Non-gaussianity in axion N-flation models

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Based on arXiv:1005.4410
by SAK, Andrew R. Liddle and David Seery (Sussex),
and earlier papers by SAK and Liddle.

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Assisted inflation

- Assisted inflation (Liddle-Mazumdar-Schnook 1998) is the observation that multiple scalar fields can cooperate to drive inflation even if each individually is unable to.

- Each field feels the acceleration from its own potential, but the collective Hubble friction from all fields.

N-flation

- N-flation (Dimopolos et al 2008) is a realization of assisted inflation using string axions.

Motivations and assumptions

- One motivation for this idea is that sufficient inflation can be obtained with all fields maintaining sub-Planckian values.

  Another is that it may be possible to relate assisted inflation to proper fundamental physics models.

- Focused on adiabatic perturbations.
- Random initial conditions for fields.
- Assumptions we made;

  Horizon crossing and Slow-roll approximations.

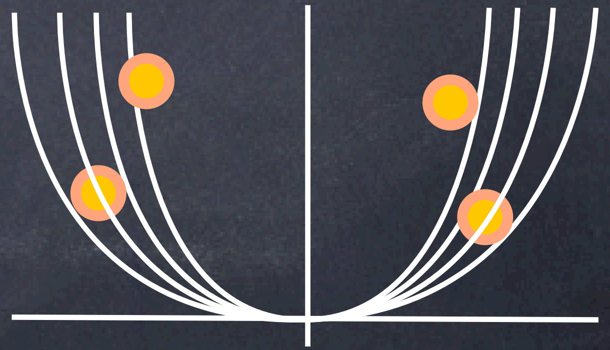
N-flation phenomenology

- The full string axion potential is

$$V_i = \Lambda_i^4 \left(1 - \cos\frac{2\pi\phi_i}{f_i}\right)$$

, where there are N_f fields with constants Λ_i and f_i. Throughout will ignore possible couplings btw the fields.

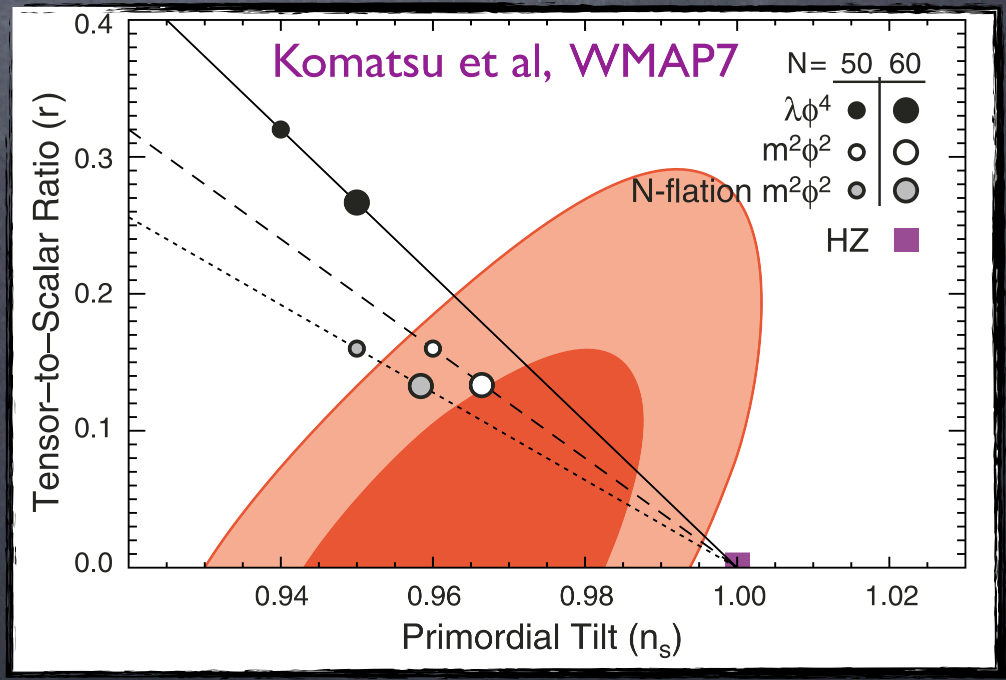
- This has been extensively explored in the quadratic approximation where all fields are close to their minima, in which case they are simply a set of massive fields with $m_i \equiv \frac{2\pi\Lambda_i^2}{f_i}$



