Leonardo Senatore
(Stanford University)

On Signatures of Inflation: high-order correlation functions

Outline:
Cosmology is ready for interactions

- Why non-Gaussianities
- The Effective Field Theory of Inflation
- Non-Gaussianities and Inflation Precision Tests
- The Effective Field Theory of Multifield Inflation

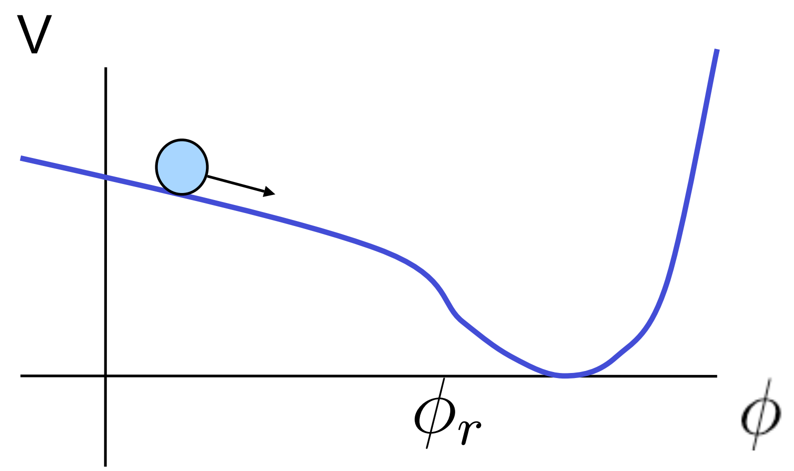
How do we probe Inflation?

How do we probe inflation?

- Simple Models

$$a \sim e^{Ht}$$

$$\dot{\phi} \sim \frac{V'}{H}$$



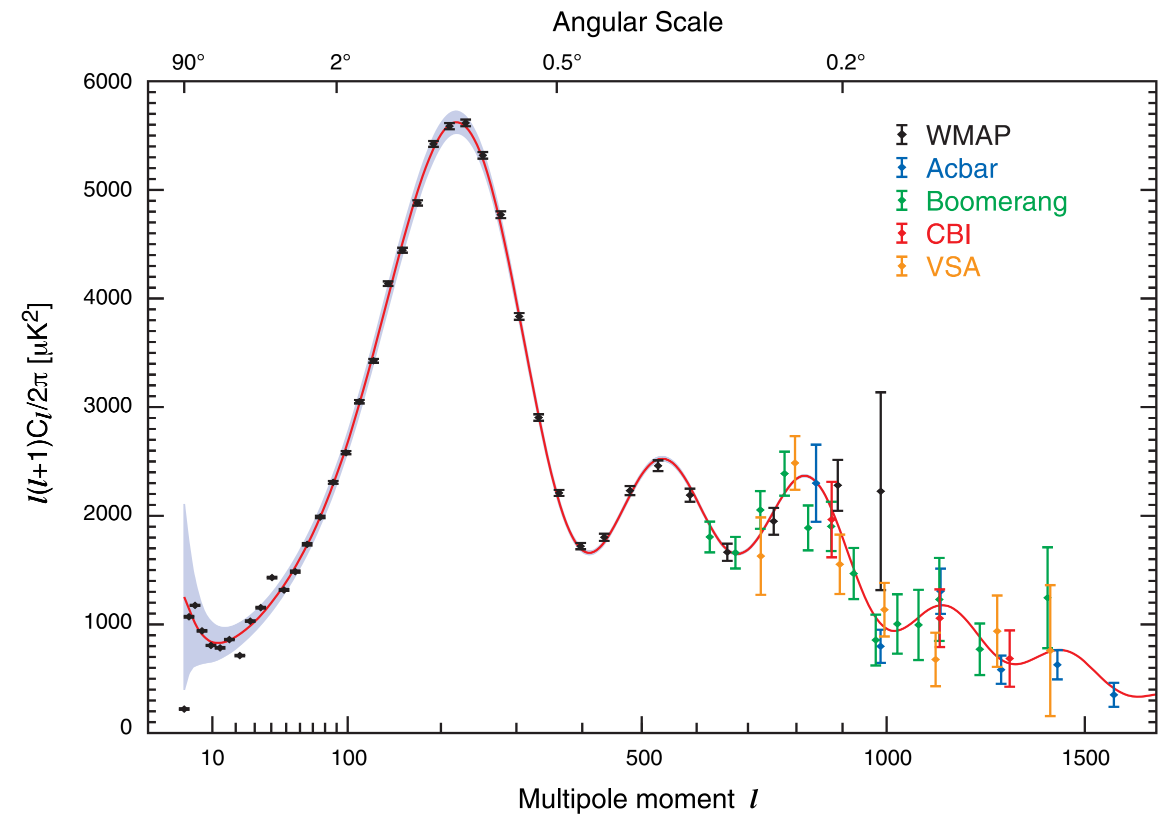
$$\epsilon \simeq \frac{\dot{\phi}^2}{V(\phi)} \ll 1$$

