Classifying models of large (local-type) non-Gaussianity

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Primordial non-Gaussianity

- The origin of primordial fluctuations is one of the important issues.

- What is the origin?

  Fluctuations of an inflaton? some other scalar field? ......

- How can we probe? (CMB, LSS, ...)

  - Power-spectrum [amplitude, scale-dependence (spectral index)]
  - Gravitational waves (tensor-to-scalar ratio)
  - Non-Gaussianity $f_{\rm NL}^{\rm local} = 32 \pm 21 \quad (68\% \ {\rm CL})$

    $f_{\rm NL}^{\rm equil} = 26 \pm 140 \quad (68\% \ {\rm CL})$

[WMAP7, Komatsu et al, 2010]

Non-Gaussianity: Bispectrum

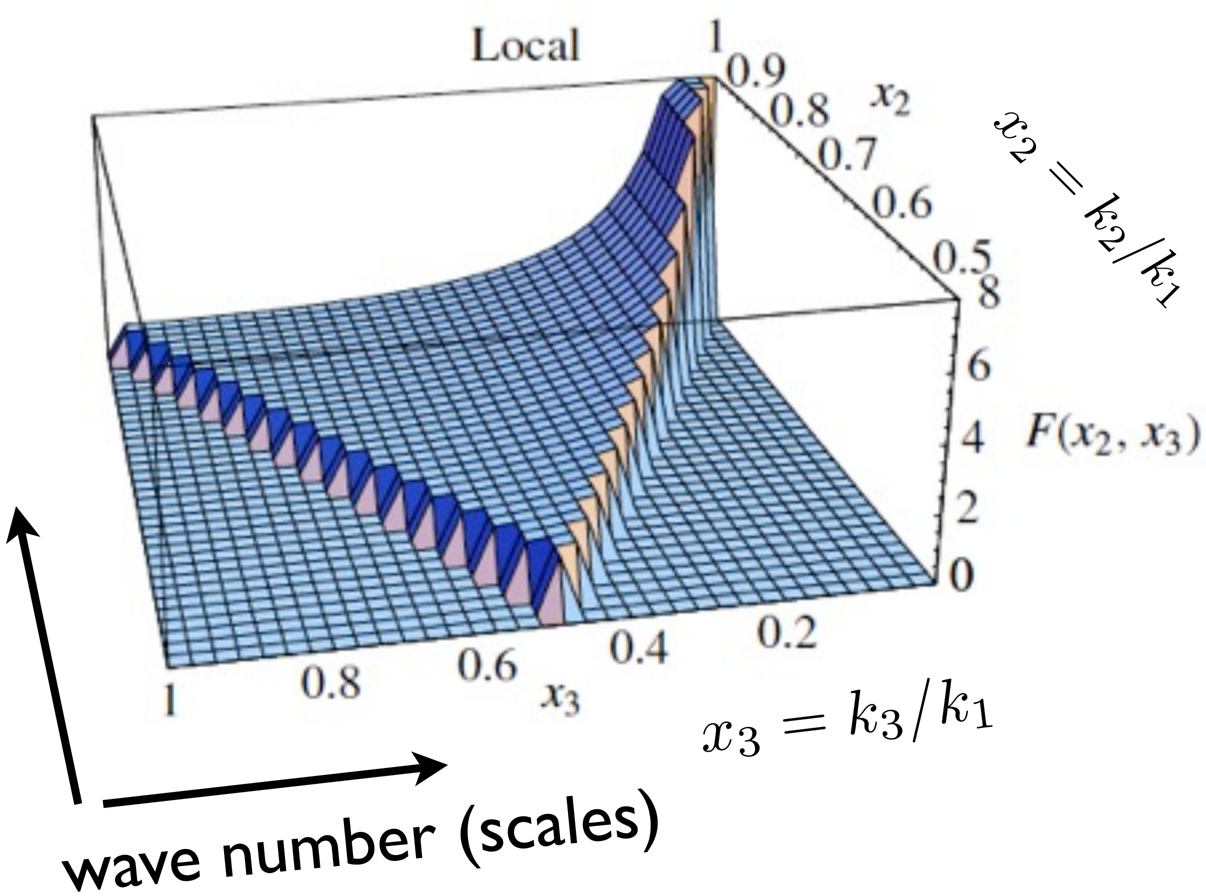
- Bispectrum:

cf. power spectrum:

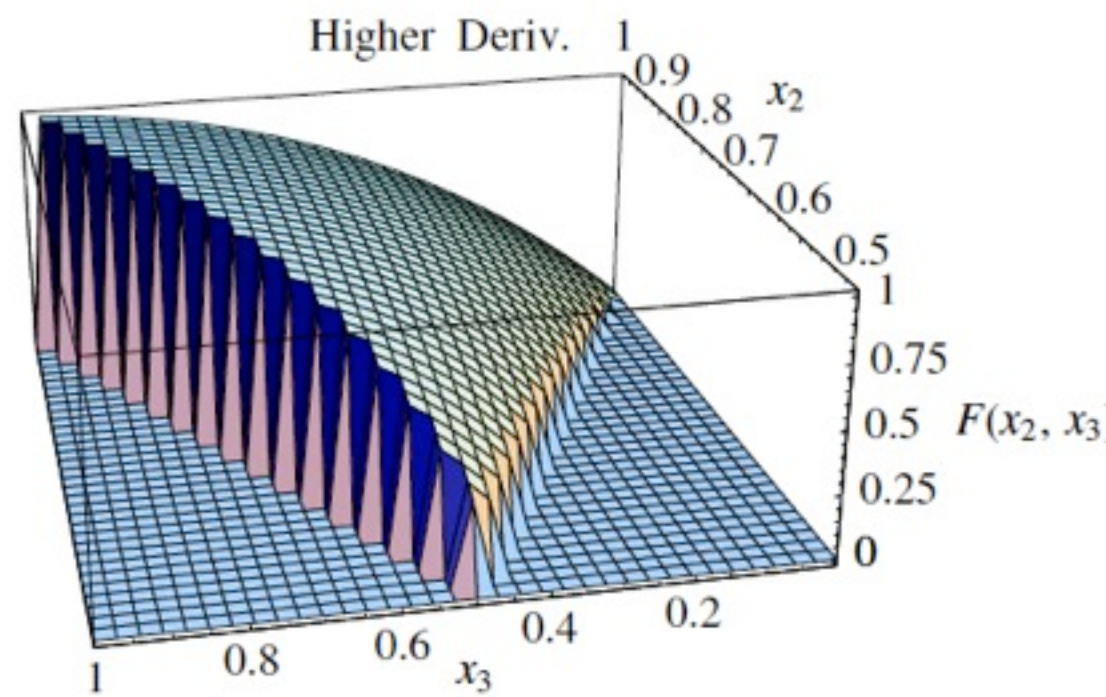
$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \right\rangle = (2\pi)^3 P(k_1)\delta(\vec{k}_1 + \vec{k}_2)$$

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \right\rangle = (2\pi)^3 B(k_1, k_2, k_3)\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

