Peculiar Velocity PDF and signature of primordial non-Gaussianity

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ABSTRACT

We study how primordial non-Gaussianity affects the pairwise velocity probability density function (PDF) using an analytical model and cosmological N-body simulations. We develop an analytical model based on the Zeldovich approximation to describe the evolution of the pairwise velocity PDF. We show thatour analytical model matches the measurements while linear theory fails to predict the PDF in the f_{nl} model. We also show explicitly how f_{nl} induces correlations between originally independent velocities along the parallel and the perpendicular to the line of separation directions.

1 Introduction

Detections of primordial non-Gaussianity can discriminate inflationary models. The local f_{nl} model parameterizes the primordial perturbation potential as,

$$\Phi = \phi + f_{nl}(\phi^2 - \langle\phi^2\rangle), \quad (1)$$

where ϕ is the Gaussian potential field.
Most of the studies on the effect of primordial non-Gaussianity on large-scale structure (LSS) focus on density-related quantities – scale-dependent halo bias, halo/void abundances, bi/tri-spectrum. The motivation of this work is two-folded:

1. Introduce a complementary LSS probe using the peculiar velocity field;
2. Develop an analytical model that can describe the change in the peculiar velocity PDF due to primordial non-Gaussianity.

2 Linear Pairwise Velocity PDF

The Poisson Equation relates the linear overdensity $\delta(k)$ and the Bardeen potential Φ while the continuity equation relates $\delta(k)$ and the peculiar velocity $\boldsymbol{u}(\boldsymbol{k}, z)$, hence:

$$u_j(k) = i\dot{D}(z)k_j M(k)\Phi(k), \text{ where } M(k) = 2c^2T(k)/3\Omega_m H_0^2 \text{ and } j = (\parallel, \perp_a, \perp_b). \quad (2)$$

Denote the pairwise velocity by $\boldsymbol{v}(\boldsymbol{r}) \equiv \boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x})$.

2.1 Uniform weighted PDF

The linear pairwise velocity PDF when $f_{nl} = 0$ is described by the multivariate Gaussian distribution:

$$p_0(\boldsymbol{v}; r) = \frac{1}{(2\pi)^{3/2}\sqrt{|A|}} \exp\left(-\frac{1}{2}\boldsymbol{v}^T A^{-1}\boldsymbol{v}\right) = p_0(v_\parallel; r)p_0(v_{\perp_a}; r)p_0(v_{\perp_b}; r), \quad (3)$$

where $\boldsymbol{v} = (v_\parallel, v_{\perp_a}, v_{\perp_b})$ and the last equality is due to no correlation between v_j's (i.e.$\langle v_i v_j\rangle \equiv 0$ when $f_{nl} = 0$).
When $f_{nl} \neq 0$, primordial bispectrum is non-zero and connected moments higher than 2nd order contribute to the PDF. The leading non-vanishing connected moments are $\langle v_\parallel^3\rangle$ and $\langle v_\parallel v_\perp^2\rangle \rightarrow$ induces correlations between $v_\parallel$ and $v_\perp$. The first order correction to the linear pairwise velocity PDF is

$$p(v_\parallel, v_{\perp_a}, v_{\perp_b}; f_{nl}, r) = p_0(v_\parallel, v_{\perp_a}, v_{\perp_b}; r)[1 + \alpha_{300}h_{300} + \alpha_{120}(h_{120} + h_{102})] \quad (4)$$

$$\text{where} \quad \alpha_{300} = \frac{1}{6}\frac{\langle v_\parallel^3\rangle}{\langle v_\parallel^2\rangle^{3/2}}, \quad \alpha_{120} = \frac{1}{2}\frac{\langle v_\parallel v_\perp^2\rangle}{\langle v_\parallel^2\rangle^{1/2}\langle v_\perp^2\rangle}, \quad h_{ijk} \equiv H_i(\nu_\parallel)H_j(\nu_{\perp_a})H_k(\nu_{\perp_b})$$

