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Non-Gaussianity from Lifshitz Scalar

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based on:

arXiv:1008.1406 with Keisuke Izumi, Shinji Mukohyama

Lifshitz scalars with z=3

- *obtain super-horizon, scale-invariant field fluctuations without the need of an inflationary era* *Mukohyama '09*

  $\Rightarrow$ *can source the primordial curvature perturbations*

- *this mechanism works regardless of the Lifshitz scalar's self-coupling strength* (*$\longleftrightarrow$ field fluctuations in the inflationary era*)

  $\Rightarrow$ *large non-Gaussianity expected*

We study the non-Gaussian nature of the Lifshitz scalar's intrinsic field fluctuations.

Lifshitz scalars

action :

$$S_\Phi = \frac{1}{2}\int dt d^3x \, \left[(\partial_t \Phi)^2 + \frac{(-1)^{z+1}}{M^{2(z-1)}}\Phi\Delta^z\Phi\right]$$

anisotropic scaling with dynamical critical exponent z *:*

$$\vec{x} \to b\vec{x} \qquad\qquad t \to b^z t$$

$$\Phi \to b^{(z-3)/2}\Phi$$

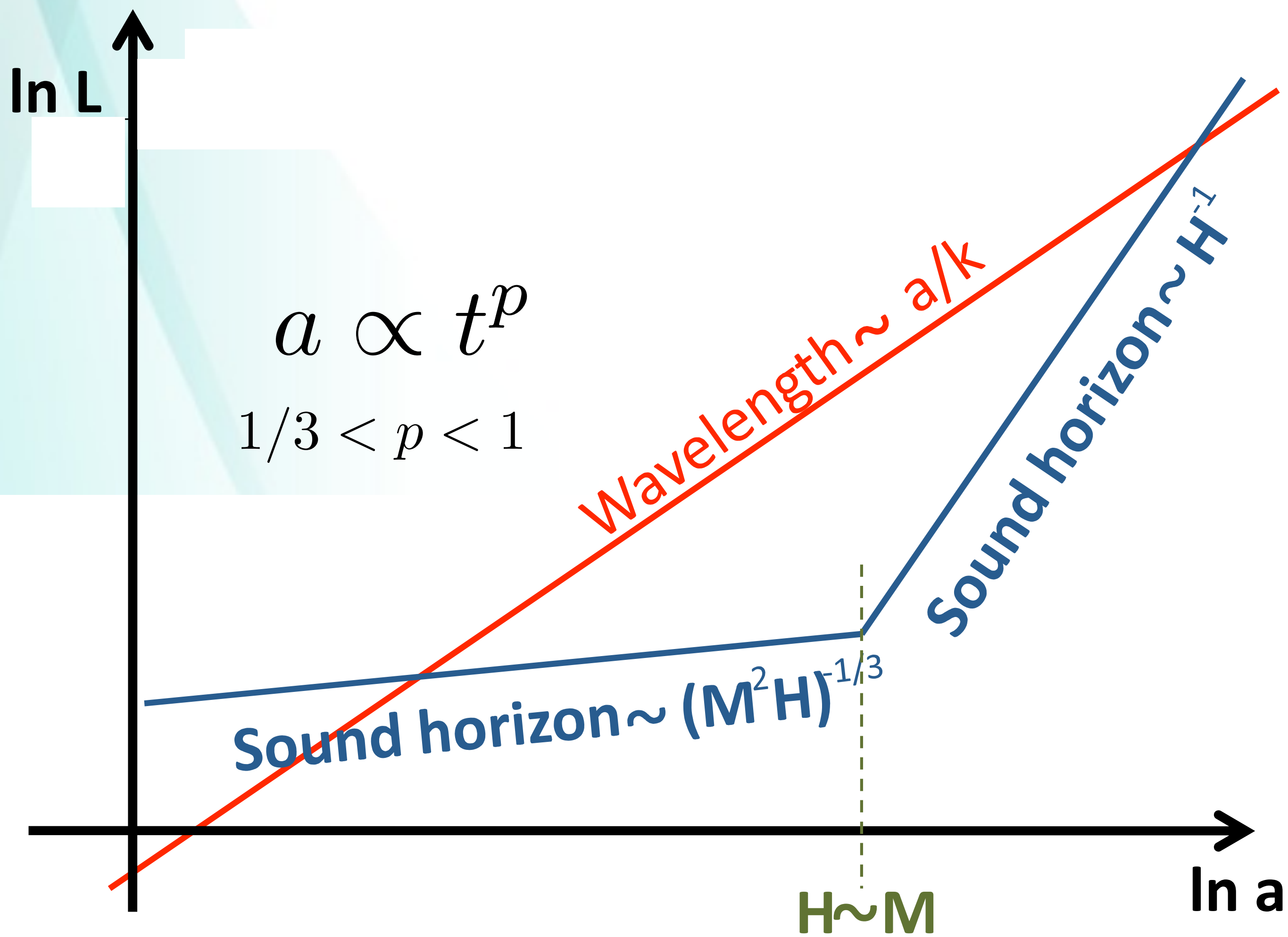
especially for $z = 3$ *,* $\quad \Phi \to b^0 \Phi$