<u>Local-type</u>



<u>Equilateral-type</u>



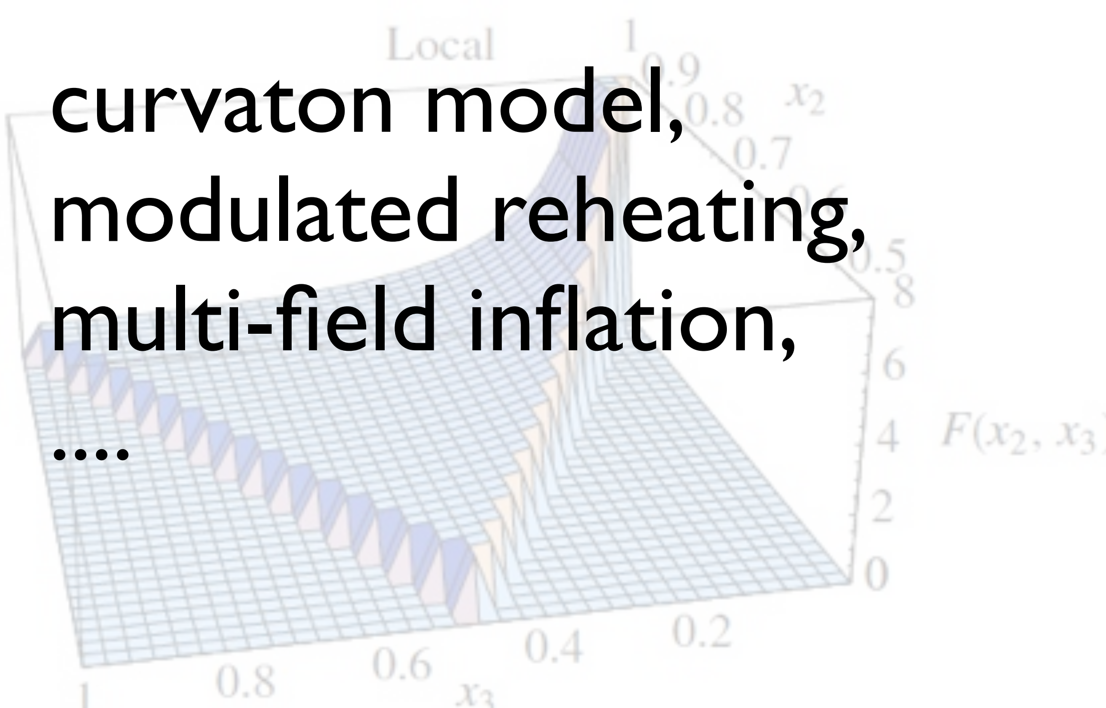
[Balbi, Creminelli, Zaldarriaga 2004]

Models can be categorized by “shapes”

- Bispectrum:

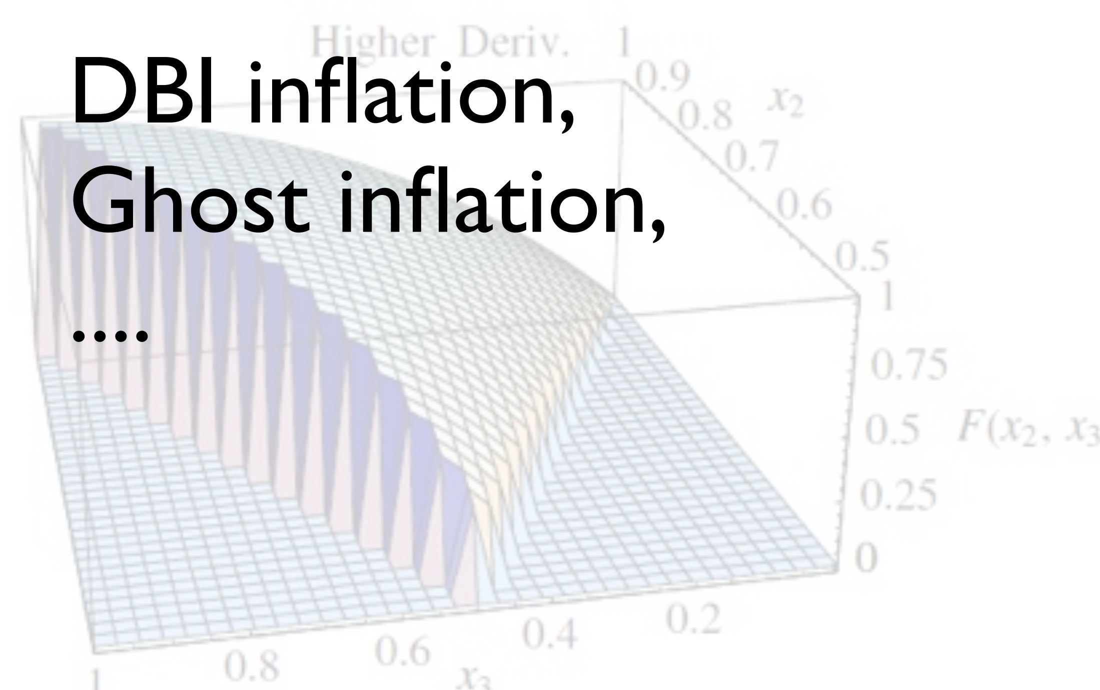
$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \right\rangle = (2\pi)^3 B(k_1, k_2, k_3)\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

<u>Local-type</u>



curvaton model,
modulated reheating,
multi-field inflation,
....

<u>Equilateral-type</u>



DBI inflation,
Ghost inflation,
....

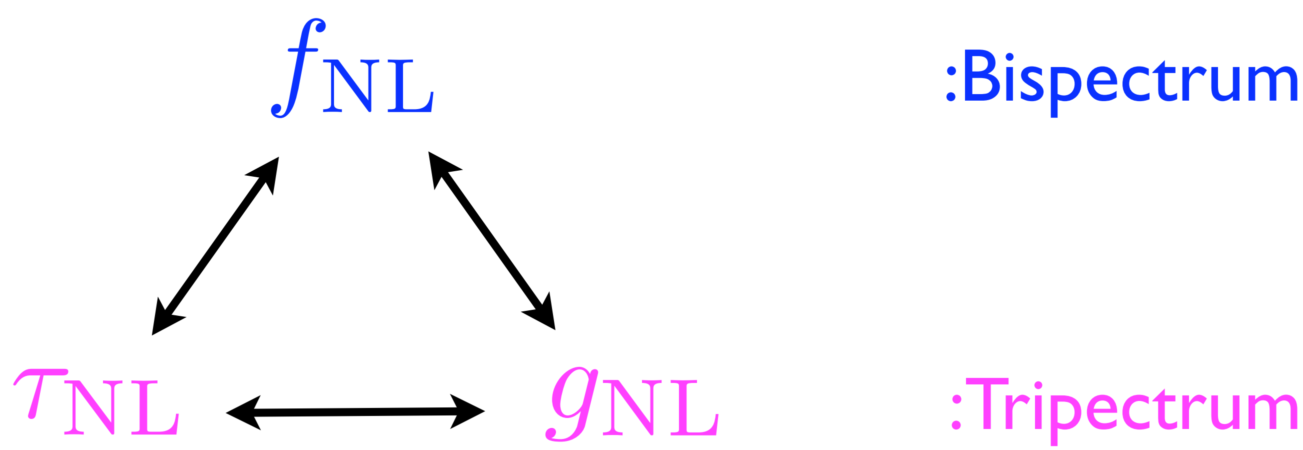
How can we differentiate models?

- Even if we limit ourselves to models of a particular type, there are a lot of possibilities......

→ Using "consistency relation" (relative size) between bispectrum and trispectrum.

Consistency relation between $f_{\rm NL}$, $\tau_{\rm NL}$ and $g_{\rm NL}$

- There are some relation between the non-linearity parameters in most models:



By using "consistency relation" between these parameters, we can divide the models into some categories.

