimum value at point other than origin.

In this model,
matters which behave as dark matter and dark energy naturally appear.

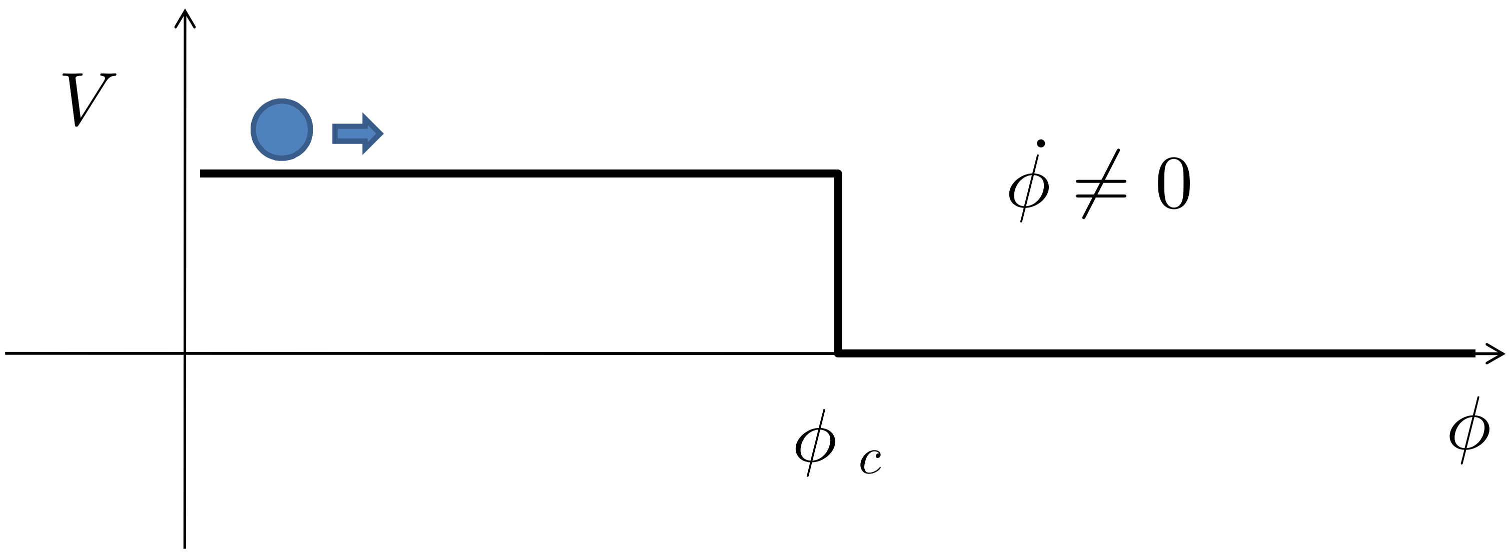


Ghost inflation

Ghost inflation is inflation model where ghost field accelerates expansion of universe.
This is realized by introducing potential of ghost field as figure below.