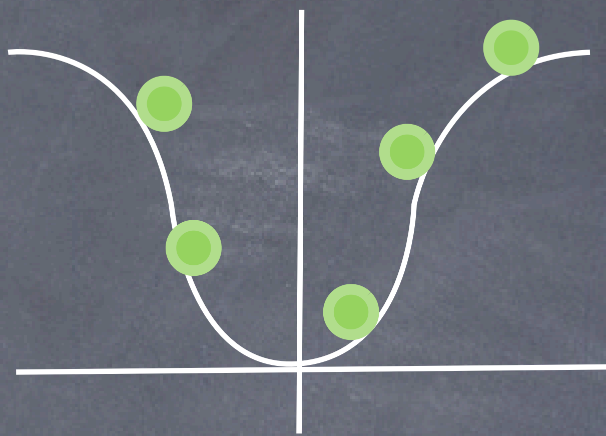
N-flation phenomenology

Regardless of these choices, the Nflation phenomenology in this approximation is remarkably simple:

- The tensor-to-scalar ratio always equals to the single-field values: $r = 8/N$ where N is the number of e-foldings.
- The scalar spectral index cannot exceed the single-field value, equalling it only in the equal-mass case: $n \leq 1-2/N$.
- The non-gaussianity f_{NL} always equals its single-field value: $f_{NL} = 2/N$ and hence is unobservably small.



The N-flation model



But..

.. in fact the quadratic approximation to the potential is unlikely to be valid. We should consider the full potential

$$V_i = \Lambda_i^4 \left(1 - \cos \frac{2\pi\phi_i}{f_i} \right)$$

Even if the potentials are all taken to be identical, assisted inflation is an attractor solution only if $d^2V/d\phi^2 > 0$ (Calcagni and Liddle 2008), which is not true near the maximum of the potential(s) where the trajectories will diverge.

[===> From now on, called the Naxion model]

Naxion equations

- With the full potential, the obervalbes can be calculated using the so-called δN formalism

$$\mathcal{P}_\zeta = \frac{H_*^2}{4\pi^2}\sum_i N_{,i}N_{,i} = \frac{H_*^2}{8\pi^2 M_{\rm P}^2}\sum_i \frac{1}{\epsilon_i^*}\,;$$

$$n-1 = -2\epsilon_* - \frac{8\pi^2}{3H_*^2}\sum_j \frac{\Lambda_j^4}{f_j^2}\frac{1}{\epsilon_j^*}\Big/\sum_i \frac{1}{\epsilon_i^*}\,;$$

$$r = \frac{2}{\pi^2\mathcal{P}_\zeta}\frac{H_*^2}{M_{\rm P}^2} = 16\Big/\sum_i \frac{1}{\epsilon_i^*}\,;$$

$$\frac{6}{5}f_{\rm NL} \simeq \frac{\sum_{ij} N_{,i}N_{,j}N_{,ij}}{\left(\sum_k N_{,k}N_{,k}\right)^2} = \frac{r^2}{128}\sum_i \frac{1}{\epsilon_i^*}\frac{1}{1+\cos\alpha_i^*}\,,$$

, where

$$\epsilon_i \equiv \frac{M_{\rm P}^2}{2}\left(\frac{V_i'}{V_i}\right)^2$$

$$\varepsilon \equiv -\dot{H}/H^2$$

$$\approx \Sigma_i (V_i/V)^2\, \varepsilon_i$$

- Here $\alpha_i=2\pi\phi_i/f_i$, ε_i is the slow-roll parameter of each field, derivatives wrt field i and indicated by a comma, and * indicates evaluation at horizon crossing

Naxion : N_{tot}

- The number of e-foldings is given by

$$N_{\rm tot} \simeq \sum_i \left(\frac{f_i}{2\pi M_{\rm P}}\right)^2 \ln\frac{2}{1+\cos\alpha_i} \simeq \frac{\ln 2}{2\pi^2}\frac{f^2}{M_{\rm P}^2}N_{\rm f}$$

where the last expression calculates the expectation value of the sum under assumption of uniformly-distributed angles α_i. For values of f of order the Planck mass, sufficient inflation requires a large number of fields, at least hundreds.