(In this talk, we focus on the <u>local-type</u> models)

Bispectrum and Trispectrum for local type

- 3-point correlation function:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

Bispectrum

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

Bispectrum and Trispectrum for local type

- 4-point correlation function:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

Trispectrum

$k_{13} = k_1 + k_3$

$$T_\zeta(k_1, k_2, k_3, k_4) = \tau_{\rm NL} (P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms.})$$

$$+ \frac{54}{25} g_{\rm NL} (P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms.})$$

Non-linearity parameters in δN formalism

- Curvature perturbation in the δN formalism

$$\zeta(t_f) = N_a \delta\phi_*^a + \frac{1}{2} N_{ab} \delta\phi_*^a \delta\phi_*^b + \frac{1}{6} N_{abc} \delta\phi_*^a \delta\phi_*^b \delta\phi_*^c + \cdots$$

where

$$N_a \equiv \frac{\partial N}{\partial \phi^a} \qquad N_{ab} \equiv \frac{\partial^2 N}{\partial \phi^a \partial \phi^b} \qquad N_{abc} \equiv \frac{\partial^3 N}{\partial \phi^a \partial \phi^b \partial \phi^c}$$

- Three non-linearity parameters:

$$\frac{6}{5} f_{\rm NL} = \frac{N_a N_b N^{ab}}{(N_c N^c)^2} \qquad \tau_{\rm NL} = \frac{N_{ab} N^{ac} N^b N_c}{(N_d N^d)^3} \qquad \frac{54}{25} g_{\rm NL} = \frac{N_{abc} N^a N^b N^c}{(N_d N^d)^3}$$

[Lyth&Rodriguez 2005, Alabidi&Lyth 2006, Byrnes et al. 2006]

Non-linearity parameters: Case with one field

- Curvature perturbation

$$\zeta = N_\phi \delta\phi_* + \frac{1}{2} N_{\phi\phi} (\delta\phi_*)^2 + \frac{1}{6} N_{\phi\phi} (\delta\phi_*)^3$$

- Non-linearity parameters:

$$\frac{6}{5} f_{NL} = \frac{N_{\phi\phi}}{N_\phi^2} \qquad \tau_{\rm NL} = \left(\frac{N_{\phi\phi}}{N_\phi^2} \right)^2 = \frac{25}{36} f_{\rm NL}^2$$

➡ "Consistency relation" for one-field case

$$\tau_{\rm NL} = \frac{36}{25} f_{\rm NL}^2$$

➡ Single-source model

Non-linearity parameters: Case with multi-source

- In multi-source models,

$$\tau_{\rm NL} \neq \left(\frac{6}{5} f_{\rm NL} \right)^2$$

➡ No definite relation between $f_{\rm NL}$ and $\tau_{\rm NL}$,

(a general situation)

➡ We classify models of this kind as "multi-source model"

(However, the "local-type inequality" should be satisfied.)

“Local-type inequality”

- (As far as the 2nd order term (loop term) does not dominate in the power spectrum, practically, $f_{\rm NL} < 100$)

Any local-type models should satisfy the “local-type inequality”

$$\tau_{\rm NL} > \left(\frac{6}{5} f_{\rm NL}\right)^2$$

(This inequality can be derived from Cauchy-Schwartz inequality *

for $\frac{6}{5} f_{\rm NL} = \frac{N_a N_b N^{ab}}{(N_c N^c)^2}$ $\quad \tau_{\rm NL} = \frac{N_{ab} N^{ac} N^b N_c}{(N_d N^d)^3}$)

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* It is first derived in [Suyama and Yamaguchi, 0709.2545] and a more general version is discussed in [Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

Another model with definite relation between $f_{\rm NL}$ and $\tau_{\rm NL}$

- Two fields, but one is Gaussian, the other is totally non-Gaussian

$$\zeta = N_\phi \delta\phi_* + \cdots + \frac{1}{2} N_{\sigma\sigma} (\delta\sigma_*)^2 + \cdots$$

➡ Ungaussiton-like model

- Non-linearity parameters:

$$\frac{6}{5} f_{\rm NL} = \frac{N_{\sigma\sigma}^3 \mathcal{P}_{\delta\sigma} \ln(k_{m1} L)}{N_\phi^4}$$

➡ $\tau_{\rm NL} = C P_\zeta^{-1/3} f_{\rm NL}^{4/3} \sim 10^3 f_{\rm NL}^{4/3}$

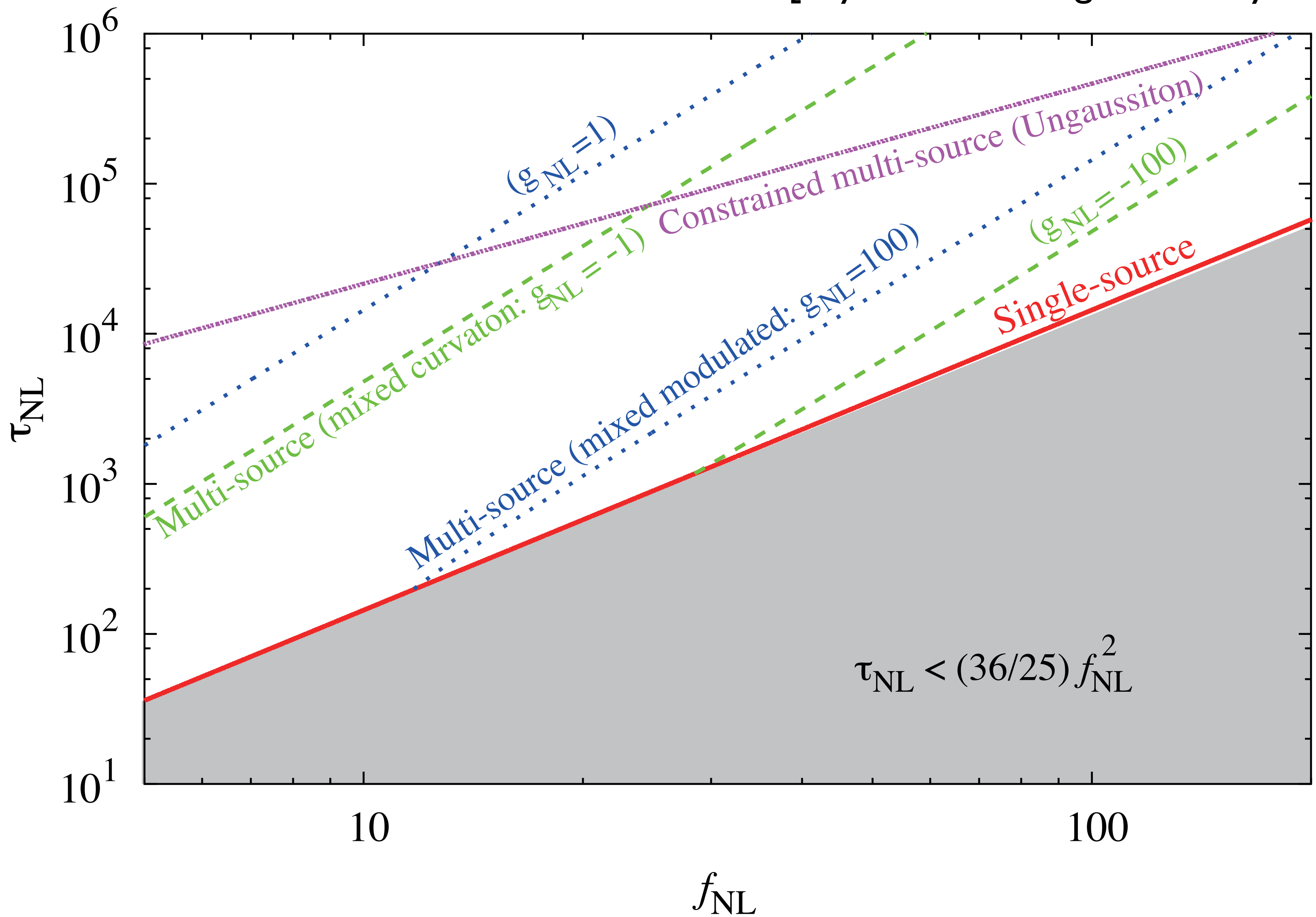
$$\tau_{\rm NL} = \frac{N_{\sigma\sigma}^4 \mathcal{P}_{\delta\sigma} \ln(k_{m1} L)}{N_\phi^6}$$

