Comments on Tensor Modes from the Inflationary era

Based on

- Work in progress with J. Polchinski, L. Senatore, M. Zaldarriaga
- Works with Westphal, McAllister, Dong, Horn, Green, Senatore

Tensor modes are a standard signature of inflation

$$\langle h_k h_{k'} \rangle = (2\pi)^3 \delta(k+k') P_h$$

$$P_h = \frac{4}{k^3} \frac{H^2}{M_p^2}$$

Lyth

$$\left(\frac{\Delta \phi}{M_p}\right) \simeq N_e \left(\underbrace{\frac{P_h}{P_g}}_{r}\right)^{\frac{1}{2}} \simeq \left(\frac{r}{0.01}\right)^{\frac{1}{2}}$$

(Slow Roll inflation)

$\gtrsim 1 \Rightarrow$ UV-sensitive

$\gtrsim 1 \Rightarrow$ detectable

$$\text{Detectability} \Leftrightarrow \underset{H/M_p}{\underset{\sim}{h}} \gtrsim 10^{-6}$$

→ It is often said that a detection of tensor modes (via CMB B-mode polarization)

⇒ • measurement of $H_{\text{inflation}} \gtrsim 10^{-6} M_p$

• determination that $\Delta\phi > M_p$

This talk:

• Quantum production of classical GW sources can potentially compete, producing

cf Chialva

$$h \geq 10^{-6} \text{ with } \frac{H}{M_p} \leq 10^{-6}$$

• First, review & update UV-complete mechanism(s) for large field inflation with tensor signature.

From one point of view, large-field "chaotic" inflation A. Linde '83 seems very simple.

cf symmetries

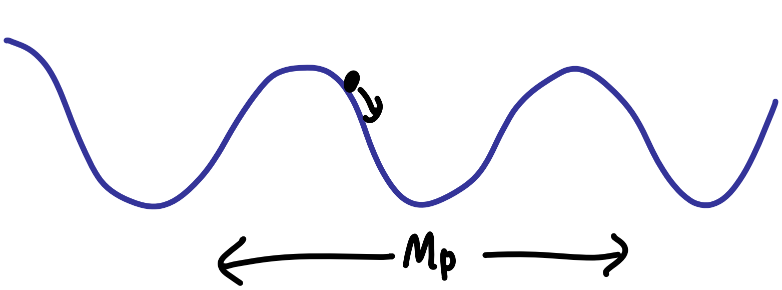
e.g. $V(\varphi)\Big|_{\text{inflation}} \sim \mu^{4-\alpha}\varphi^{\alpha}$

If this is the leading effect breaking $\varphi \to \varphi + c$ symmetry, it's radiatively stable. → Does the UV completion respect this symmetry?
(relevant since $\varphi \sim 15 M_p > M_p$)

Freese, Frieman, Olinto '90; +Adams, Bond '93

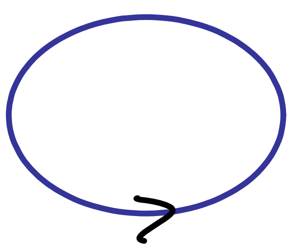
Axions naturally respect an (approximate) shift symmetry

→ Natural Inflation



$\Lambda^4 \sim e^{-\frac{\text{const}}{g^2}}$

Naturally small



$a \cong a + (2\pi)^2$

$\phi_a = f_a a$ — canonical scalar field

→ Does $\frac{\Delta\phi}{M_p} \gtrsim 1$, protected by shift symmetry, arise in string theory?

In string theory, the basic period $f_\theta (2\pi)^2$ a priori turns out $\ll M_p$ at weak curvature + coupling

Banks/Dine/Fox/Gorbatov
Svrcek/Witten

cf Arkani-Hamed et al

e.g. <u>Axions</u>

$$a = \int_{\Sigma_p} \underbrace{A_{i_1 \cdots i_p}}_{\text{potential field}} dx^{i_1} \wedge \cdots \wedge dx^{i_p}$$