(cf. Hořava '09, "Quantum gravity at a Lifshitz point")

super-horizon field flucuations of Lifshitz scalars with z=3

$$S_\phi = \frac{1}{2}\int dt d^3x \; a(t)^3 \left[(\partial_t \phi)^2 + \phi \mathcal{O} \phi + O(\phi^3)\right]$$

$$\mathcal{O} = \frac{1}{M^4 a(t)^6}\Delta^3 - \frac{s}{M^2 a(t)^4}\Delta^2 + \frac{c_s^2}{a(t)^2}\Delta - m^2$$



curvature perturbations via curvaton mechanism/ modulated reheating

scale-invariant field flucuations of Lifshitz scalars with z=3

$$S_\phi = \frac{1}{2}\int dtd^3x\ a(t)^3\left[(\partial_t\phi)^2 + \phi\mathcal{O}\phi + O(\phi^3)\right]$$

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Since $[\phi] = 0$ *, field fluctuations are independent of H:*

$$\langle\phi\phi\rangle \sim M^2,$$

regardless of its self-coupling strength.

scale-invariant field flucuations of Lifshitz scalars with z=3

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Since $[\phi] = 0$ *, field fluctuations are independent of H:*

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regardless of its self-coupling strength.

Fluctuations can be scale-invariant & non-Gaussian.

order estimate of f_{NL}

$$S_\phi = \int dt d^3x\, a^3 \left[\frac{1}{2}(\partial_t \phi)^2 + \frac{1}{2}\frac{\phi \Delta^3 \phi}{M^4 a^6} + \alpha \frac{\phi^2 \Delta^3 \phi}{M^5 a^6} + \cdots\right]$$

$$\langle \phi\phi \rangle \sim M^2 \qquad \langle \phi\phi\phi \rangle \sim \alpha M^3$$

for linear conversion to curvature pert. $\zeta = \frac{\phi}{\mu},$

$$\langle \zeta\zeta \rangle \sim \frac{M^2}{\mu^2} \qquad \langle \zeta\zeta\zeta \rangle \sim \alpha \frac{M^3}{\mu^3}$$

$$f_{\rm NL} \sim \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^2} \sim \alpha \frac{\mu}{M}$$

order estimate of f$_{\rm NL}$

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for linear conversion to curvature pert. $\zeta = \frac{\phi}{\mu},$

$$10^{-10} \sim \langle\zeta\zeta\rangle \sim \frac{M^2}{\mu^2} \qquad \langle\zeta\zeta\zeta\rangle \sim \alpha\frac{M^3}{\mu^3}$$

$$f_{\rm NL} \sim \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^2} \sim \alpha\frac{\mu}{M} \sim 1000\,\alpha$$

order estimate of $\mathrm{f_{NL}}$

$$S_\phi = \int dt d^3x\, a^3 \left[\frac{1}{2}(\partial_t \phi)^2 + \frac{1}{2}\frac{\phi \Delta^3 \phi}{M^4 a^6} + \alpha \frac{\phi^2 \Delta^3 \phi}{M^5 a^6} + \cdots \right]$$

$$\langle \phi\phi \rangle \sim M^2 \qquad \langle \phi\phi\phi \rangle \sim \alpha M^3$$

Large f_{NL} produced as α approaches unity.