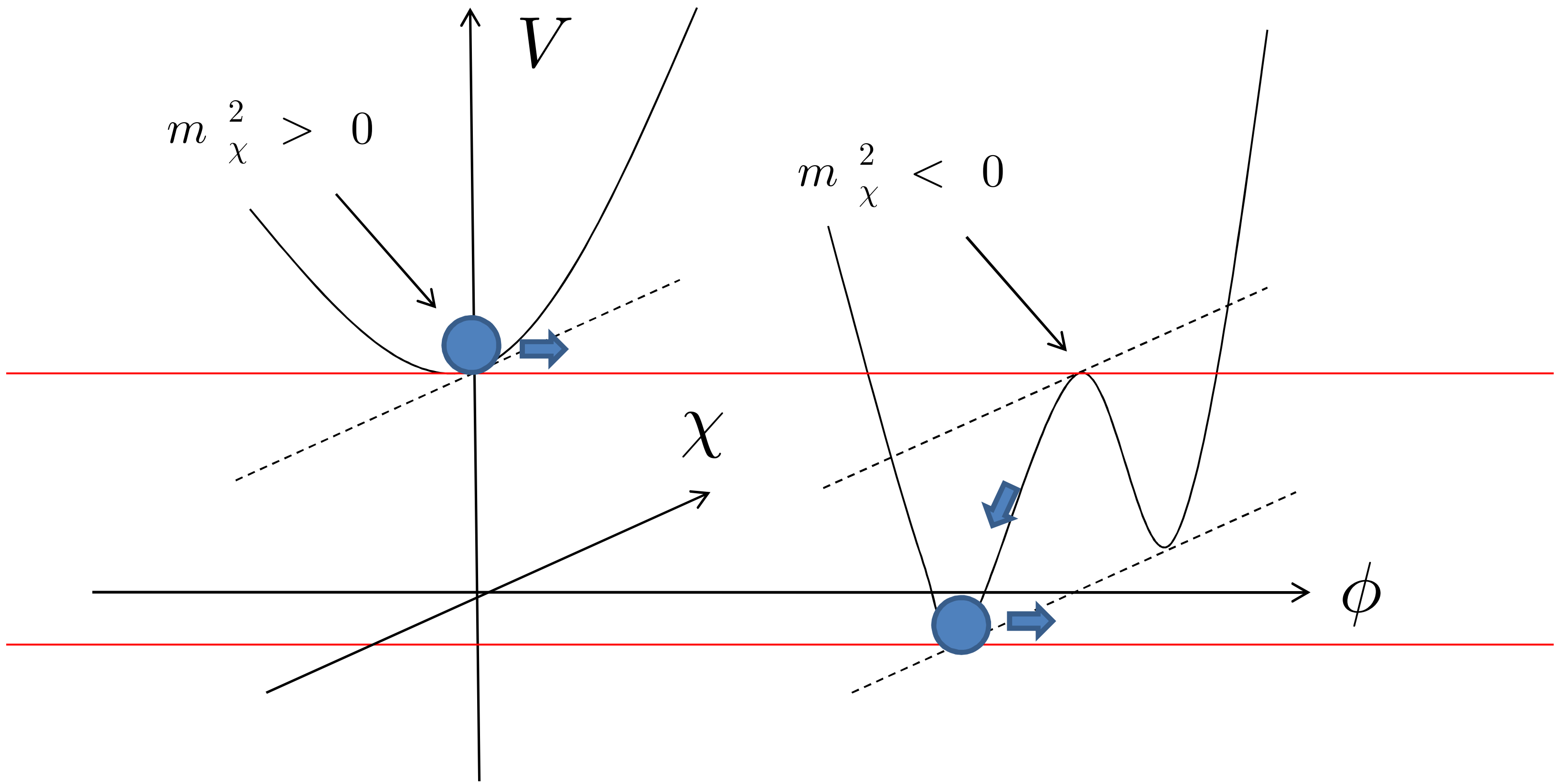
Because of timelike vacuum expectation value of ghost field,
ghost field is always rolling even without gradient of potential.

For $\phi < \phi_c$, ghost field has potential energy and it drives inflation.
After ghost field rolls through $\phi = \phi_c$, potential energy becomes zero and inflation ends.



Ghost inflation

(Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004)



Not slow roll $\dot{\phi} \neq 0$

Similar to hybrid inflation

Contents

Scaling dimensions

We perturb ghost field and metric.

Ghost field $$\phi = M^2 t + \delta\phi \equiv M^2 t + \pi$$

Metric $$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

After taking the decoupling limit $M/M_{pl} \to 0$, for $H/M \ll 1$
quadratic action can be reduced to

$$S_\pi = \int dt d^3x \left(\frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{2} M^2 (\vec{\nabla}^2 \pi)^2 + \cdots \right)$$

In order to see the leading cubic and quartic interactions,
we should identify the scaling dimensions.