Σ_p: p-dim'l closed submanifold

potential field (higher-dim'l analogue of Maxwell A_μ)

f_a comes from kinetic term:

$$\int d^D x \sqrt{G_{(D)}}\, F_{i_1 \cdots i_{p+1}} G_{(D)}^{i_1 i_1'} \cdots G_{(D)}^{i_p i_p'} F_{i_1' \cdots i_{p+1}'}$$

$$= \int d^4 x \sqrt{g_4}\, f_a^2 (\partial a)^2 = \int d^4 x \sqrt{g_4} (\partial \theta_a)^2$$

$\Rightarrow$ for all sizes $\sim R$, this yields

$$f_a \sim M_p \left(\frac{\sqrt{\alpha'}}{R}\right)^p \ll M_p$$

$\sqrt{\alpha'}$ = string length

- Similar statement for certain brane collective coordinates

  (Bauman, McAllister)

- Generalization to multiple fields

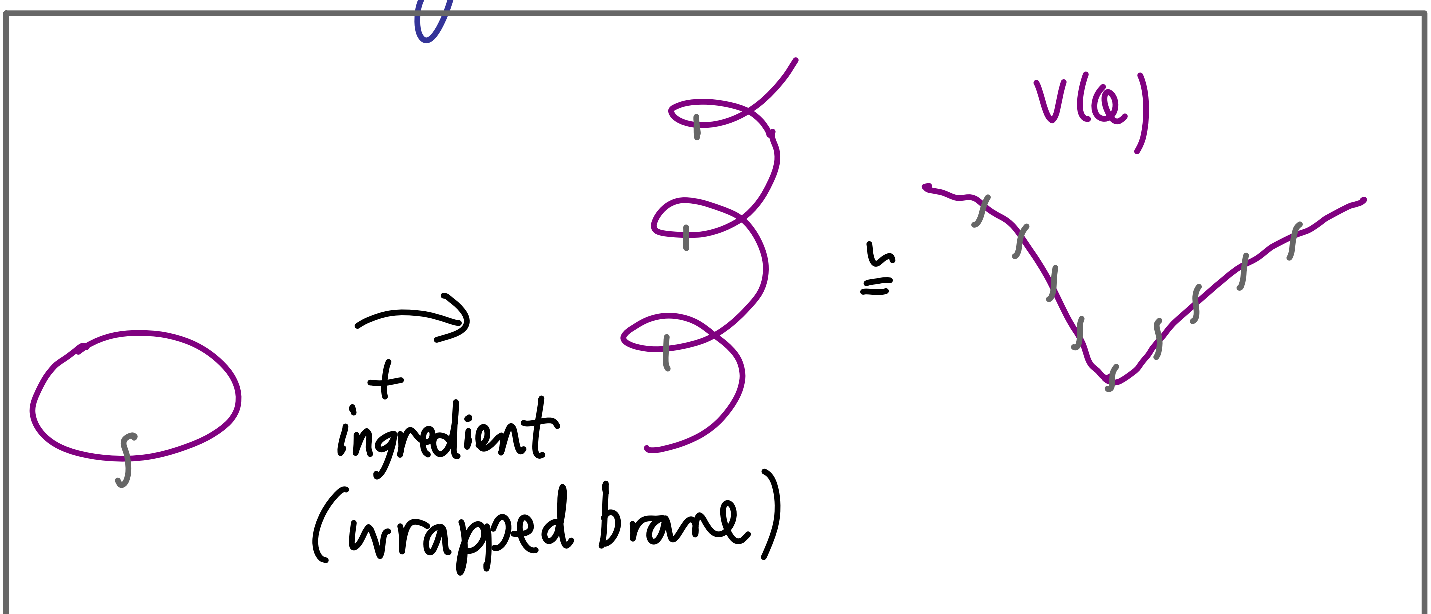
  - axions: N-flation (Dimopoulos, Kachru, McGreevy, Wacker); Easther, McAllister, Grimm; Kallosh, Sivanandam, Soroush
  - branes: Becker, Leblond, Shandera; Lidsey-Huston, Ward; Kobayashi et al, ...

Kinematically extends range, but yields significant back reaction. May well be UV-completable.