Naxion: n and r

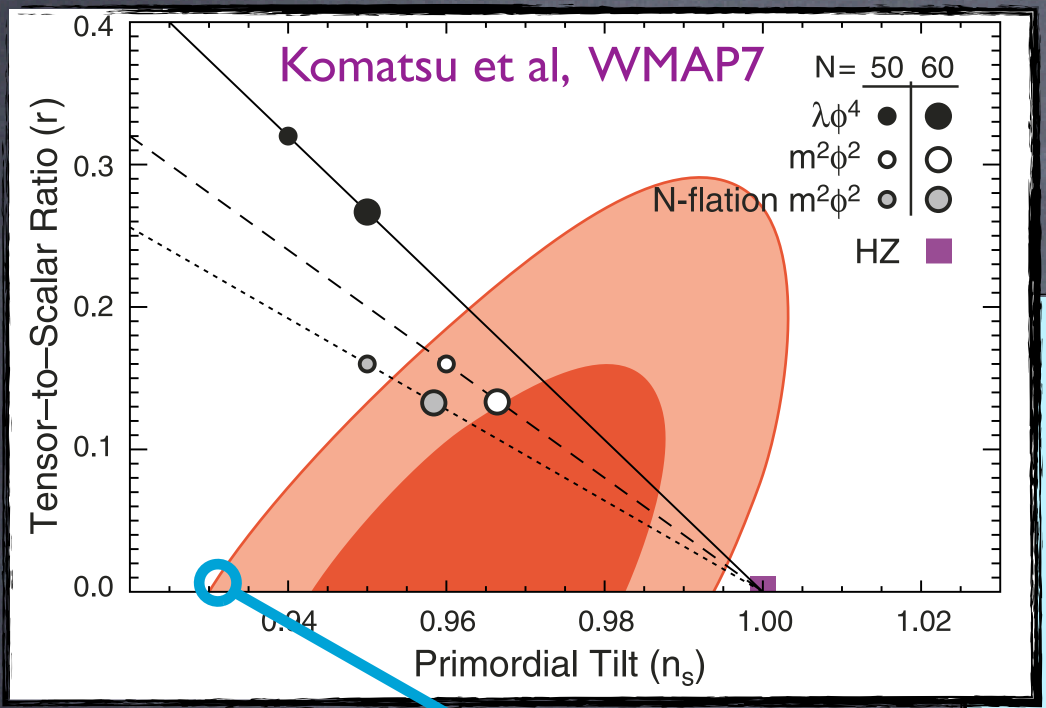
- A similar summation trick, assuming uniformly distributed fields, gives an analytic estimate of the spectral index as