We suppose $E \to bE \quad (t \to b^{(-1)} t)$

Then, scalings of spatial coordinates and perturbation of ghost field become

$$x \to b^{-1/2} x \qquad\qquad \pi \to b^{1/4} \pi$$

Most relevant coupling terms

Scaling dimension

$$t \rightarrow b^{-1}t \quad (E \rightarrow bE)$$

$$x \rightarrow b^{-1/2}x \qquad S_\pi = \int dt\, d^3x \left(\frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{2}M^2(\vec{\nabla}^2\pi)^2 + \cdots\right)$$

$$\pi \rightarrow b^{1/4}\pi$$

Assumption: Ghost field ϕ has shift symmetry

$$\phi \rightarrow \phi + \epsilon \quad (\pi \rightarrow \pi + \epsilon)$$

π appears in only the derivative forms $\dot{\pi}\,,\ \vec{\nabla}\pi$

Leading nonlinear operator in infrared: $\int dt\, d^3x \frac{\dot{\pi}(\vec{\nabla}\pi)^2}{M^2}$

Scaling dimension 1/4 $\Rightarrow$ Good low-energy effective theory

Most relevant 4-th order term: $\int dt\, d^3x \frac{(\vec{\nabla}\pi)^4}{M^4}$

Scaling dimension 1/2

Trispectrums (result)

$$S_\pi = \int dt\, d^3x \left(\frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{2M^2}(\vec{\nabla}^2 \pi)^2 - \frac{\beta}{2M^2}\dot{\pi}(\vec{\nabla}\pi)^2 - \frac{\gamma}{8}(\vec{\nabla}\pi)^4 \right)$$

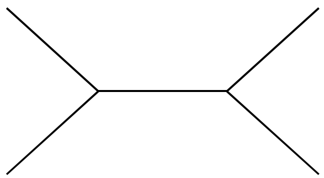
Bispectrum (Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004)

$$f_{nl} \sim \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^2} \sim 10^2 \times \beta\alpha^{-8/5}$$

Trispectrum

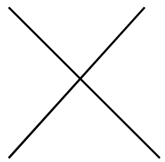
There are two contributions to the four-point function:
a scalar exchange contribution and a contact term contribution.

Scalar exchange contribution:
two three-point vertices



$$\tau_{nl} \sim \frac{\langle\zeta\zeta\zeta\zeta\rangle_{SE}}{\langle\zeta\zeta\rangle^3} \sim 10^4 \times \beta^2\alpha^{-8/5}$$

Contact term contribution:
one four-point vertex



$$\tau_{nl} \sim \frac{\langle\zeta\zeta\zeta\zeta\rangle_{CT}}{\langle\zeta\zeta\rangle^3} \sim 10^4 \times \tilde{\gamma}\alpha^{-8/5}$$

Non-linear parameter τ_{nl} becomes $\sim 10^4 \times$ dimensionless parameters.

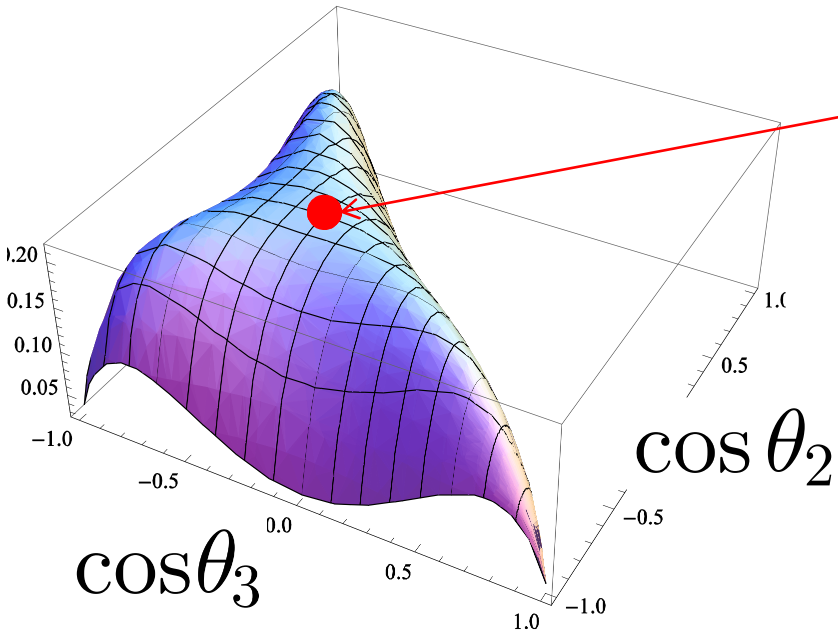
Momentum dependence (result)