In any case, a rather generic structure

- Monodromy -

in string compactifications



unwraps the would-be periodic direction. → Large field range with distinctive potential, with

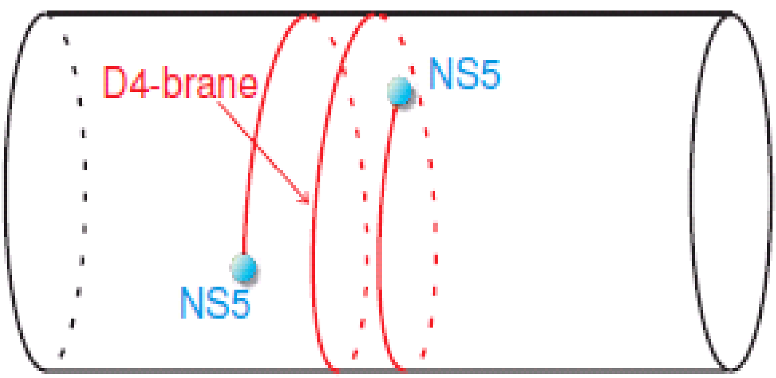
$$V(\phi > M_p) \sim \begin{cases} \phi^{2/3} & \text{twisted torus} \\ \phi & \text{axions} \end{cases}$$

ES, AW '08

LMcA, ES, AW '08

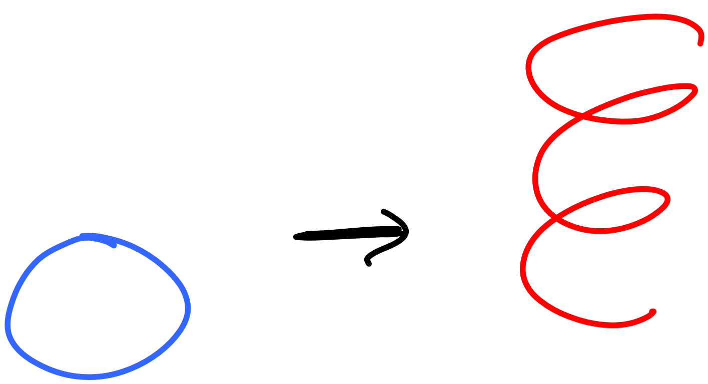
the so far worked-out examples.

The basic mechanism is very simple :



["T-dual" to axions]