Multi-source model, but definite relation between $f_{\rm NL}$ and $\tau_{\rm NL}$

➡ Constrained multi-source model

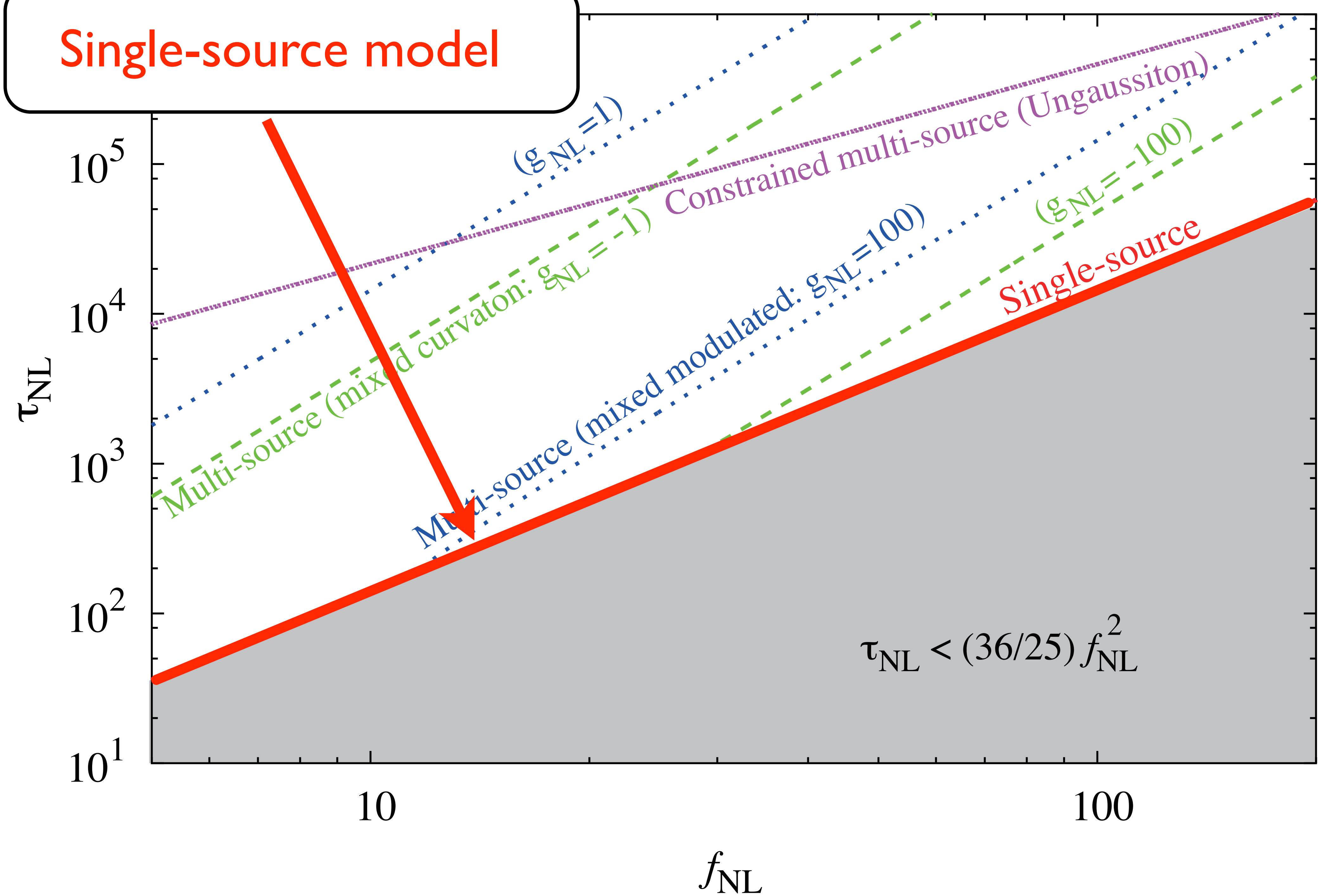
f_{NL} - τ_{NL} diagram

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

