Loop corrections to cosmological correlation functions

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Based on

- JG, J.-c. Hwang and H. Noh, arXiv:1010.XXXX
- D. Jeong, JG, J.-c. Hwang and H. Noh, arXiv:1010.XXXX

Outline

Higher order contributions to correlation functions

Why?

- Non-linear perturbations
- Higher order effects
- Recent claims: breakdown of perturbation theory?

B. Losic and W. G. Unruh, Phys. Rev. Lett. **101**, 111101 (2008) [arXiv:0804.4296 [gr-qc]]

H. Noh, D. Jeong and J. c. Hwang, Phys. Rev. Lett. **103**, 021301 (2009) [arXiv:0902.4285 [astro-ph.CO]]

We consider 2 representative power spectra with simplifications

1. Curvature perturbation $\mathscr{R}$
   - Single field inflation
   - Large scales in de Sitter limit
2. Matter density contrast δ
   - Einstein-de Sitter universe
   - Curl-free velocity

Basic strategy

We proceed as follows

1. Choose a gauge: comoving gauge
2. Set up the equations
   - 3+1 decomposition: 2 constraint + dynamical equations
3. Expand up to appropriate order
   - Let $g(\mathbf{k}) = g_1(\mathbf{k}) + g_2(\mathbf{k}) + g_3(\mathbf{k}) + \cdots$
   - For power spectrum,

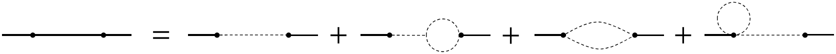
$$\langle g(\mathbf{k}_1)g(\mathbf{k}_2)\rangle \sim \langle g_1 g_1\rangle + \langle g_1 g_2\rangle + [\langle g_2 g_2\rangle + \langle g_1 g_3\rangle] + \cdots$$

   We need up to 3rd order

4. Calculate solutions up to that order
5. Calculate the power spectrum
   - Inflation: $\langle\cdots\rangle \to$ vacuum expectation value
   - EdS: $\langle\cdots\rangle \to$ ensemble average

One-loop corrected power spectrum

Higher order corrections are given as momentum loop integrals



$$P(k) = P^{(11)}(k) + P^{(12)}(k) + P^{(22)}(k) + P^{(13)}(k)$$

With $g(\mathbf{k}) = g_1(\mathbf{k}) + g_2(\mathbf{k}) + g_3(\mathbf{k}) + \cdots$,

$$\langle g(\mathbf{k}_1) g(\mathbf{k}_2) \rangle \sim \langle g_1 g_1 \rangle \quad \to P^{(11)}(k)$$

$$+ \langle g_1 g_2 \rangle \quad \to P^{(12)}(k) \sim \int \underbrace{\langle g_1(\mathbf{k}) g_1(\mathbf{q}) g_1(-\mathbf{k}-\mathbf{q}) \rangle}_{= B_g(\mathbf{k},\mathbf{q},-\mathbf{k}-\mathbf{q})}$$

$$+ \left[\langle g_2 g_2 \rangle + \langle g_1 g_3 \rangle \right] \quad \to \begin{cases} P^{(22)}(k) \sim \int P^{(11)}(\mathbf{q}) P^{(11)}(\mathbf{k}-\mathbf{q}) \\ P^{(13)}(k) \sim \int P^{(11)}(\mathbf{k}) P^{(11)}(\mathbf{q}) \end{cases}$$

$$+ \cdots$$

Next-to-leading (bispectrum) while next-next-to-leading (rest)

Curvature perturbation up to 3rd order

Working in the comoving gauge: $\delta\phi = 0$

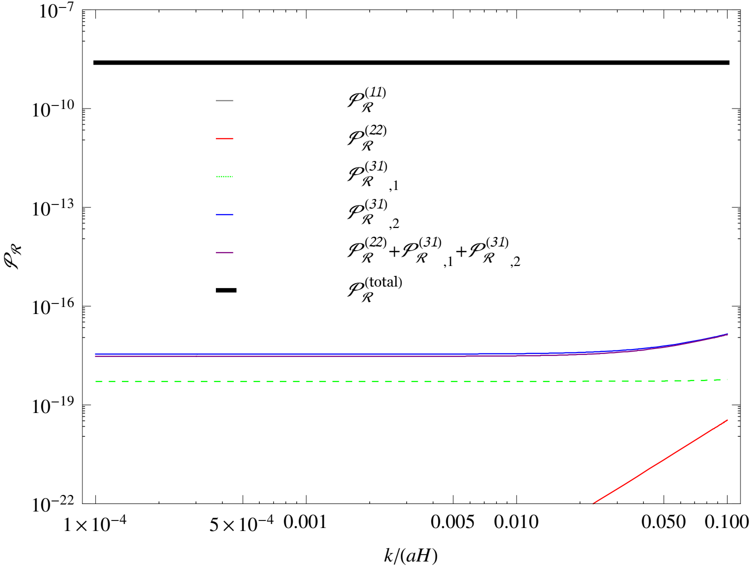
$$
\begin{aligned}
&\mathcal{P}_\mathcal{R}(k) = \\
&\mathcal{P}_\mathcal{R}^{(11)} + \frac{1}{4}\left[\mathcal{P}_\mathcal{R}^{(11)}\right]^2\left(\frac{k}{k_H}\right)^4\int_0^\infty dr\frac{1}{105r^2|1+r||1-r|} \\
&\qquad\times\left[\left(-41+64r+14r^2+84r^3+70r^4+14r^5+14r^6-6r^7-6r^8\right)|1-r|\right. \\
&\qquad\left.+\left(41+64r-14r^2+84r^3-70r^4+14r^5-14r^6-6r^7+6r^8\right)|1+r|\right] \\
&-\frac{1}{8}\left[\mathcal{P}_\mathcal{R}^{(11)}\right]^2\left(\frac{k}{k_H}\right)^4\int_0^\infty\frac{dr}{r}\left[\frac{1}{24}\left(-3r^6+75r^4+323r^2+213\right)-\frac{1}{16r}\left(r^2-1\right)^2\left(r^4-2r^2-7\right)\log\left|\frac{1-r}{1+r}\right|\right] \\
&-\frac{1}{2}\left[\mathcal{P}_\mathcal{R}^{(11)}\right]^2\left(\frac{k}{k_H}\right)^2\int_0^\infty\frac{dr}{r}\left[\frac{1}{12}\left(9r^4-24r^2-115\right)+\frac{1}{8r}\left(3r^6-5r^2+2\right)\log\left|\frac{1-r}{1+r}\right|\right]
\end{aligned}
$$