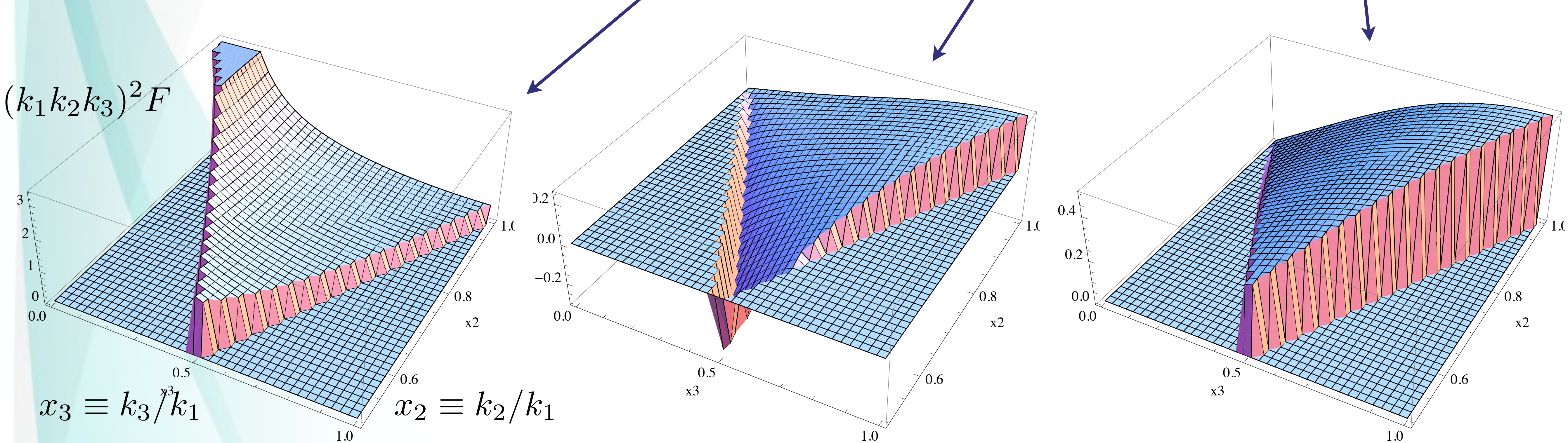
$$10^{-10} \sim \langle \zeta\zeta \rangle \sim \frac{M^2}{\mu^2} \qquad \langle \zeta\zeta\zeta \rangle \sim \alpha \frac{M^3}{\mu^3}$$

$$f_{\mathrm{NL}} \sim \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^2} \sim \alpha \frac{\mu}{M} \sim 1000\, \alpha$$

shapes of bispectra

three independent marginal cubic operators:

$$S_3 = \int dt d^3x \, \frac{1}{M^5 a(t)^3} \left\{ \alpha_1 \phi^2 \Delta^3 \phi + \alpha_2 (\Delta^2 \phi)(\partial_i \phi)^2 + \alpha_3 (\Delta \phi)^3 \right\}$$



$$\langle \phi_{\boldsymbol{k_1}}(t) \phi_{\boldsymbol{k_2}}(t) \phi_{\boldsymbol{k_3}}(t) \rangle = (2\pi)^3 M^3 \delta^{(3)}(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3}) F(k_1, k_2, k_3)$$

Only the α_1 term breaks the shift symmetry $\phi \rightarrow \phi + const.$,
and blows up in the squeezed traiangle limit.

observational constraints

$$S_3 = \int dt d^3x \, \frac{1}{M^5 a(t)^3} \left\{ \alpha_1 \phi^2 \Delta^3 \phi + \alpha_2 (\Delta^2 \phi)(\partial_i \phi)^2 + \alpha_3 (\Delta\phi)^3 \right\}$$

WMAP7 constraints:

$-10 < f_{\rm NL}^{\rm local} < 74$ $\quad -410 < f_{\rm NL}^{\rm orthog.} < 6$ $\quad -214 < f_{\rm NL}^{\rm equil.} < 266$ *(95% CL)*

for $\zeta \propto \phi$,

$$-0.03 < \alpha_1 < 0.004 \qquad -1 < \alpha_2 < 0.4 \qquad -0.5 < \alpha_3 < 0.3$$

observational constraints

$$S_3 = \int dt d^3x \, \frac{1}{M^5 a(t)^3} \left\{ \alpha_1 \phi^2 \Delta^3 \phi + \alpha_2 (\Delta^2 \phi)(\partial_i \phi)^2 + \alpha_3 (\Delta \phi)^3 \right\}$$

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NOTE: $|\alpha_i| \lesssim 1$ *required for validity of perturbative expansions*

significant orthogonal/equilateral-type non-Gaussianity produced within perturbatively controllable regime

observational constraints

$$S_3 = \int dt d^3x \, \frac{1}{M^5 a(t)^3} \left\{ \alpha_1 \phi^2 \Delta^3 \phi + \alpha_2 (\Delta^2 \phi)(\partial_i \phi)^2 + \alpha_3 (\Delta \phi)^3 \right\}$$

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NOTE: $|\alpha_i| \lesssim 1$ *required for validity of perturbative expansions*

significant orthogonal/equilateral-type non-Gaussianity produced within perturbatively controllable regime

α_1 *required to be further suppressed, e.g. by shift symmetry*

trispectra

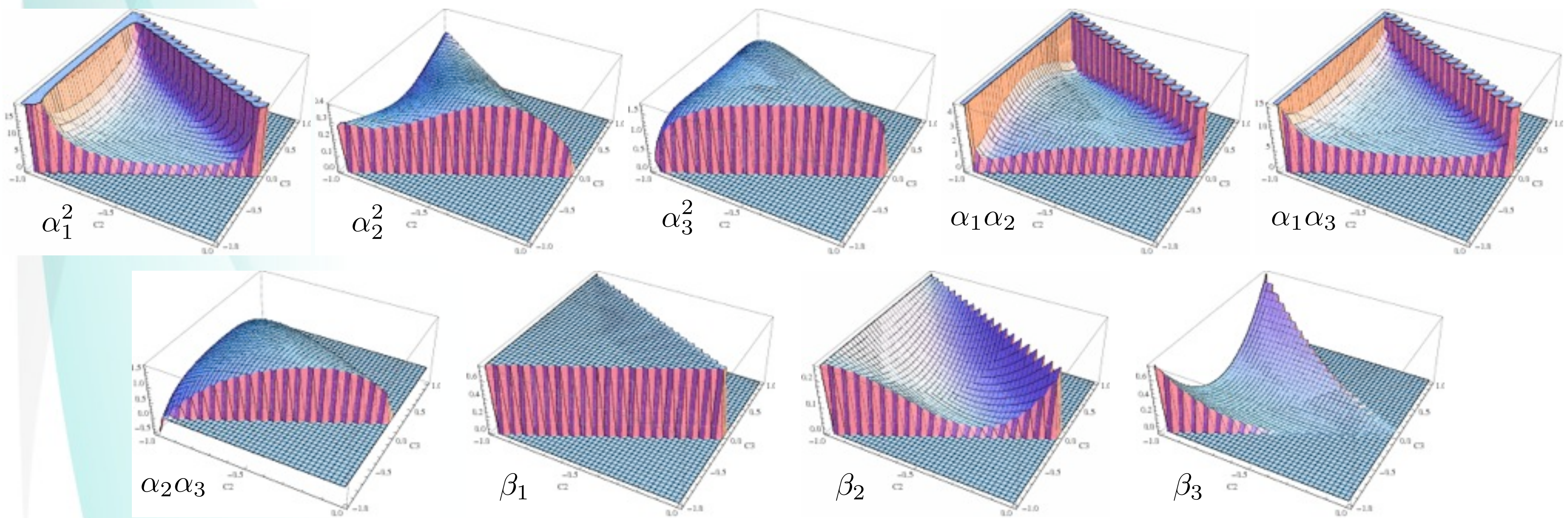
$$S_3 = \int dt d^3x \, \frac{1}{M^5 a(t)^3} \left\{ \alpha_1 \phi^2 \Delta^3 \phi + \alpha_2 (\Delta^2 \phi)(\partial_i \phi)^2 + \alpha_3 (\Delta \phi)^3 \right\}$$