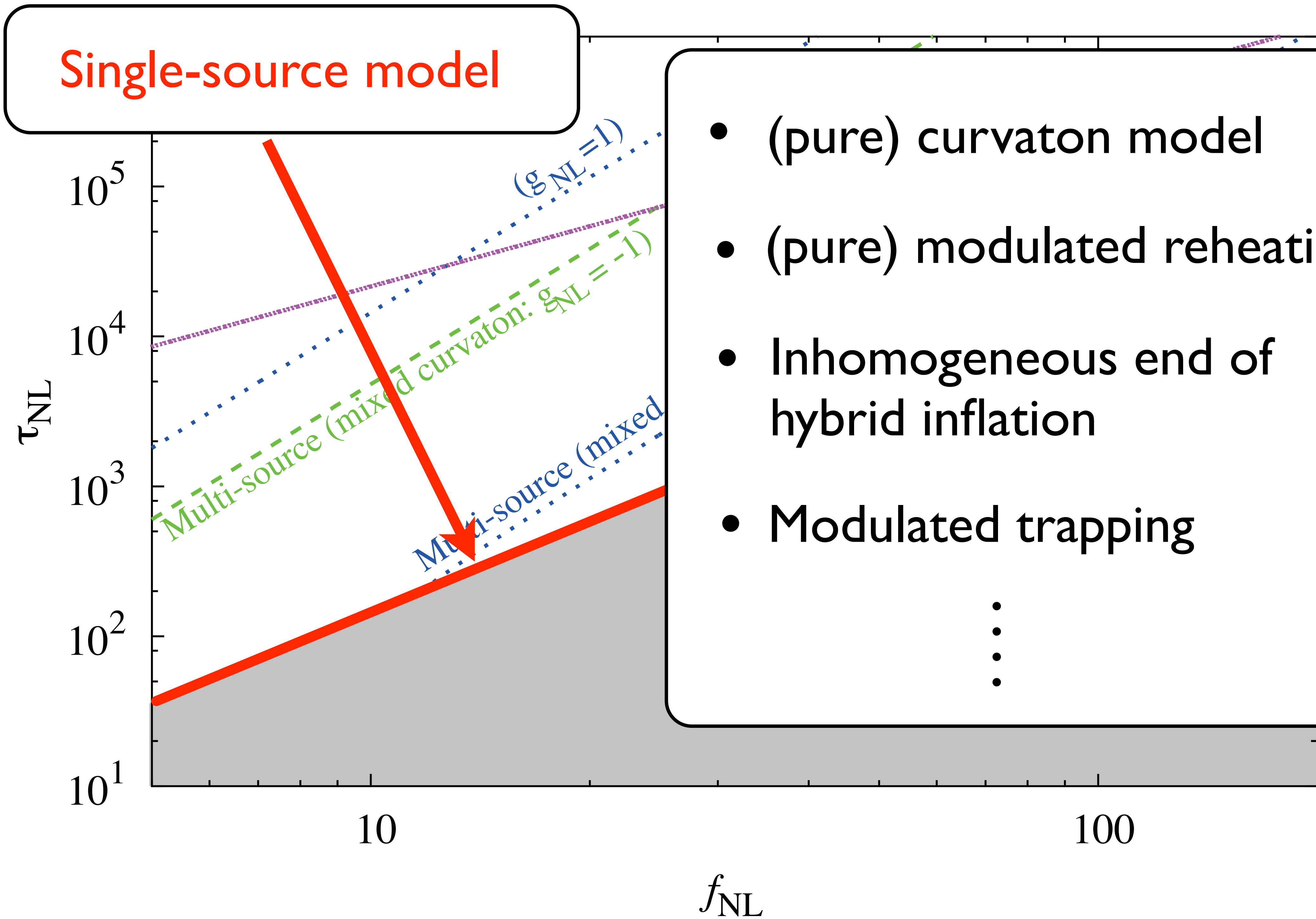


f_{NL} - τ_{NL} diagram

Single-source model



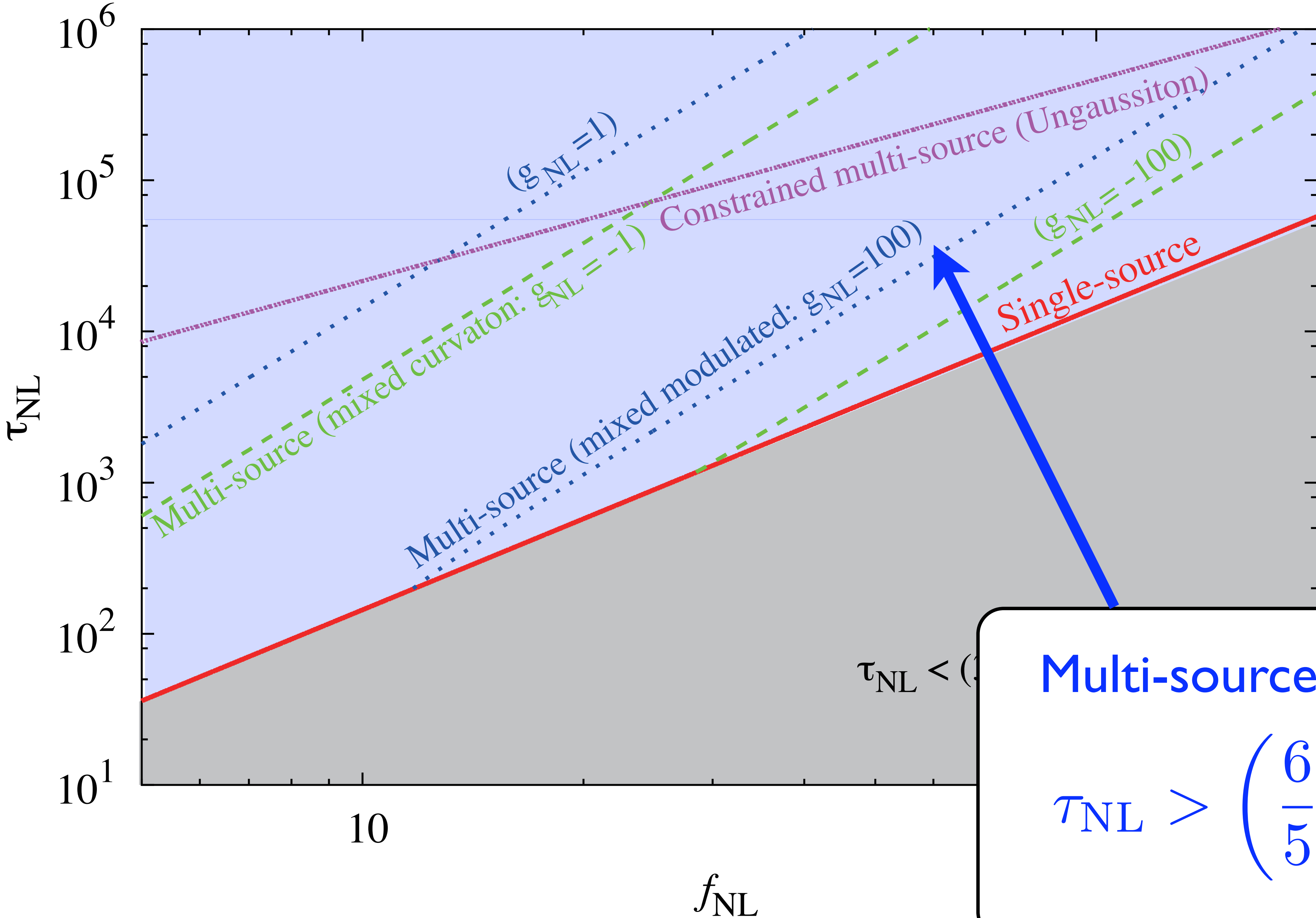
f_{NL} - τ_{NL} diagram



- (pure) curvaton model
- (pure) modulated reheating
- Inhomogeneous end of hybrid inflation
- Modulated trapping