- Dimensionless power spectrum

$$
\left\langle\mathcal{R}_\mathbf{k}\mathcal{R}_\mathbf{q}\right\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})\frac{2\pi^2}{k^3}\mathcal{P}_\mathcal{R}(k)
$$

- Valid on super-horizon scales: $k \gg k_H \equiv aH$
- Slow-roll limit: $\mathcal{P}_\mathcal{R}^{(11)} \approx 2.5\times10^{-9}$ =constant

Power spectrum of $\mathscr{R}(\mathbf{k})$



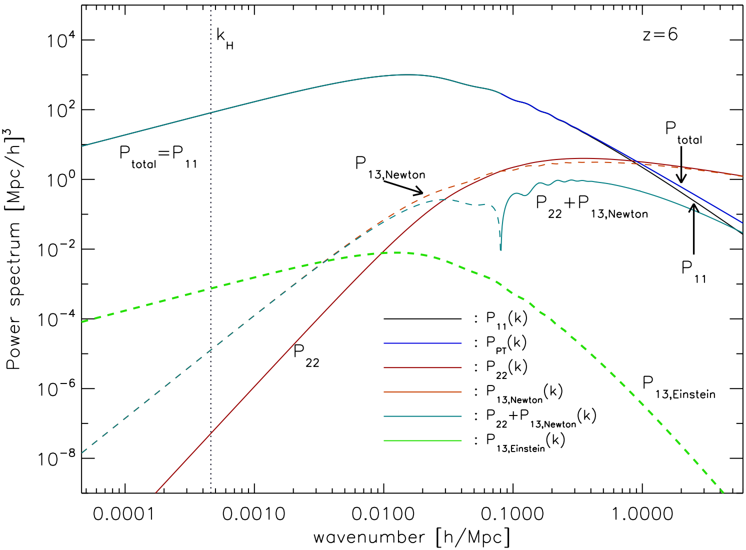
Matter density contrast up to 3rd order

Comoving gauge: $T^i{}_0 = 0$

$$
\begin{aligned}
P(k) = & P_L(k) + \frac{1}{98}\frac{k^3}{(2\pi)^2}\int_0^\infty drP_L(kr)\int_{-1}^1 dxP_L\left(k\sqrt{1+r^2-2rx}\right)\frac{\left(3r+7x-10rx^2\right)^2}{\left(1+r^2-2rx\right)^2} \\
& + \frac{1}{252}\frac{k^3}{(2\pi)^2}P_L(k)\int_0^\infty drP_L(kr)\left[-42r^4+100r^2-158+\frac{12}{r^2}+\frac{3}{r^3}\left(r^2-1\right)^3\left(7r^2+2\right)\log\left|\frac{1+r}{1-r}\right|\right] \\
& + \frac{10}{224}\left(\frac{k_H}{k}\right)^2\frac{k^3}{(2\pi)^2}P_L(k) \\
& \quad \times\int_0^\infty drP_L(kr)\left[172r^2-260-\frac{144}{r^2}+\frac{2}{r^3}\left(36+53r^2-46r^4-43r^6\right)\log\left|\frac{1+r}{1-r}\right|\right]
\end{aligned}
$$

- Valid on all scales: no large scale approximation
- Precisely the same results up to 2nd order as Newtonian
- GR correction (term with k_H) appears only at 3rd order

Power spectrum of $\delta(\mathbf{k})$



Conclusions and outlooks

1. Non-linear contributions to the power spectrum
   - 3rd order perturbation theory
   - Comoving gauge
2. $\mathscr{R}$ from single field inflation, δ in Einstein-de Sitter universe
   - Very small on all scales
   - No divergence: both at IR and UV
3. To consider
   - Bispectrum: up to 4th order (to appear)
   - Higher order effects: $f_{\rm NL}$, bias, etc (in preparation)
   - More realistic models: slow-roll, cosmological constant, etc
   - Different spatial gauge conditions: no non-locality