Equilateral case $(k_1 = k_2 = k_3 = k_4 \equiv k)$

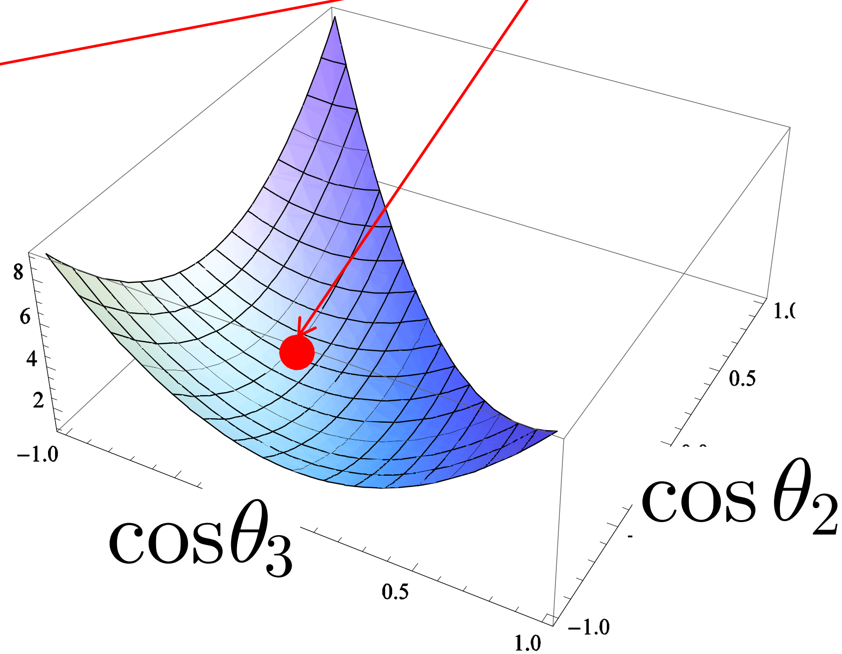
Residual parameters $\vec{k}_1 \cdot \vec{k}_2 / k^2 \equiv \cos\theta_2,\ \vec{k}_1 \cdot \vec{k}_3 / k^2 \equiv \cos\theta_3$

Most symmetric point

$\cos\theta_2 = \cos\theta_3 = -1/3$



Scalar exchange contribution



Contact term contribution

Conclusion

Non-gaussianity is important to identify inflation model in our Universe.

We calculated equilateral trispectrum in ghost inflation.

$\tau_{NL} \sim 10^4 \times$ dimensionless parameters

At most symmetric point $\theta_2 = \theta_3 = -1/3$,
equilateral trispectrum from scalar exchange has maximum value
and equilateral trispectrum from contact term has minimum value

Since trispectrum of contact term contribution is proportional to
dimensionless coupling constant $\tilde{\gamma}$,
trispectrum can become negative.