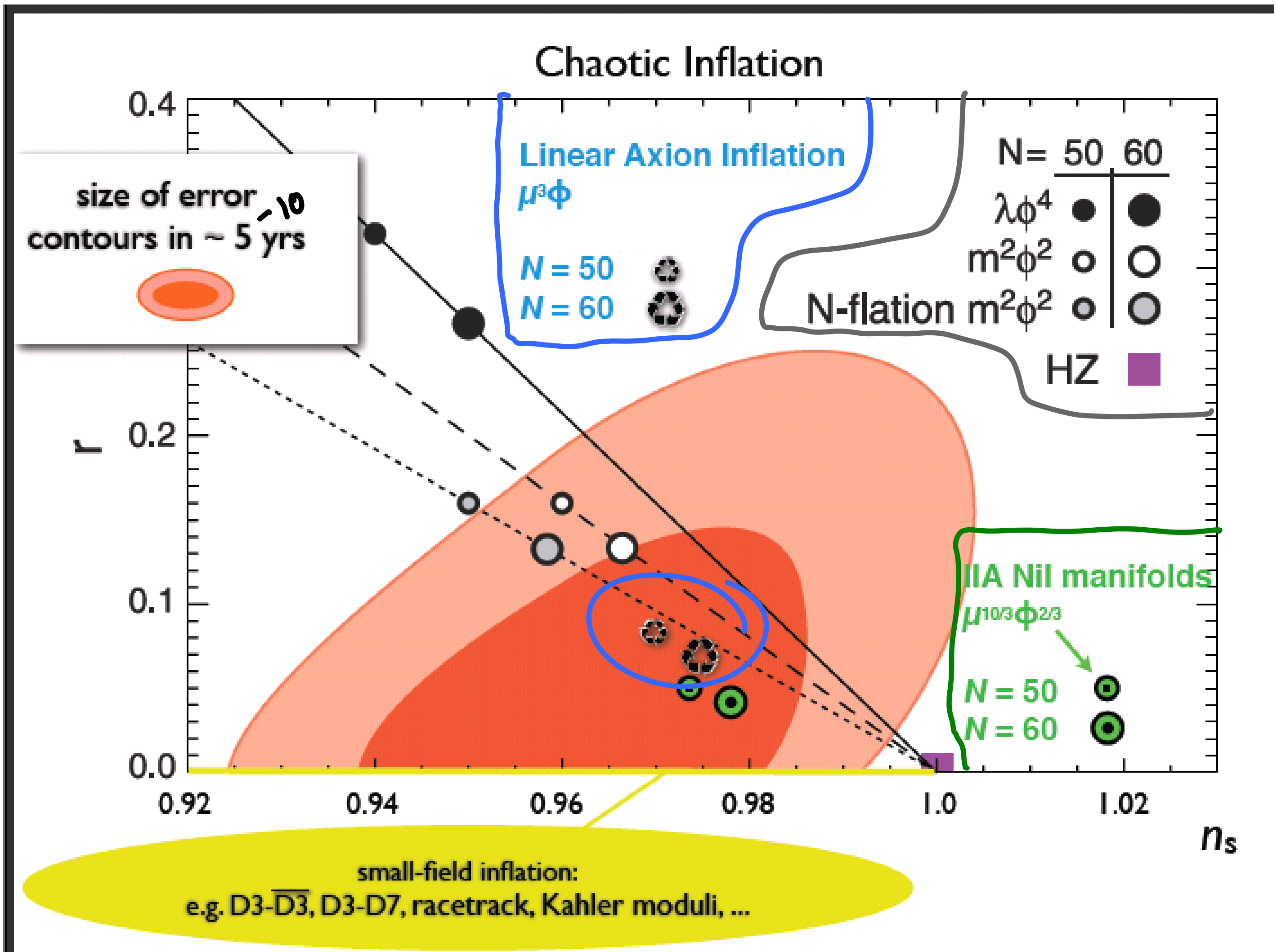
- "NS5" branes position periodic on this circle, until add stretched "D4" brane



detailed analysis: 0808.0206 [hep-th] LMcA ES AW

. . .

Result:



$$\boxed{\begin{aligned} r &= 0.07 \\ n_s &\simeq 0.98 \end{aligned} \qquad V(\varphi) \simeq \mu^3 \varphi} + \Lambda^4 \cos\left(\frac{\varphi}{2\pi f}\right)$$

Because of the symmetry, and oscillating nature of the (instanton-suppressed) corrections, these predictions are precise $\Rightarrow$ falsifiable

Updates:

- Additional (but model-dependent) signatures from $\Delta V \sim \Lambda^4 \cos \frac{\phi}{f}$

Flauger et al

- Flux Monodromy and $V \sim \phi^p$

$$S = \cdots - \int \sqrt{G}\, \left| dC_p + B \wedge dC_{p-2} + \cdots \right|^2$$

→ flux monodromy inflation X. Dong, B. Horn, E.S., A.W.
cf Kaloper, Lawrence

Naively, this gives $m^2\phi^2$, but back reaction automatically pushes down the power!

$$\int_\Sigma B = \theta \qquad L_\Sigma = L_\Sigma(B)$$

$$\int \frac{\dot{B}^2}{L(B)^4} - \alpha B^2 = \int \dot{\phi}^2 - \mu^{4-p}\phi^p$$

New Sources of GW's

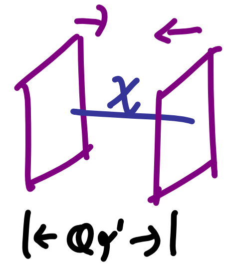
The inflaton ϕ generically couples to other degrees of freedom (e.g. for reheating)

(Kofman + many)

For example,

(1) $\delta \mathcal{L} = g^2 \phi^2 \chi^2 \rightarrow m_\chi^2 = m_0^2 + \phi^2(t, \vec{x})$

$\Rightarrow$ particle production

(brane picture: 