⋮

f_{NL} - τ_{NL} diagram



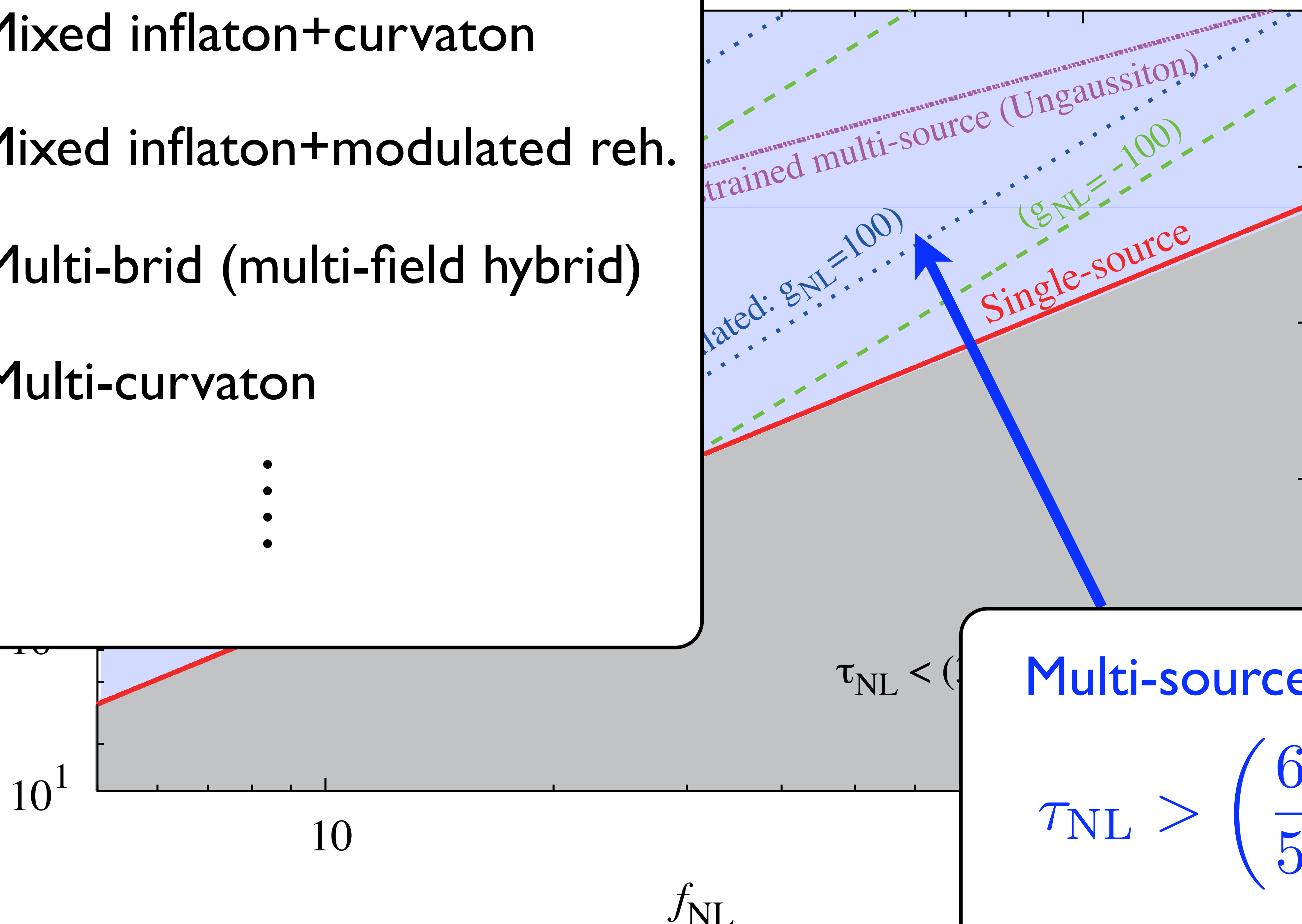
Multi-source model

$$\tau_{NL} > \left(\frac{6}{5} f_{NL}\right)^2$$

$f_{\rm NL}$ - $\tau_{\rm NL}$ diagram

- Mixed inflaton+curvaton
- Mixed inflaton+modulated reh.
- Multi-brid (multi-field hybrid)
- Multi-curvaton

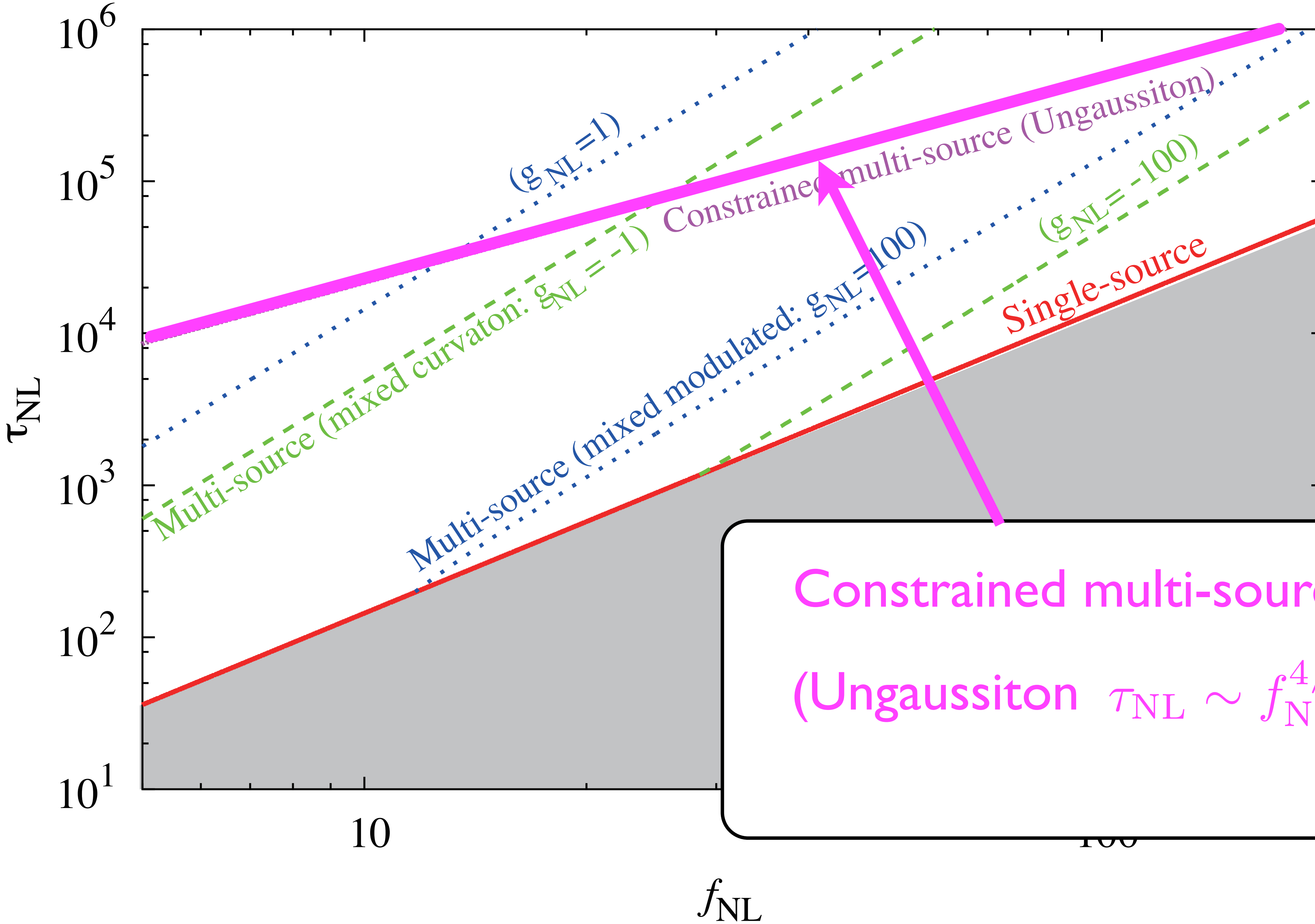
⋮



Multi-source model

$$\tau_{\rm NL} > \left(\frac{6}{5} f_{\rm NL}\right)^2$$

$f_{\rm NL}$ - $\tau_{\rm NL}$ diagram



Constrained multi-source

(Ungaussiton $\tau_{\rm NL} \sim f_{\rm NL}^{4/3}$)

Can we differentiate the model?

f_{NL} - τ_{NL} diagram

Single-source

Multi-source

Constrained multi-source

- There are still some (many) possibilities for each categories....

➡ We can further look at the relation between f_{NL} and g_{NL}.