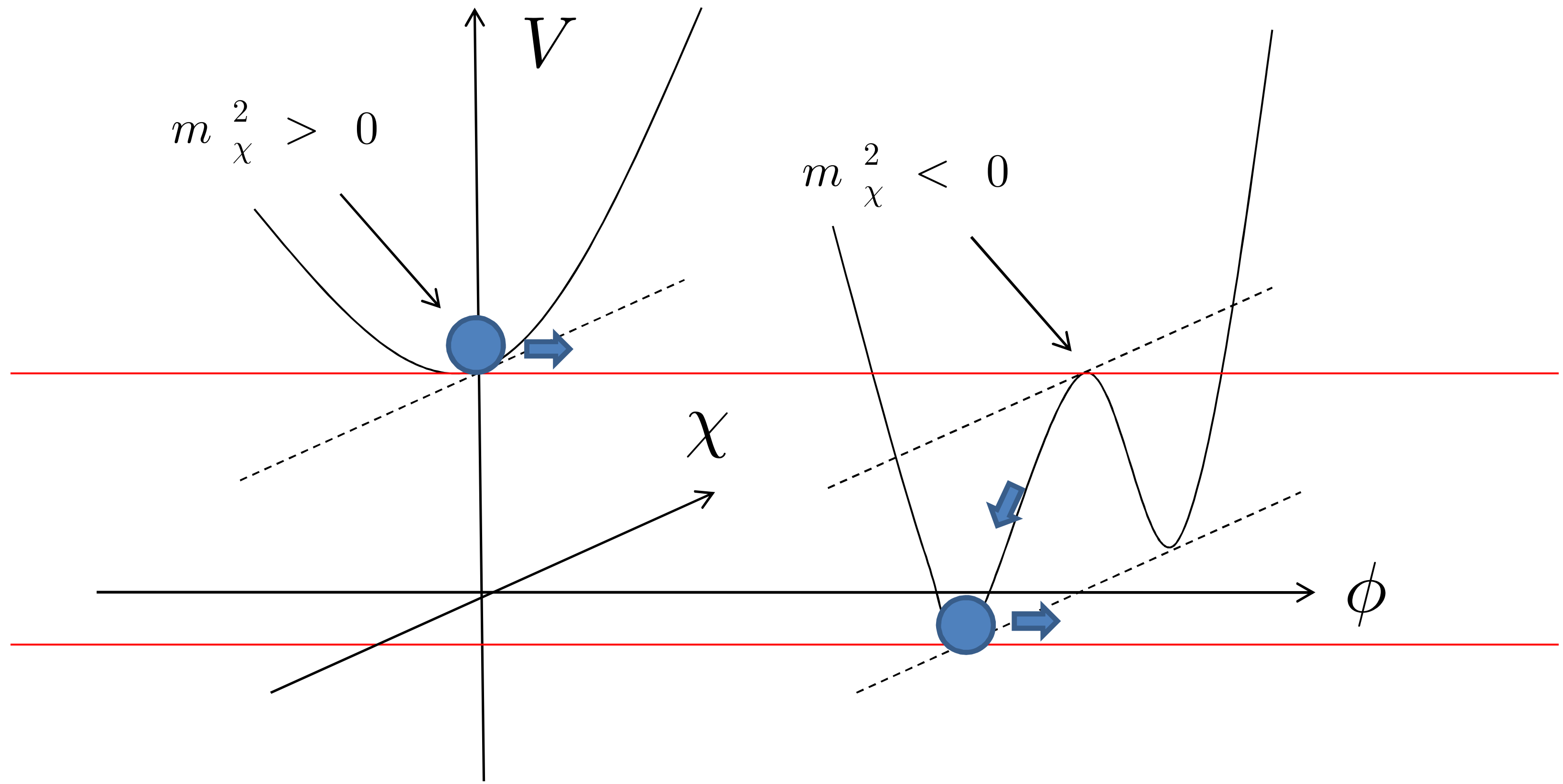
Bound on scale of ghost condensation

Arkani-Hamaed, Cheng, Luty, Mukohyama and Wiseman, JHEP 0701:036,2007

Twinkling from Lensing gives the most tighten constraint .
The scale of ghost condensation must be below 100 GeV.