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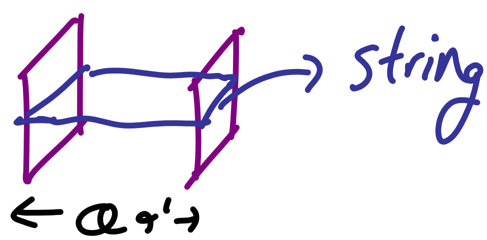
(2) $T_{\text{string}}^2 = T_{\text{min}}^2 + \phi^2 M_0^2$

(brane picture: 

)

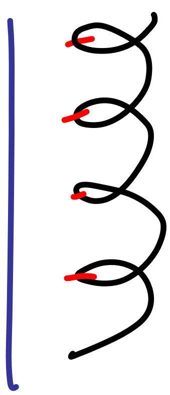
→ Do these produce competitive GWs?

Of course inflation dilutes exotic high-scale relics;

Produced particles, strings dilute in a Hubble time. However,

★ In e.g. monodromy, any production events are repeated many times

Axion monodromy → string production



cf Trapped Infl

Dong Horn Senatore ES

Kofman, Linde

Kim

→ repleshinishing supply of GWs

This, plus the more general question of B-mode degeneracy, motivates analyzing this question.

First, given these classical sources of GW's, is $h_{\text{source}} \gtrsim 10^{-6}$?

$$\sim \left(\frac{\rho_{GW}}{H^2 M_p^2}\right)^{\frac{1}{2}}\Bigg|_{\text{freeze-out},\ \omega \sim \frac{k}{a} \sim H}$$

$$\rho_{GW} \sim \rho_{\text{production}} \cdot \underbrace{\left(\frac{H}{\omega}\right)^4}_{\text{inflationary dilution}}$$

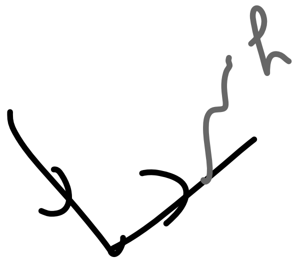
Low-frequency sources ($\omega \gtrsim H$) most efficient

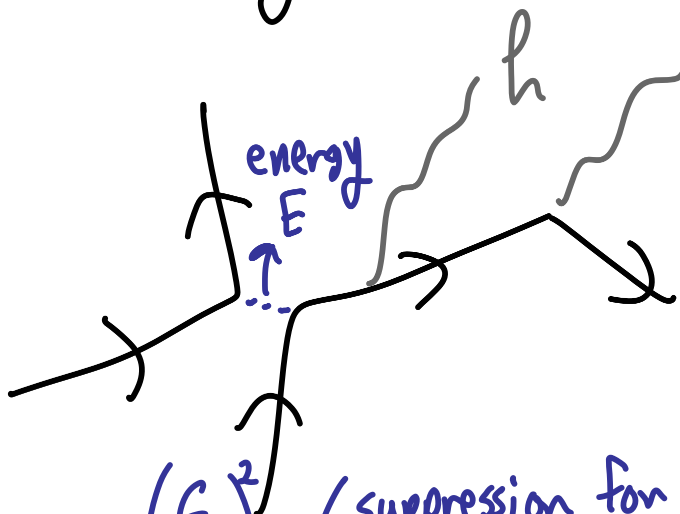
zeroeth-order check: $\rho_{\text{sources}} < \overset{10^{-2}}{\overset{\backsim}{\varepsilon}} H^2 M_p^2$.

If converted $\rho_{\text{sources}} = \varepsilon H^2 M_p^2 \to \rho_{GW}\big|_{\omega = H}$

would get $h \sim 10^{-1} \gg 10^{-6}$

- Particles: Bremsstrahlung





$$\frac{dP_{GW}}{d\omega} \sim n_p \left(\Gamma_{int}\, t\right)\left[\frac{dE_g}{d\omega} = \left(\frac{E}{M_p}\right)^2 \times \left(\begin{matrix}\text{suppression for}\\ >1 \text{ interaction}\end{matrix}\right)\right]$$

cf σ

various regimes

e.g. $\Gamma \sim H \Rightarrow \lesssim 1$ interaction (production itself)

$$h^2 \sim \underbrace{\frac{\rho_{particles}}{\rho_{Total}}}_{f<1} \frac{EH}{M_p^2} \Rightarrow \frac{H}{M_p} \sim h\left(h \underbrace{\left[\frac{M_p}{fE}\right]}_{>1}\right)$$

$$\Rightarrow \text{for } h \gtrsim 10^{-6}, \quad \frac{H}{M_p} > 10^{-12} = 10^{-6} \times H_{GUT}$$

small window, not parametric

Similar results in other interesting regimes.