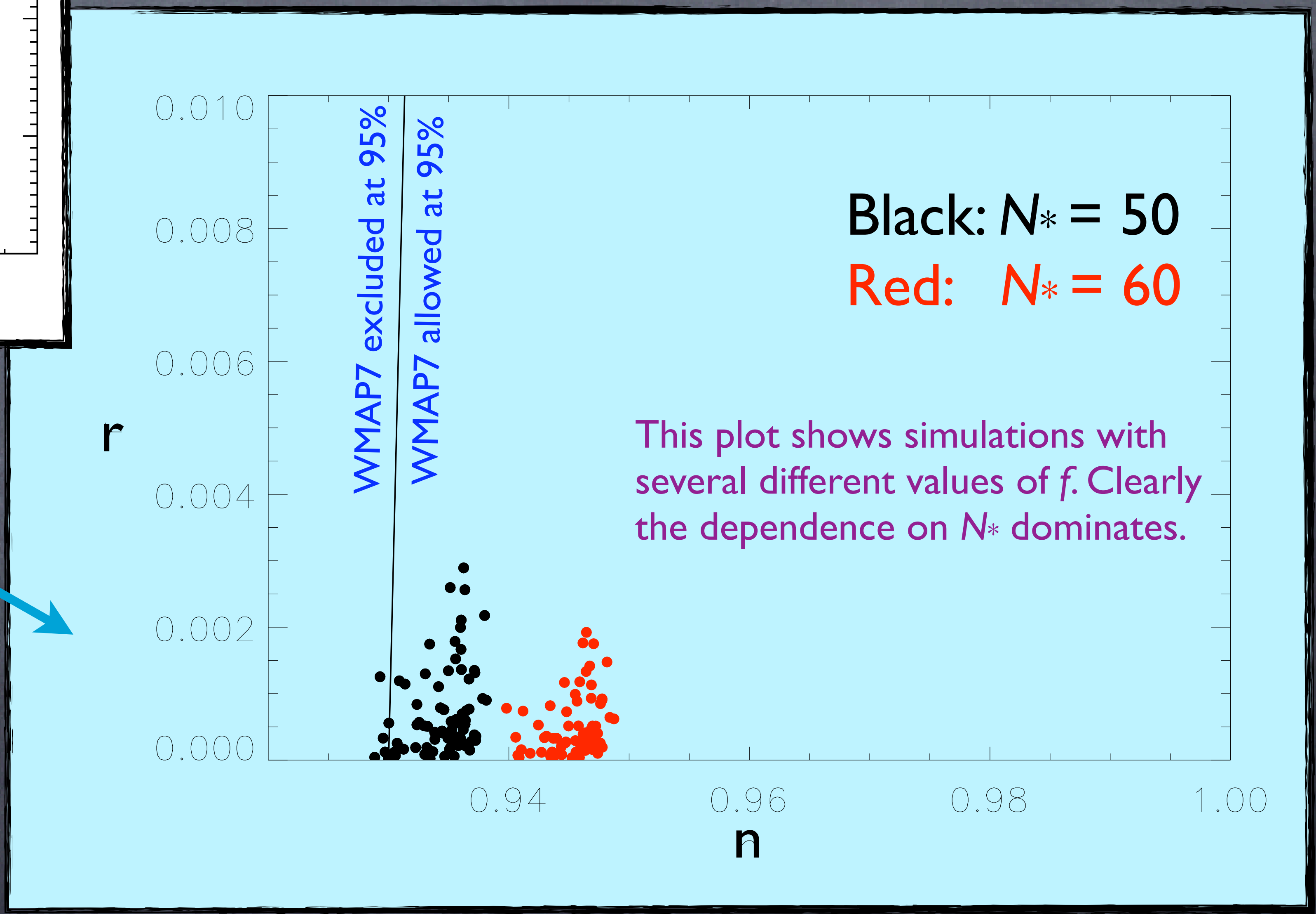
$$\langle n-1 \rangle \simeq -5 \ln 2 / N_*$$

NB: $5 \ln 2 \simeq 3.5$

- At the same time, the tensor-to-scalar ratio is highly suppressed by the small ε_i of fields close to the maximum.

Naxion: n and r





Naxion: non-gaussiaity

- The interesting aspect of the model is the behavior of the non-gaussianity:

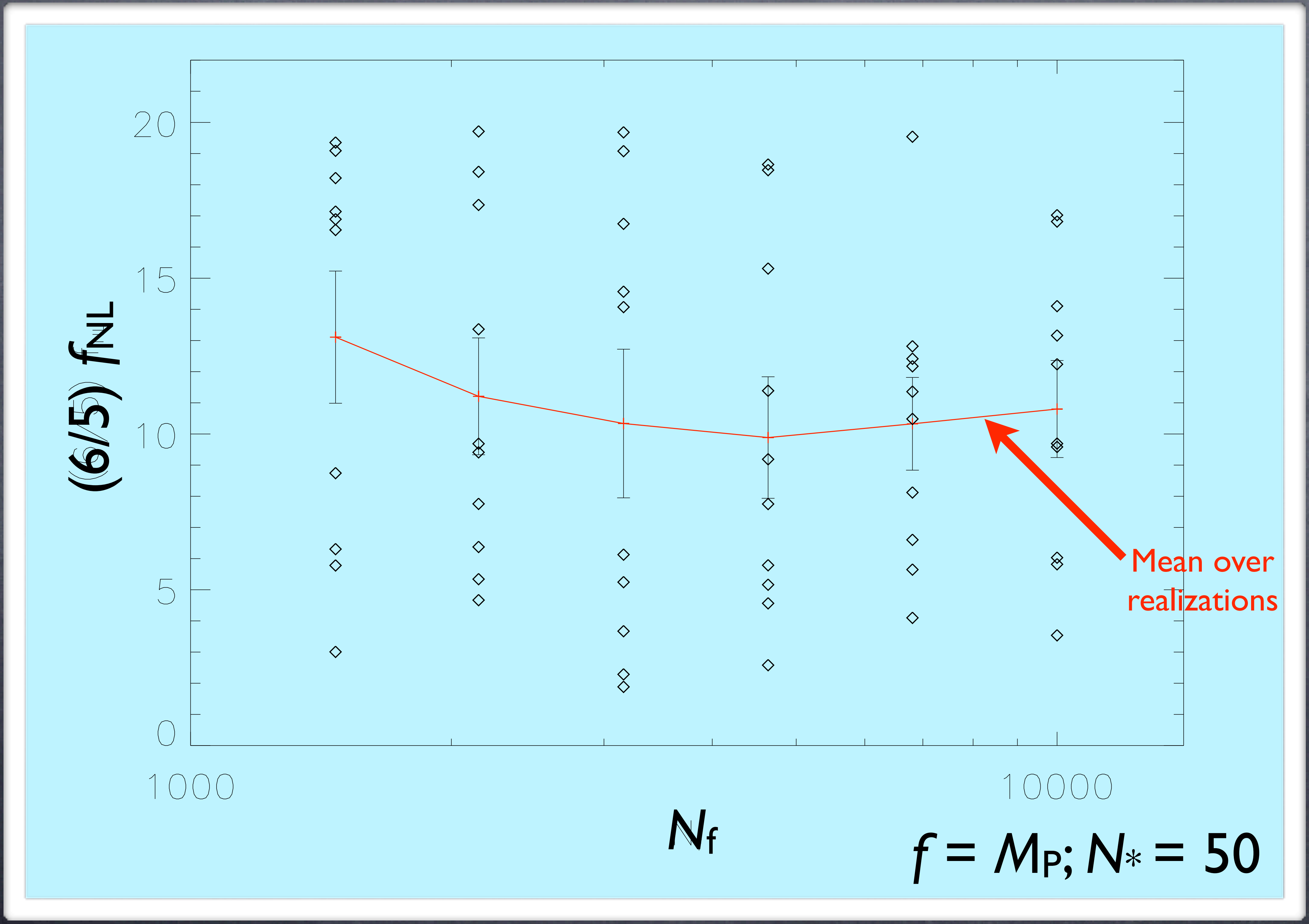
$$\frac{6}{5} f_{\rm NL} \simeq \frac{\sum_{ij} N_{,i} N_{,j} N_{,ij}}{\left(\sum_k N_{,k} N_{,k}\right)^2} = \frac{r^2}{128} \sum_i \frac{1}{\epsilon_i^*} \frac{1}{1+\cos\alpha_i^*},$$

- The sum may be dominated by a small number of fields whose α_i is very close to π. If there are $\tilde{N}$ fields which dominate with comparable ε_i, there is an approximate form

$$\frac{6}{5} f_{\rm NL} \approx \frac{2\pi^2}{\bar{N}} \left(\frac{M_{\rm P}}{f}\right)^2,$$

hence for f of oder M_p the f_{NL} may be of order tens.