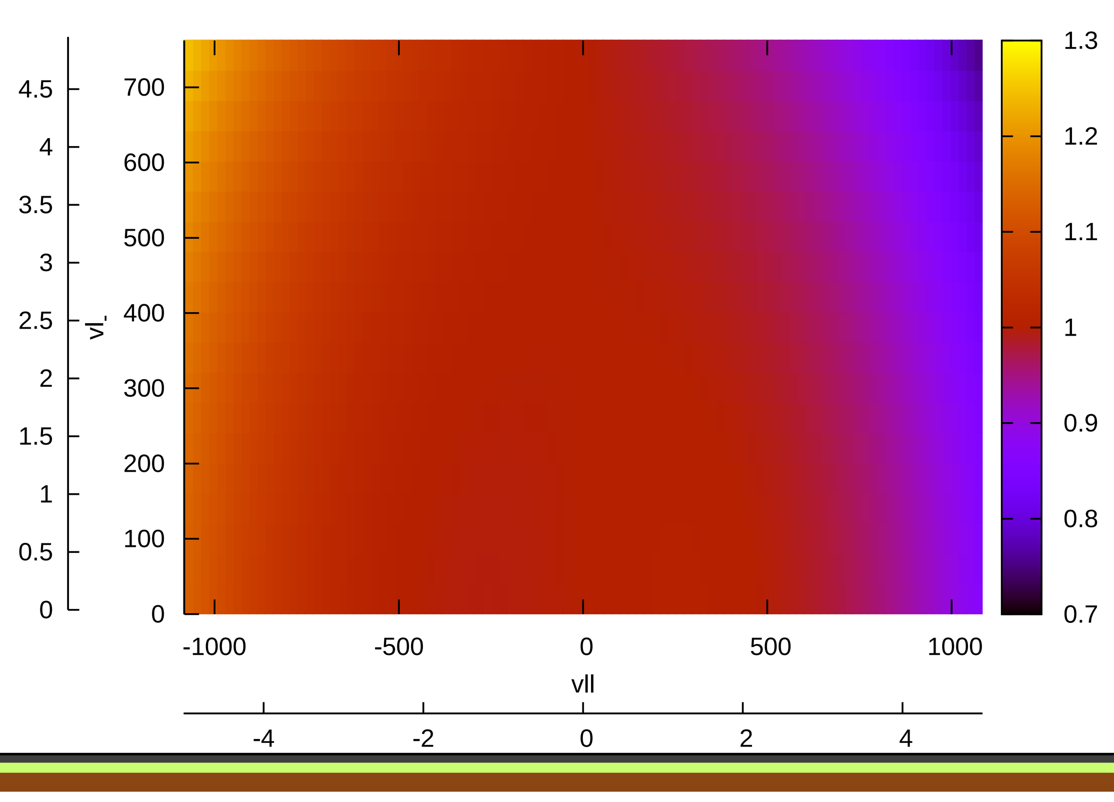


Fig 1: Ratio of uniform weighted linear pairwise PDF for $r = 8\ h^{-1}$Mpc and $f_{nl} = 100$ to the associated PDF for $f_{nl} = 0$ (equation (4)).

2.2 Pair weighted linear PDF

The uniform weighted linear PDF does not take into account the pair weighting which would be important (Sheth & Zehavi 2009). Scoccimarro (2004) discussed how to generalize the uniform weigthed linear pairwise PDF when f_{nl} to include pair weighting:

$$[1 + \xi(r)]\frac{q_0(\nu_\parallel, \nu_{\perp_a}, \nu_{\perp_b}; r)}{p_0(\nu_\parallel, \nu_{\perp_a}, \nu_{\perp_b}; r)} = 1 + \xi(r) + h_{100}\beta_{100} + h_{200}\beta_{200}, \quad (5)$$

where β_{100} and β_{200} are associated with the correlation between the pairwise velocity and the linear overdensity. When $f_{nl} \neq 0$ additional terms (related to 3pt function) contribute (see arXiv:*1008.0406* for details).

3 Evolution model of pairwise velocity PDF

The analytical evolution model is based on Zeldovich approximation which assumes the comoving velocity remains unchanged. At some initial z_i two particles separated by r_i had a relative velocity $(v_\parallel^i, v_{\perp_a}^i, v_{\perp_b}^i)$. At a later redshift z_0 the separation and the relative velocity of the two particles become

$$r^2 = \left(r_i + \frac{D_0}{\dot{D}_i}v_\parallel^i\right)^2 + \left(\frac{D_0}{\dot{D}_i}\right)^2 \left({v_{\perp_a}^i}^2 + {v_{\perp_b}^i}^2\right) \quad (6)$$

$$v_\parallel = \frac{\dot{D}_0}{r}\left(\frac{r_i v_\parallel^i}{\dot{D}_i} + \frac{D_0}{\dot{D}_i^2}{v^i}^2\right) \quad (7)$$

$$|v_\perp|^2 = v_{\perp_a}^2 + v_{\perp_b}^2 = \left(\frac{\dot{D}_0}{\dot{D}_i}v^i\right)^2 - v_\parallel^2. \quad (8)$$