- Standard predictions

$$\zeta \sim \frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \qquad \zeta \simeq \frac{H}{\dot{\phi}} \delta\phi$$

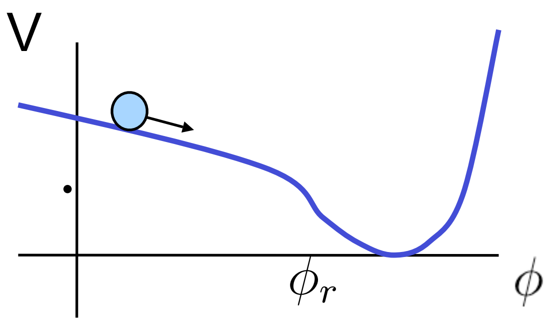
$$\langle \zeta^2 \rangle \sim \frac{1}{\epsilon} \frac{H^2}{M_{\rm Pl}^2}$$

$$\langle \gamma^2 \rangle \sim \frac{H^2}{M_{\rm Pl}^2}$$



- WMAP, Planck, SDSS, Bicep,... Now we can look for more!
  - Is it there something more?

Statistics of the fluctuations

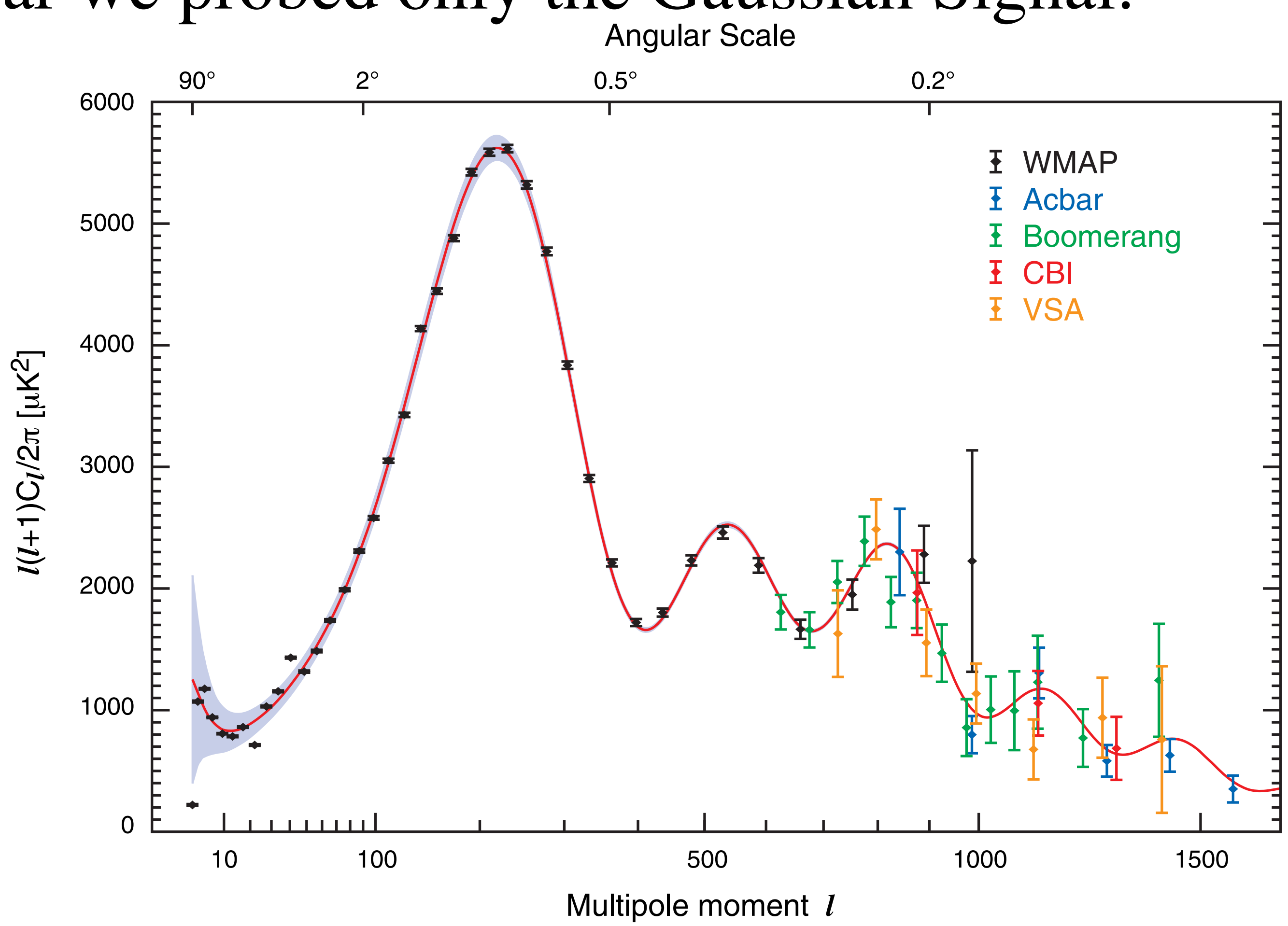


- We started from inflationary fluctuations, which induced $\zeta \simeq \frac{H}{\dot{\phi}}\delta\phi$
- The distribution is Gaussian

$$P(\{\zeta_{\vec{k}}\}) = N \text{ Exp}\left(-\sum_{\vec{k}_i}\frac{\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}}{P(k_1)}\right)$$ where $$P_k \sim \langle\zeta_{\vec{k}}\zeta_{-\vec{k}}\rangle_{\text{vac}} \sim \frac{H^4}{\dot{\phi}^2}\langle\delta\phi_{\vec{k}}\delta\phi_{-\vec{k}}\rangle_{\text{vac}}$$

- Because we solved linear equations $\ddot{\delta\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0$
  (like QM harmonic oscillator $\hat{\delta\phi} \to \hat{x}$)

- So far we probed only the Gaussian Signal:



$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle = \delta^3(\vec{k}_1 + \vec{k}_2)P_\zeta(k_1)$$