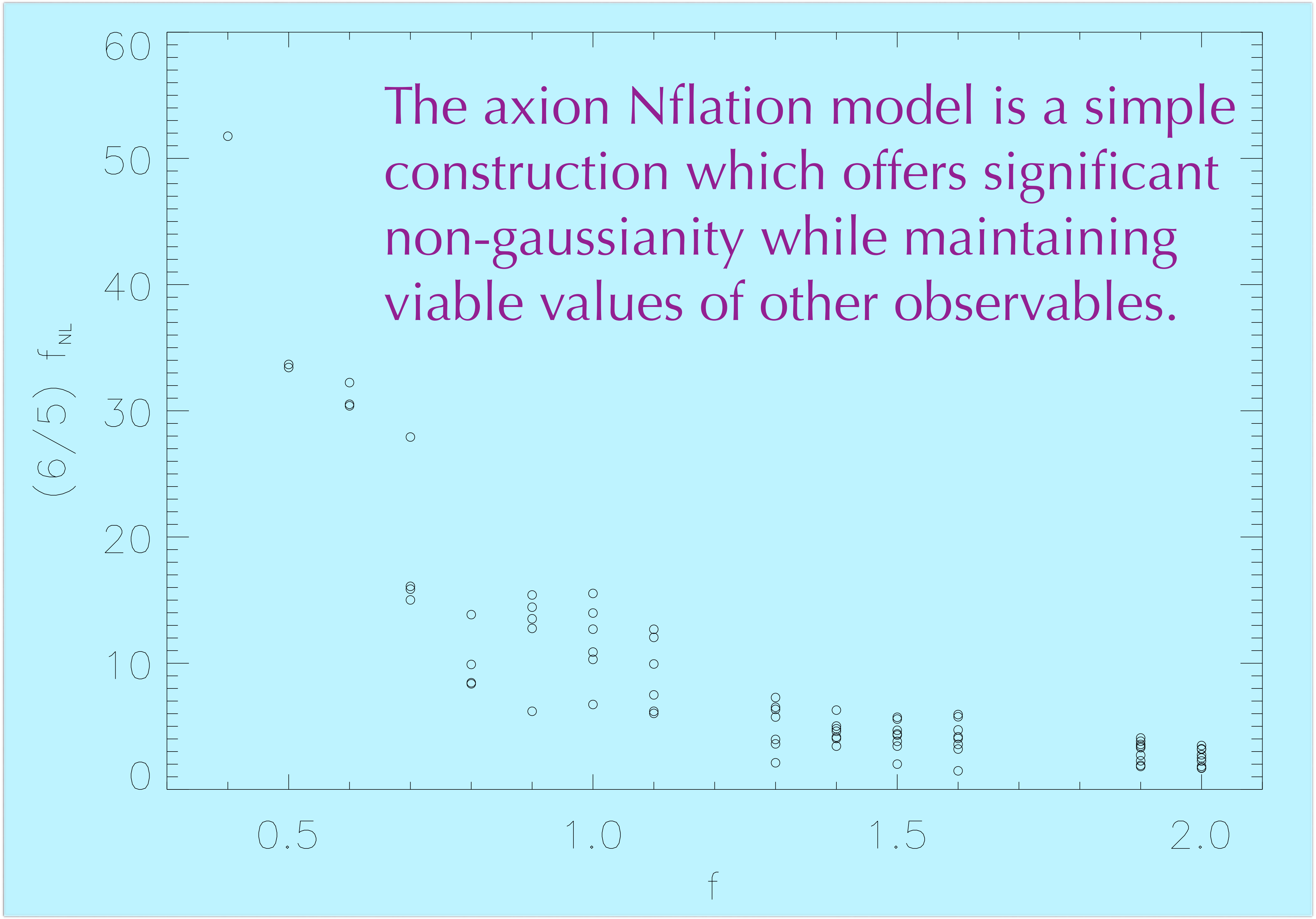
Naxion: non-gaussianity



Interpretation

- There is nothing particularly unusual about the predictions of the non-gaussianity in this model; it is in fact what one would get from a singe field evolving in the axion potential.
- However a single-field model with those parameters would not be satisfactory, as it would not give sufficient inflation and the spectral index would be far from unity.
- The scenario works because the assisted inflation mechanism strongly alters the predicted spectral index, but has only a marginal effect on the non-gaussianity

Conclusion


The axion Nflation model is a simple construction which offers significant non-gaussianity while maintaining viable values of other observables.
(6/5) f_NL
0
10
20
30
40
50
60
0.5
1.0
1.5
2.0
f

Naxion: trispectrum

- A similar analysis yields an estimate of the trispectra

$$\tau_{\rm NL} = (4\pi^4/\bar{N}^2)(M_{\rm P}^4/f^4)$$

$$(54/25)g_{\rm NL} = (8\pi^4/\bar{N}^2)(M_{\rm P}^4/f^4)$$

$= (6/5 f_{NL})^2$

- As seen in the bispectrum plot, there is a large spread of predictions due to the randomness of initial conditions.

  However there are predicted correlations within a realization, for instance between r and f_{NL}.