$$S_4 = \int dt d^3x \, \frac{1}{M^6 a(t)^3} \left\{ \beta_1 \phi^3 \Delta^3 \phi + \beta_2 \phi^2 (\Delta \phi)(\Delta^2 \phi) + \beta_3 \phi (\Delta \phi)^3 + \beta_4 \phi^2 (\Delta \partial_i \phi)^2 + \beta_5 \phi^2 (\partial_i \partial_j \partial_k \phi)^2 + \beta_6 (\partial_i \partial_j \partial_k \phi)(\partial_i \phi)(\partial_j \phi)(\partial_k \phi) \right\}$$

$$\langle \phi_{\boldsymbol{k_1}}(t) \phi_{\boldsymbol{k_2}}(t) \phi_{\boldsymbol{k_3}}(t) \phi_{\boldsymbol{k_4}}(t) \rangle = (2\pi)^3 M^4 \delta^{(4)}(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3} + \boldsymbol{k_4}) \mathcal{T}(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}, \boldsymbol{k_4})$$

plots of $(k_1 k_2 k_3 k_4)^{9/4} \mathcal{T}$ *in the "equilateral" limit:* $k_1 = k_2 = k_3 = k_4$

in terms of $C_2 \equiv \boldsymbol{k_1} \cdot \boldsymbol{k_2} / k_1 k_2$, $C_3 \equiv \boldsymbol{k_1} \cdot \boldsymbol{k_3} / k_1 k_3$



summary

- *a Lifshitz scalar with $z = 3$ obtains super-horizon, scale-invariant field fluctuations, regardless of its self-coupling strength*
- *the resulting curvature perturbations necessarily leave large non-Gaussianity in the sky, unless the field's self-couplings are somehow suppressed*
- *significant non-Gaussianity of order $f_{NL} = O(100)$ is generated, within the regime of validity of perturbative expansions*
- *self-coupling terms containing spatial derivatives produce non-Gaussianities with various shapes, including the local, equilateral, and orthogonal shapes*

marginal cubic terms in the UV

cubic terms with six spatial derivatives:

$$A_1 \equiv \left(\Delta^3\phi\right)\phi^2, \quad A_2 \equiv \left(\Delta^2\partial_i\phi\right)(\partial_i\phi)\,\phi, \quad A_3 \equiv \left(\Delta^2\phi\right)(\Delta\phi)\,, \quad A_4 \equiv (\Delta\partial_i\partial_j\phi)(\partial_i\partial_j\phi)\,\phi$$

$$A_5 \equiv (\Delta\partial_i\phi)(\Delta\partial_i\phi)\,\phi, \quad A_6 \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\partial_j\partial_k\phi)\,\phi, \quad A_7 \equiv \left(\Delta^2\phi\right)(\partial_i\phi)(\partial_i\phi)\,, \quad A_8 \equiv (\Delta\partial_i\partial_j\phi)(\partial_i\phi)(\partial_j$$

$$A_9 \equiv (\Delta\partial_i\phi)(\Delta\phi)(\partial_i\phi)\,, \quad A_{10} \equiv (\Delta\partial_i\phi)(\partial_i\partial_j\phi)(\partial_j\phi)\,, \quad A_{11} \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\partial_j\phi)(\partial_k\phi)\,,$$

$$A_{12} \equiv (\Delta\phi)(\Delta\phi)(\Delta\phi)\,, \quad A_{13} \equiv (\Delta\phi)(\partial_i\partial_j\phi)(\partial_i\partial_j\phi)\,, \quad A_{14} \equiv (\partial_i\partial_j\phi)(\partial_j\partial_k\phi)(\partial_k\partial_i\phi)\,.$$

total derivatives with six spatial derivatives:

$$\begin{aligned}
&\partial_i\left[\left(\Delta^2\partial_i\phi\right)\phi^2\right] = A_1 + 2A_2,\\
&\partial_i\left[(\Delta\partial_i\partial_j\phi)(\partial_j\phi)\,\phi\right] = A_2 + A_4 + A_8,\\
&\partial_i\left[(\Delta\partial_j\phi)(\partial_i\partial_j\phi)\,\phi\right] = A_4 + A_5 + A_{10},\\
&\partial_i\left[(\Delta\partial_i\phi)(\partial_j\phi)(\partial_j\phi)\right] = A_7 + 2A_{10},\\
&\partial_i\left[(\partial_i\partial_j\partial_k\phi)(\partial_j\phi)(\partial_k\phi)\right] = A_8 + 2A_{11},\\
&\partial_i\left[(\Delta\phi)(\partial_i\partial_j\phi)(\partial_j\phi)\right] = A_9 + A_{10} + A_{13},\\
&\partial_i\left[(\partial_j\partial_k\phi)(\partial_j\partial_k\phi)(\partial_i\phi)\right] = 2A_{11} + A_{13}.
\end{aligned}$$

$$\begin{aligned}
&\partial_i\left[\left(\Delta^2\phi\right)(\partial_i\phi)\,\phi\right] = A_2 + A_3 + A_7,\\
&\partial_i\left[(\Delta\partial_i\phi)(\Delta\phi)\,\phi\right] = A_3 + A_5 + A_9,\\
&\partial_i\left[(\partial_i\partial_j\partial_k\phi)(\partial_j\partial_k\phi)\,\phi\right] = A_4 + A_6 + A_{11},\\
&\partial_i\left[(\Delta\partial_j\phi)(\partial_i\phi)(\partial_j\phi)\right] = A_8 + A_9 + A_{10},\\
&\partial_i\left[(\Delta\phi)(\Delta\phi)(\partial_i\phi)\right] = 2A_9 + A_{12},\\
&\partial_i\left[(\partial_i\partial_j\phi)(\partial_j\partial_k\phi)(\partial_k\phi)\right] = A_{10} + A_{11} + A_{14},
\end{aligned}$$

three independent terms:

$$S_3 = \int dt d^3x\, \frac{1}{M^5 a(t)^3}\left\{\alpha_1\phi^2\Delta^3\phi + \alpha_2(\Delta^2\phi)(\partial_i\phi)^2 + \alpha_3(\Delta\phi)^3\right\}$$

observational constraints on α_i

$$\langle \phi_{\boldsymbol{k_1}}(t)\phi_{\boldsymbol{k_2}}(t)\phi_{\boldsymbol{k_3}}(t)\rangle = (2\pi)^3 M^3 \delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3})F(k_1,k_2,k_3)$$

scalar product: $$F_1 \cdot F_2 \equiv \sum_{\boldsymbol{k_i}} \frac{F_1(k_1,k_2,k_3)F_2(k_1,k_2,k_3)}{P_{k_1}P_{k_2}P_{k_3}}$$

$$F_{\rm template} = c_{\rm NL}^{\rm local} F_{\rm local} + c_{\rm NL}^{\rm orthog.} F_{\rm orthog.} + c_{\rm NL}^{\rm equil.} F_{\rm equil.}$$

$$c_{\rm NL}^{\rm local} = -0.125\alpha_1$$
$$c_{\rm NL}^{\rm orthog.} = 0.226\alpha_1 + 0.0186\alpha_2 - 0.00334\alpha_3$$
$$c_{\rm NL}^{\rm equil.} = -0.223\alpha_1 + 0.0280\alpha_2 - 0.0876\alpha_3$$

for $\zeta = \frac{\phi}{\mu}$, $$f_{\rm NL}^i = \frac{20}{3}\frac{\mu}{M}c_{\rm NL}^i \simeq 2.2\times 10^4 c_{\rm NL}^i$$

necessary conditions for $-10 < f_{\rm NL}^{\rm local} < 74$ $\quad -410 < f_{\rm NL}^{\rm orthog.} < 6$ $\quad -214 < f_{\rm NL}^{\rm equil.} < 266$

are $-0.03 < \alpha_1 < 0.004$ $\quad -1 < \alpha_2 < 0.4$ $\quad -0.5 < \alpha_3 < 0.3$