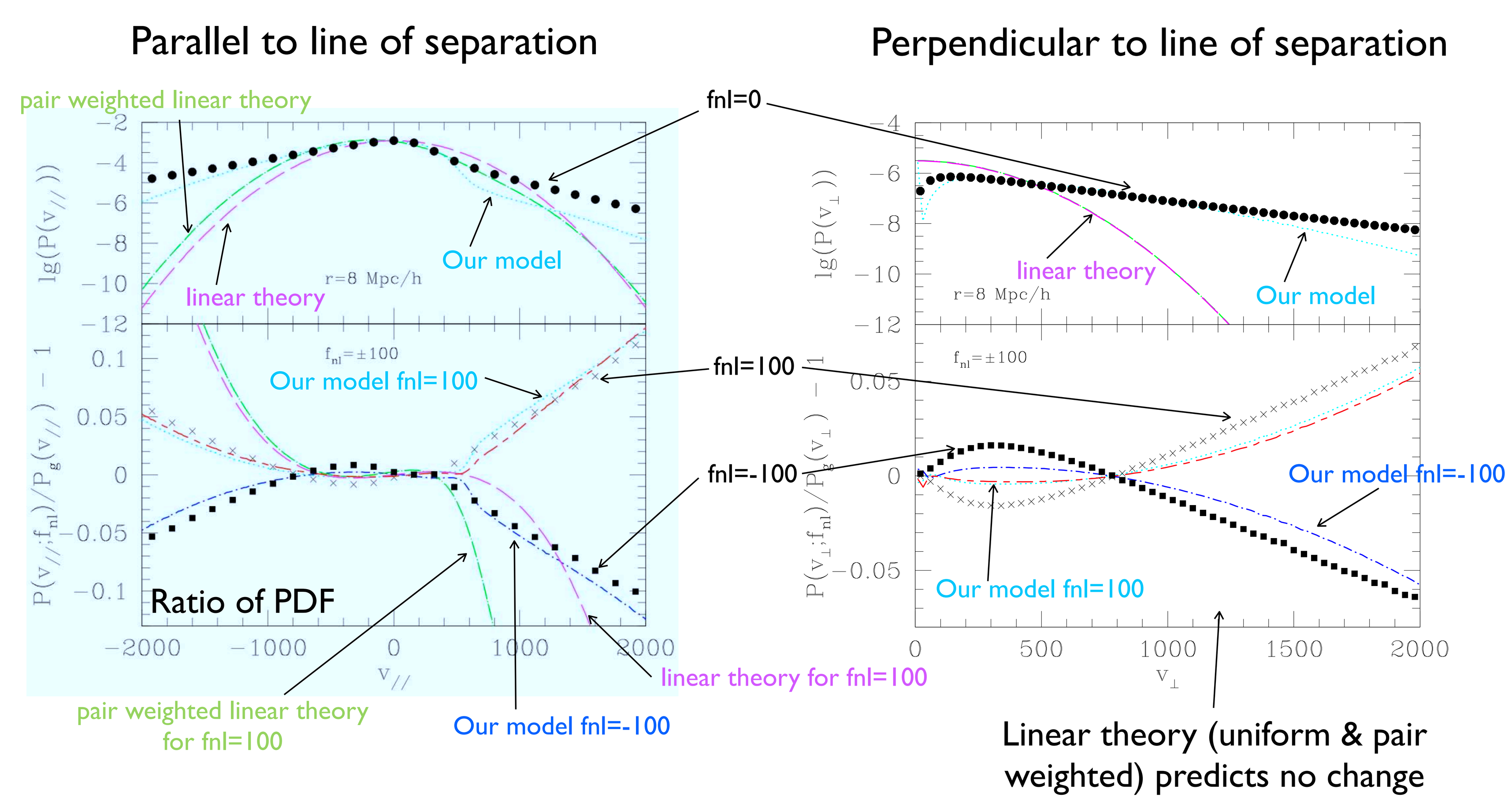
The evolved pairwise velocity PDF is therefore

$$p(V_\parallel, V_\perp; R) = \int \mathrm{d}r_i \mathrm{d}v_\parallel^i \mathrm{d}v_{\perp_a}^i \mathrm{d}v_{\perp_b}^i \frac{r_i^2}{R^2} p(v_\parallel^i, v_{\perp_a}^i, v_{\perp_b}^i; r)\delta_{\mathrm{D}}(r-R)\delta_{\mathrm{D}}(v_\parallel - V_\parallel)\delta_{\mathrm{D}}(v_\perp - V_\perp), \quad (9)$$

4 Comparisons with simulations

Our analytical predictions are then compared to measurements from N-body simulations. The simulations were performed in boxes of 2000 h^{-1}Mpc on a side, each containing 512^3 particles with *WMAP* 5-years ΛCDM best bit parameters. Pairwise velocity PDFs were measured from simulation outputs at $z = 0.5$ at various separations ($r = 4$, 8, 12, 50 h^{-1}Mpc). Here only the PDFs at $r = 8\ h^{-1}$Mpc are shown:

Why evolution is important



- Linear theory predictions fail in both parallel and perpendicular directions
- Our model agrees with measurements, both the profile when f_{nl}=0 and the ratio of PDF

Fig 2: The pairwise velocity PDF ($p(v_\parallel)$ on right and $p(v_\perp)$ on left for $r = 8\ h^{-1}$Mpc. Upper panels show the PDF profile when $f_{nl} = 0$; lower panels show the ratio of the PDF to the associated $f_{nl} = 0$ PDF.