$f_{\rm NL}$ - $g_{\rm NL}$ relation

Single-source model

Category	$f_{\rm NL}$–$\tau_{\rm NL}$ relation	Examples and $f_{\rm NL}$–$g_{\rm NL}$ relation
Single-source	$\tau_{\rm NL} = (6 f_{\rm NL}/5)^2$	(pure) curvaton (w/o self-interaction) $[g_{\rm NL} = -(10/3) f_{\rm NL} - (575/108)]^{(a)}$
		(pure) curvaton (w/ self-interaction) $[g_{\rm NL} = A_{\rm NQ} f_{\rm NL}^2 + B_{\rm NQ} f_{\rm NL} + C_{\rm NQ}]^{(b)}$
		(pure) modulated reheating $[g_{\rm NL} = 10 f_{\rm NL} - (50/3)]^{(c)}$
		modulated-curvaton scenario $\left[g_{\rm NL} = 3 r_{\rm dec}^{1/2} f_{\rm NL}^{3/2}\right]^{(d)}$
		Inhomogeneous end of hybrid inflation $[g_{\rm NL} = (10/3) \eta_{\rm cr} f_{\rm NL}]$
		Inhomogeneous end of thermal inflation $[g_{\rm NL} = -(10/3) f_{\rm NL} - (50/27)]^{(e)}$
		Modulated trapping $[g_{\rm NL} = (2/9) f_{\rm NL}^2]^{(f)}$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

$f_{\rm NL}$ - $g_{\rm NL}$ relation

Multi-source model

Multi-source	$\tau_{\rm NL} > (6f_{\rm NL}/5)^2$	mixed curvaton and inflaton $[g_{\rm NL} = -(10/3)(R/(1+R))f_{\rm NL} - (575/108)(R/(1+R))^3]^{(g)}$
		mixed modulated and inflaton $[g_{\rm NL} = 10(R/(1+R))f_{\rm NL} - (50/3)(R/(1+R))^3]^{(h)}$
		mixed modulated trapping and inflaton $[g_{\rm NL} = (2/9)((1+R)/R)f_{\rm NL}^2 = (25/162)\tau_{\rm NL}]^{(i)}$
		multi-curvaton $[g_{\rm NL} = C_{\rm mc}f_{\rm NL},\ \ g_{\rm NL} = (4/15)f_{\rm NL}^2]^{(j)}$
		Multi-brid inflation (quadratic potential) $[g_{\rm NL} = -(10/3)\eta f_{\rm NL},\ \ g_{\rm NL} = 2f_{\rm NL}^2]^{(k)}$
		Multi-brid inflation (linear potential) $[g_{\rm NL} = 2f_{\rm NL}^2]^{(l)}$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

We can divide into three types by using the relation between $f_{\rm NL}$ and $g_{\rm NL}$

- "Linear" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}$$

(with O(1) coefficient)

- "Suppressed" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim (\text{suppression factor}) \times f_{\rm NL}$$

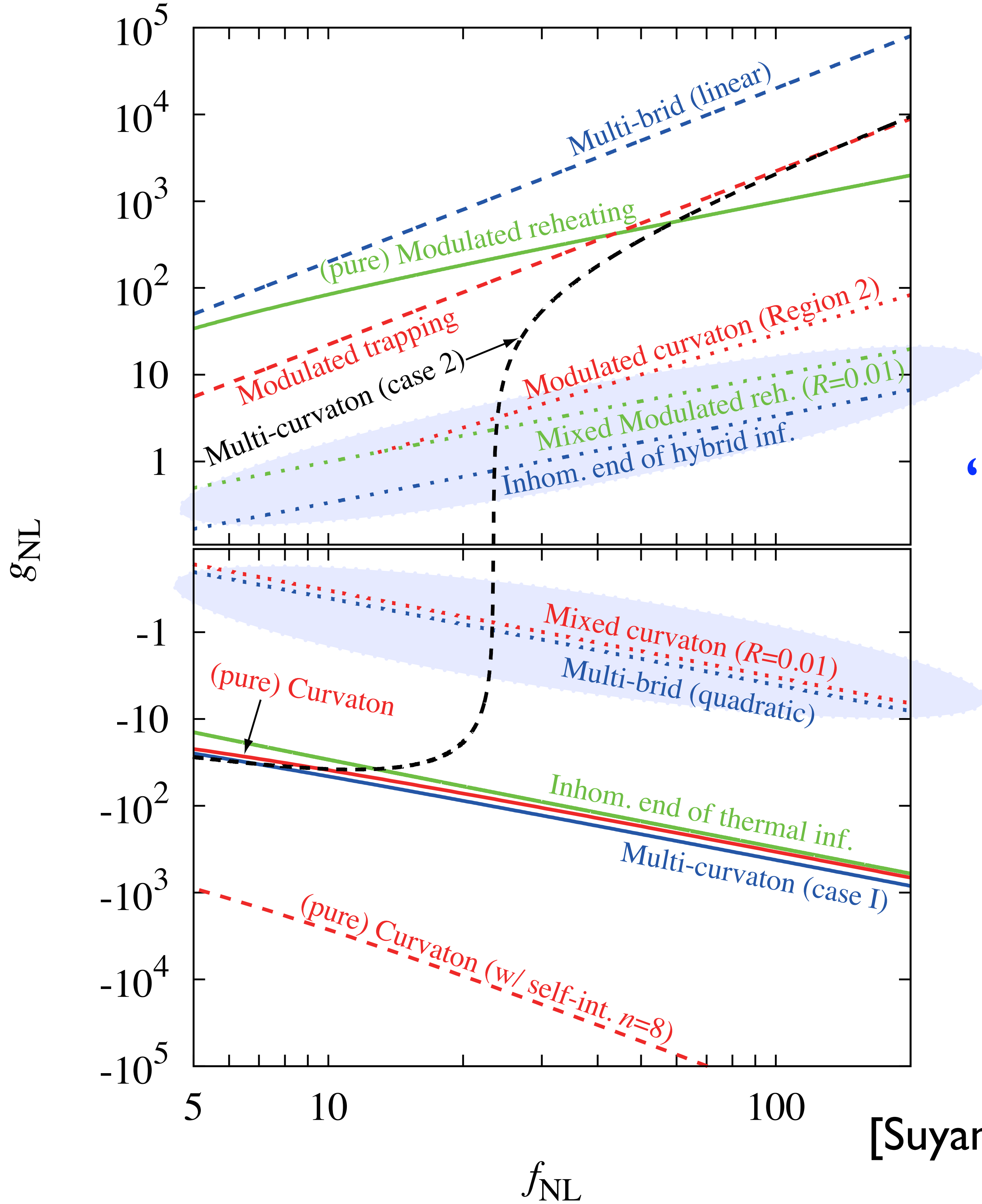
(e.g., suppressed by the slow-roll params. ε, η)

- "Enhanced" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}^n$$

(n > 1, in many models n=2)