- strings : individually produce GWs as oscillate

"$G\mu$"

Vaschaspati Vilenkin ...

$$\frac{dE}{dt} \simeq 50 \left(\frac{\text{Tension}}{M_p^2}\right) \times \text{Tension}$$

$$\rightarrow \quad \rho_{GW} \simeq 50\, N_H \left(\frac{\Upsilon}{M_p^2}\right) (HL)^4$$

- consistency with inflation requires

$$\rho_{str} \sim N_H\, \Upsilon\, L H^3 \ll H^2 M_p^2$$

$$\Rightarrow \quad N_H \ll \frac{M_p^2}{H \Upsilon L}$$

- detectability : $\rho_{GW} > 10^{-12} H^2 M_p^2$

$$\Rightarrow (HL) \gg 10^{-5} \left(\frac{M_p^2}{\Upsilon}\right)^{1/3}$$

In words: Large $(10^{-3}H^{-1} \lesssim L \lesssim H^{-1})$

high-tension $\left(\frac{\mathcal{T}}{M_p^2} \sim 10^{-6}\right)$

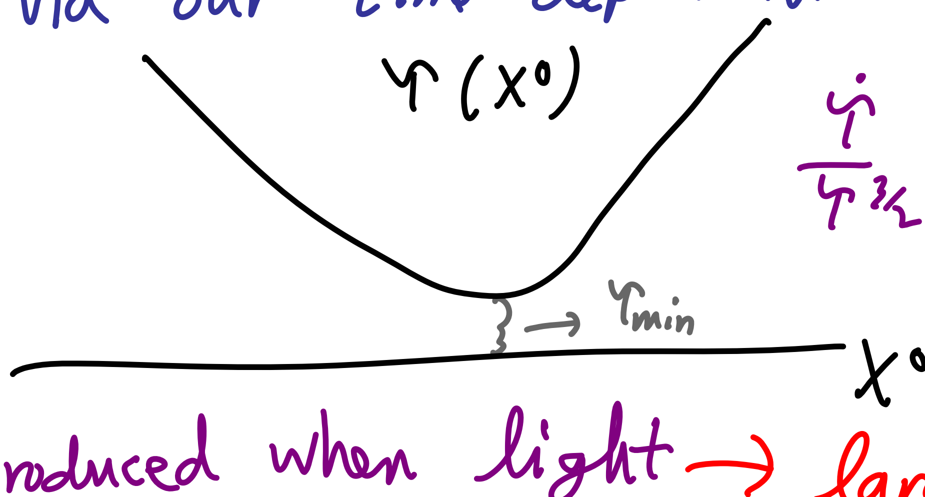
strings produce detectable GWs

* independently of $\frac{H}{M_p}$ *

(a parametric effect)

cf strings from reheating: Tye et al, Copeland, Myers, Polchinski, Jackson, ...

- Are such strings produced via our time-dependent tension?



$\frac{\dot{\mathcal{T}}}{\mathcal{T}^{3/2}}$ can be > 1

- produced when light → large ?
- $\mathcal{T}$ then ramps up, can → $\mathcal{T} \sim 10^{-6} M_p^2$ easily in e.g. monodromy models

In particular, for the parametric effect one needs

$$L \overset{\lesssim H^{-1}}{} \gg \text{timescale of production } (\Delta t)_{\text{non-adiabatic}}$$

This is not inconsistent with causality (cf Kibble), but must be analyzed anew for this quantum production mechanism

cf Lawrence, Martinec
Tolley, Wesley
Strominger, Takayanagi
McGreevy, ES
Gubser et al

- This is an interesting theoretical problem in its own right.

$$S_{string} = \int d^2\sigma\, \Upsilon(X^0) \left(\partial_\alpha X^M \partial^\alpha X^N G_{MN} \right)$$

nonlinear worldsheet action

- Naive generalization of particle production is inadequate a priori:

Υ can increase so rapidly that string cannot causally track its naive oscillator spectrum $m^2_{naive} \sim N_{osc} \Upsilon^{\frac{1}{2}}$

To start, review particle production in particle case. $\Delta\mathcal{L} = g^2\phi^2\chi^2$

$\phi \simeq vt$ during production

$$\omega_\chi^2(t) \simeq v^2t^2+k^2$$

→ Heisenberg equation of motion for the field is

$$\left\{ -\partial_t^2 - \omega^2(t) \right\}\psi = 0$$

WKB $\psi \xrightarrow[t\to-\infty]{} \frac{1}{\sqrt{2\omega}} e^{-i\int^t dt'\omega(t')}$

$|t|\to\infty$

$$\psi \to \frac{\alpha}{\sqrt{2\omega}} e^{-i\int^t \omega} + \frac{\beta}{\sqrt{2\omega}} e^{+i\int^t \omega}$$

→ number density $n_p = \int |\beta|^2$

$$|\beta_k|^2 = e^{-\frac{2\pi k^2}{gv}}$$

To anticipate string case, derive from world line action

$$S = \int d\lambda \sqrt{g_{\lambda\lambda}} \left(\left(\frac{dX^0}{d\lambda}\right)^2 g^{\lambda\lambda} - (v^2 X^{0^2} + k^2) \right)$$

$$\Pi_{X^0} = g^{\lambda\lambda} \partial_\lambda X^0 \longrightarrow \frac{\partial}{\partial X^0}$$

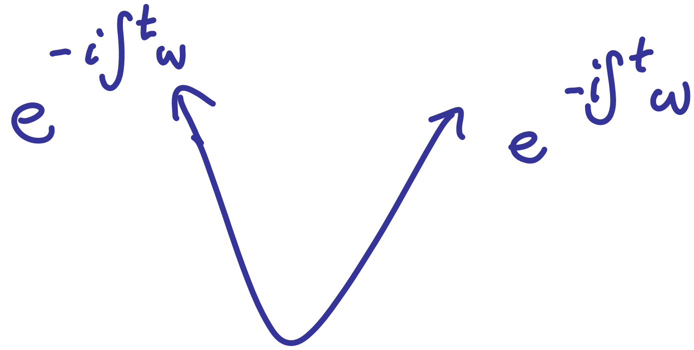
Hamiltonian constraint is

$$(\partial_\lambda X^0)^2 = v^2 X^{0^2} + k^2$$

equivalently

$$\left\{ -\frac{\partial^2}{\partial X^{0^2}} - (v^2 X^{0^2} + k^2) \right\} \psi = 0$$

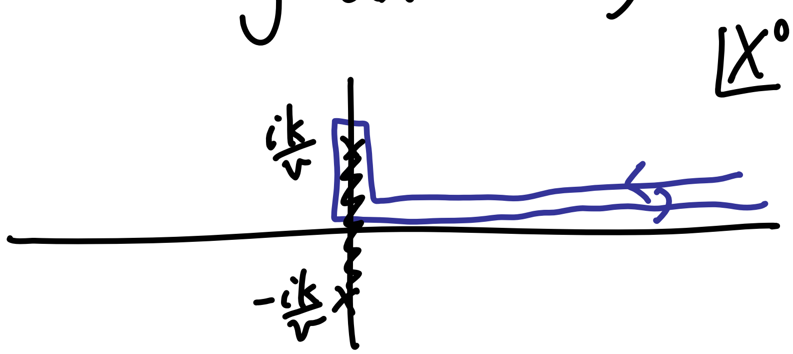
One can compute the Bogoliubov coefficient β as a propagator



$$S = \int d\lambda \left(-\left(\frac{dX^0}{d\lambda}\right)^2 - \omega^2(X^0) \right)$$

$$= -2 \int d\lambda \left(\frac{dX^0}{d\lambda}\right)^2 = -2 \int^t dX^0 \, \frac{dX^0}{d\lambda}$$

$$= -2 \int^t dX^0 \, \omega(X^0)$$



Can also compute $\mathrm{Im}\left(Z_{1\text{-loop}}\right)$

Generalization to strings: $\varphi^2 = \varphi_{min}^2 + b^2 x_0^2$

Hamiltonian constraint (circular, size $L \sim r$)