Ghost inflation

(Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004)



$$2\,\partial_t(a^3 P(\dot\phi^2)\dot\phi) + a^3 V_\phi = 0$$

$$V = m^2(\phi)\chi^2 + F(\chi)$$

$$V_\phi = m^2(\phi)'\chi^2$$

Perturbation

Ghost field

$$\phi = M^2 t + \delta\phi \equiv M^2 t + \pi$$

Metric on Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Under transformation: $x^{\mu} \rightarrow x^{\mu} + \xi^{\mu}$

$$\delta h_{\mu\nu} = \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}$$

$$\delta\pi = M^{2}\xi^{0}$$

Unitary gauge

$$\pi = 0$$

Residual symmetry

$$\overrightarrow{x} \rightarrow \overrightarrow{x}'(t, \overrightarrow{x})$$

Under residual transformation

$$\delta h_{00} = 0$$

$$\delta h_{0i} = \partial_{0}\xi_{i}$$

$$\delta h_{ij} = \partial_{j}\xi_{j} + \partial_{j}\xi_{i}$$

Action in second order level

strategy

Construct most general action invariant under residual symmetry in unitary gauge

Undo unitary gauge, get action for π

Under residual transformation

$$\delta h_{00} = 0$$
$$\delta h_{0i} = \partial_0 \xi_i$$
$$\delta h_{ij} = \partial_j \xi_j + \partial_j \xi_i$$

Invariant operators

$(h_{00})^2$ OK

$\cancel{(h_{0i})^2}$

$K^2, (K_{ij})^2$ OK

Invariant action

$$L_{eff} = L_{EH} + M^4\left((h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K_{ij} + \cdots\right)$$

Action for π

Action in unitary gauge

$$L_{eff} = L_{EH} + M^4\{ (h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K_{ij}K^{ij} + \cdots\}$$

Undo unitary gauge

$$\xi^0 \to M^{-2}\pi \Rightarrow \begin{cases} h_{00} \to h_{00} - 2M^{-2}\partial_t\pi \\ K_{ij} \to K_{ij} + M^{-2}\partial_i\partial_j\pi \end{cases}$$

$$L_{eff} = L_{EH} + M^4\{ (h_{00} - 2M^{-4}\dot{\pi})^2 - \frac{\alpha_1}{M^2}(K + M^{-2}\vec{\nabla}^2\pi)^2$$
$$- \frac{\alpha_2}{M^2}(K_{ij} + M^{-2}\vec{\nabla}_i\vec{\nabla}_j\pi)(K^{ij} + M^{-2}\vec{\nabla}^i\vec{\nabla}^j\pi) + \cdots\}$$

$$L_\pi = \frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{2}M^2(\vec{\nabla}^2\pi)^2 + \cdots$$

Estimation of non-gaussianity

$$\zeta = -\frac{H}{\dot{\phi}}\pi \simeq -(H/M)^{5/4}$$

Scale dimension

$$\dot{\phi} = M^2 \qquad\qquad \pi \simeq M(H/M)^{1/4}$$

Power spectrum

$$\mathcal{P}_\zeta^{1/2} = \langle\zeta\zeta\rangle^{1/2} \simeq (H/M)^{5/4} \simeq 10^{-5} \qquad\qquad H/M \simeq 10^{-4}$$