f_{NL} - g_{NL} diagram

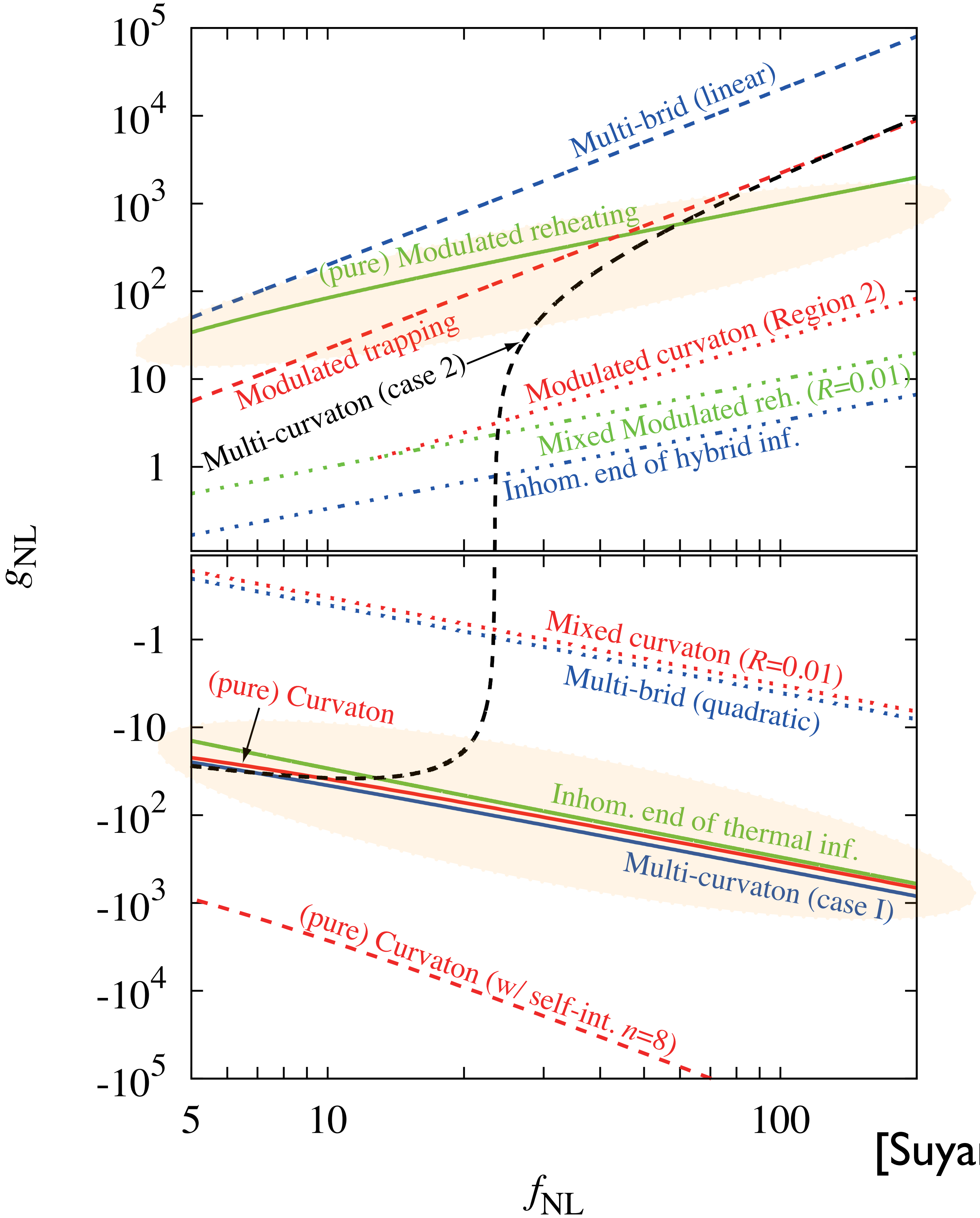


"Suppressed" g_{NL} Type

$$g_{NL} \sim (\text{suppression factor}) \times f_{NL}$$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

$f_{\rm NL}$ - $g_{\rm NL}$ diagram

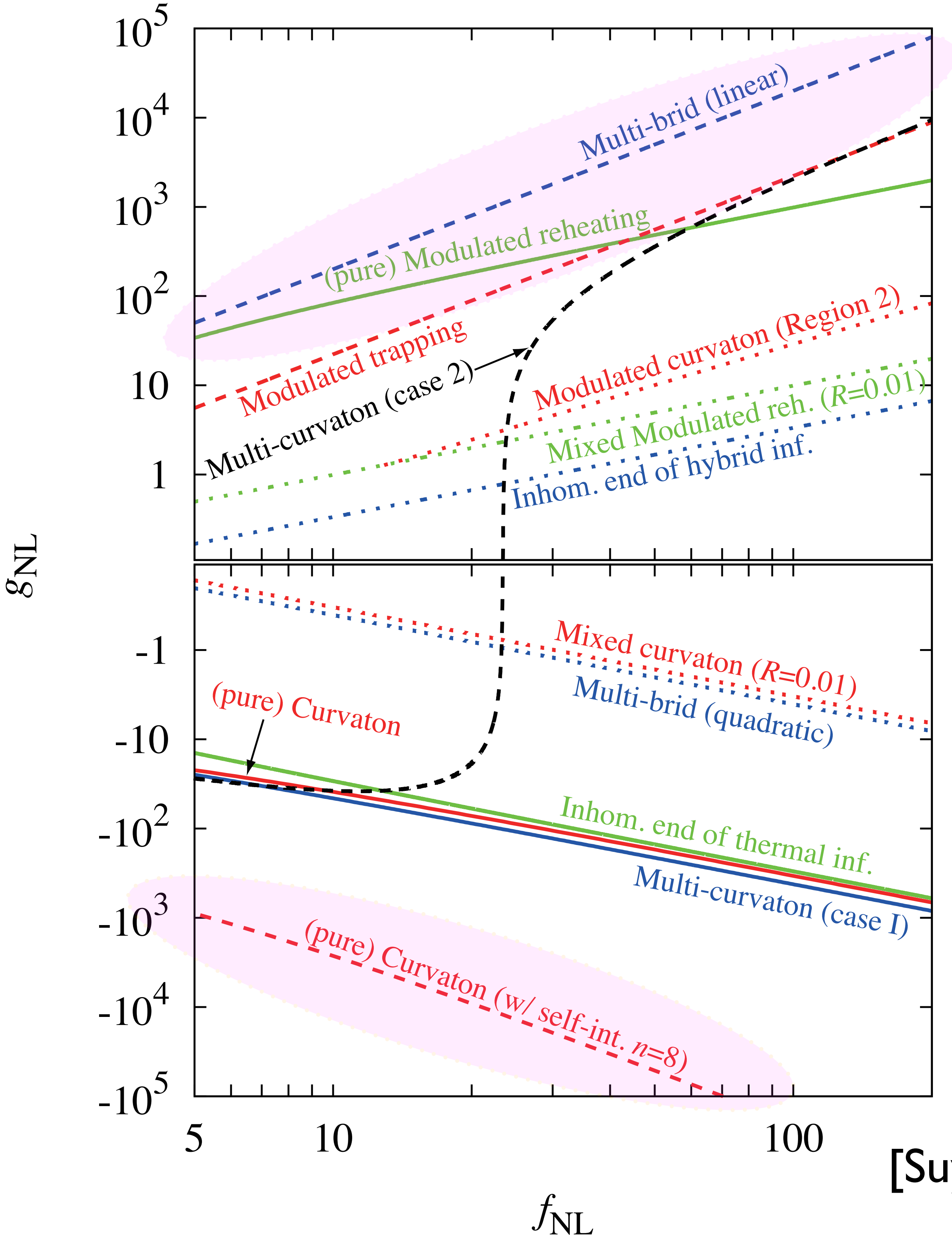


"Linear" $g_{\rm NL}$ Type

$$g_{\rm NL} \sim f_{\rm NL}$$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

f_{NL} - g_{NL} diagram



"Enhanced" g_{NL} Type

$$g_{NL} \sim f_{NL}$$

[Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

Summary

- We can classify models of large (local-type) non-Gaussianity using $f_{\rm NL}$, $\tau_{\rm NL}$ and $g_{\rm NL}$:

▶ $f_{\rm NL}$ - $\tau_{\rm NL}$ relation

- single-source
- multi-source
- constrained multi-source

▶ $f_{\rm NL}$ - $g_{\rm NL}$ relation

- "Linear" $g_{\rm NL}$ Type
- "Suppressed" $g_{\rm NL}$ Type
- "Enhanced" $g_{\rm NL}$ Type

- Local-type models should satisfy the "local-type inequality."

$$\tau_{\rm NL} > \left(\frac{6}{5} f_{\rm NL}\right)^2$$