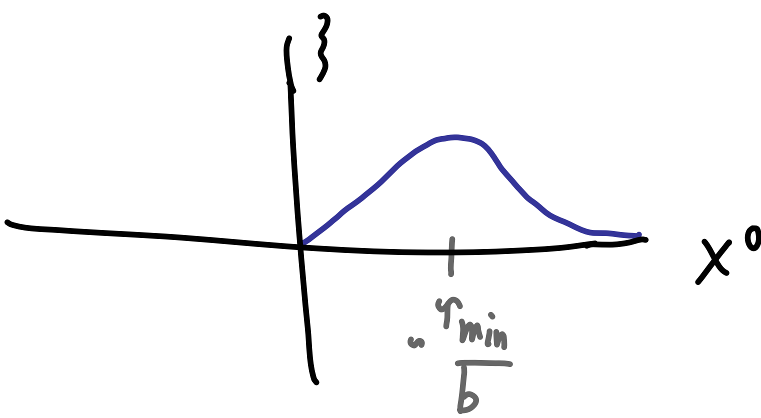
$$\left(\frac{dX^0}{d\tau}\right)^2 = \left(\frac{dr}{d\tau}\right)^2 + r^2\varphi^2$$

i.e. $$\left(-\partial_{X^0}^2 + \partial_r^2 - r^2\varphi^2\right)\Psi = 0$$

(1) Warmup: adiabatic regime

$$\xi \equiv L\,\frac{\dot\varphi}{\varphi} < 1$$

(small loops, or $X^0 \to \infty$ long loops)



$$\rightarrow \quad \mathrm{Im}\, S \sim N_{osc}^{1/2}\, \varphi^{3/2}/\dot\varphi$$

(2) Long loops $r \gg X^0$:

↑ Tension too fast for string to relax

$$\left(-\partial_{X^0}^2 + \partial_r^2 - r^2\left(b^2 X^{0\,2} + \gamma_{min}^2 + k^2\right)\right)\psi = 0$$

↳ find $\partial_r^2 \ll \partial_{X^0}^2$ $\left(\frac{1}{r} \ll \frac{1}{X^0}\right)$

$|\dot{r}| \ll |\dot{X}^0|$

↳ $$|\beta|^2 = e^{-\frac{2\pi}{b}\left(\frac{k^2}{r} + \gamma_{min}^2 r\right)}$$

Need to integrate over phase space (all shapes) and understand r-dependence here.

At face value: long loops formed for sufficiently small γ_{min}.

(3) For the case $\Upsilon(X^0) = \mu_0 + \mu_1 e^{-\kappa X^0}$, can compute $\mathrm{Im}(Z_{1\text{-loop}})$ via a fun trick : $\hookrightarrow$ path integral on a flat torus

$$S = \int d\sigma^1 d\sigma^0 \left(\mu_0 + \mu_1 e^{-\kappa X^0}\right)\left(\partial_\sigma X^\mu \partial^\sigma X_\mu\right)$$

$$X^0 \equiv \underset{\substack{\uparrow \\ \text{0-mode}}}{t} + \hat{X}^0$$

t appears only in $e^{-\kappa t} \quad e^{-\kappa \hat{X}^0} = e^{-\kappa X^0}$

By computing $\partial_{\mu_1} Z$ and doing t integration first, find

$$Z = V_{D-1}\left(\log \frac{\mu_1}{\mu_*} + i\pi\right) \hat{Z}_{\mu_0}$$

indicating Hagedorn spectrum of tension-μ_0 strings, created in time

$$\Delta t \ll \left(\text{size} = \mu_0^{-\frac{1}{2}}\right)$$

Summary

- Long strings, (if) produced during inflation, can $\rightarrow$ tensor modes parametrically > the usual $h^2 \sim \left(\frac{H}{M_p}\right)^2$
- particles can compete

$\rightarrow$ measurement of primordial B-modes not necessarily indicative of high-scale inflation ... but can be a new signature of exotics.

- break degeneracy with further details? (e.g. non-Gaussianity).