Bispectrum (Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga, JHEP 0404:001,2004)

$$\langle\zeta\zeta\zeta\rangle \sim (H/M^2)^3\langle\int dt d^3x \dot{\pi}(\vec{\nabla}\pi)^2, \pi\pi\pi\rangle \sim (H/M)^4$$

$$\sim (H/M)^{1/4}$$

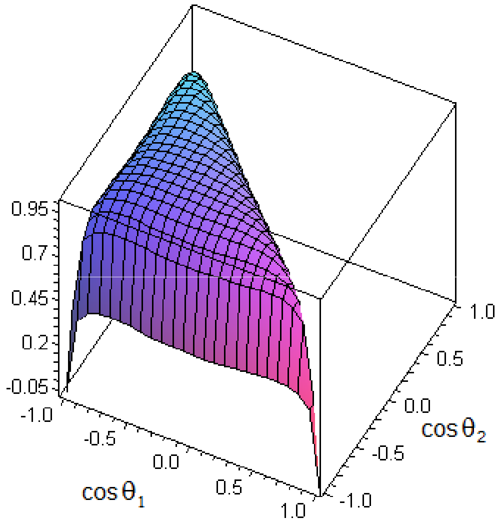
$$f_{NL} \sim \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^2} \sim (H/M)^{-1} \sim 10^4 \quad \Rightarrow \quad \sim 80$$

Trispectrum

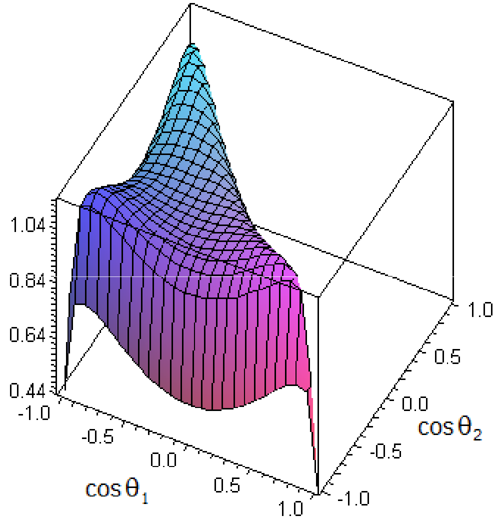
$$\langle\zeta\zeta\zeta\zeta\rangle \sim (H/M^2)^4\langle\int dt d^3x (\vec{\nabla}\pi)^4, \pi\pi\pi\pi\rangle \sim (H/M)^{11/2}$$

$$g_{NL} \sim \frac{\langle\zeta\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^3} \sim (H/M)^{-2} \sim 10^8 \quad \Rightarrow \quad \sim 10^4 \;\; (\sim f_{NL}^2)$$

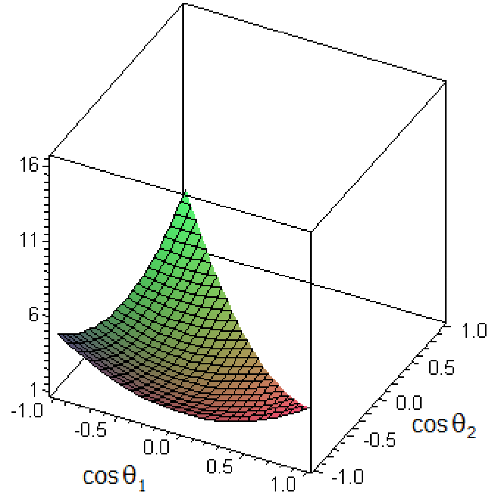
Arroja's talk (Trispectrum in DBI)



Adiabatic



Mixed



Entropy

Maximum for $\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = -1/3$

- Different shapes implies that the trispectrum can distinguish multiple from single field DBI inflation

Most relevant coupling terms

Scaling dimension

$$t \rightarrow b^{-1} t \quad (E \rightarrow bE)$$

$$x \rightarrow b^{-1/2} x$$

$$\pi \rightarrow b^{1/4} \pi$$

$$S_\pi = \int dt d^3x \left(\frac{1}{2}\dot{\pi}^2 - \frac{\alpha}{2} M^2 (\vec{\nabla}^2 \pi)^2 + \cdots \right)$$

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Most relevant 4-th order term: $\int dt d^3x \frac{(\vec{\nabla}\pi)^4}{M^4}$

Scaling dimension 1/2