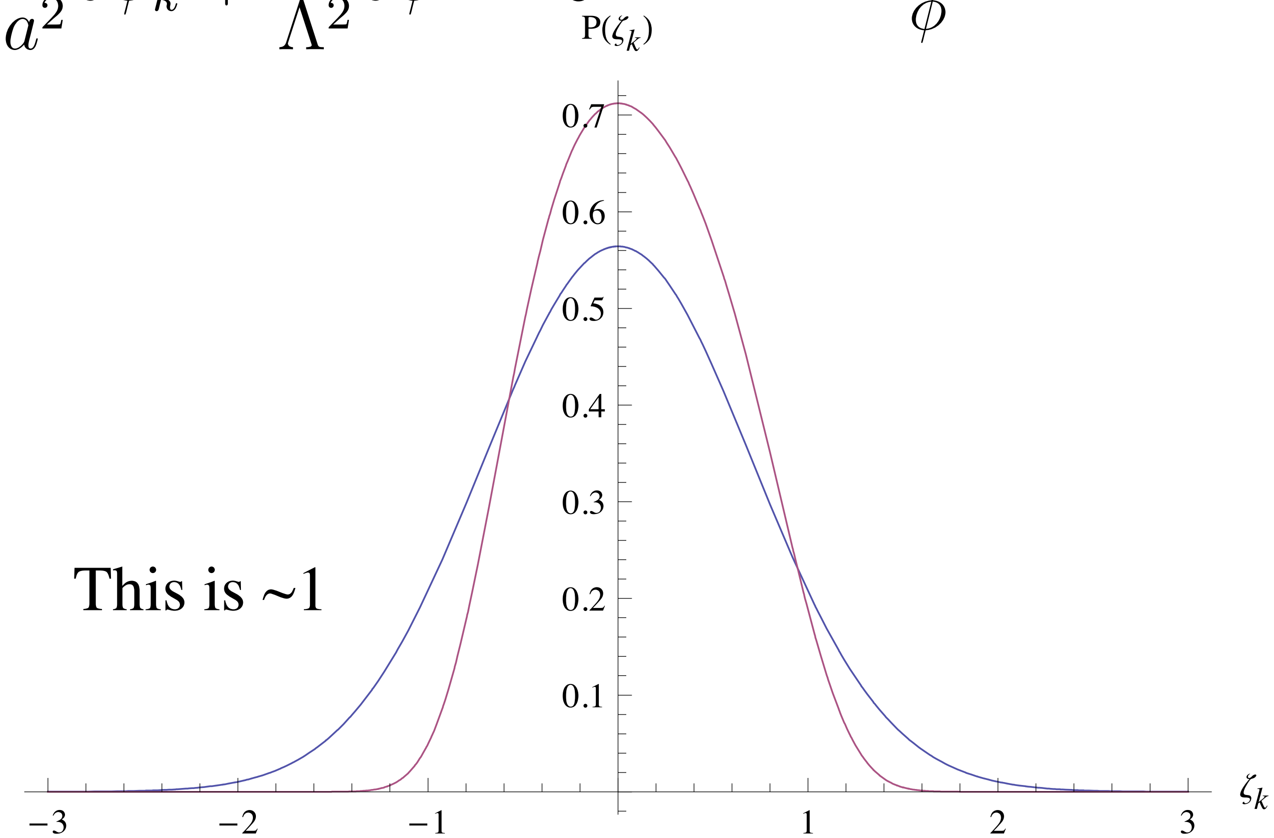
$$C_l = \langle a_{lm}a^*_{lm}\rangle$$

$$C_l^{\text{exp.}} = \frac{1}{2l+1}\sum_{m=-l}^{l} a^*_{lm}a_{lm}$$

Non-Gaussianities

- The distribution can be non-Gaussian $P(\{\zeta_{\vec{k}}\}) = N \text{ Exp}\left(-\sum_{\vec{k}_i}\left(\frac{\zeta_{\vec{k}_1}\zeta_{-\vec{k}_1}}{P(k_1)} + \frac{\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{-\vec{k}_1-\vec{k}_2}}{C(k_1,k_2,|\vec{k}_1+\vec{k}_2|)} + \ldots\right)\right)$
- This would have come if we had solved $\ddot{\delta\phi}_k + \frac{k^2}{a^2}\delta\phi_k + \frac{1}{\Lambda^2}\dot{\delta\phi}^2 = 0$ $\qquad \zeta \simeq \frac{H}{\dot{\phi}}\delta\phi$
- This would come from interactions
- Non-Gaussian Signal:

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = \delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)F(\vec{k}_1,\vec{k}_2,\vec{k}_3)$$



- So far: $\frac{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle}{\langle \zeta_{\vec{k}}\zeta_{-\vec{k}}\rangle^{3/2}} \lesssim 10^{-2} \sim \frac{1}{N_{\text{pix}}^{1/2}}$

$$N_{\text{pix}}^{\text{WMAP}} \sim 10^5$$

- Free Field: Gaussian
- Interacting field: Non-Gaussian
- Interactions of Inflation!
- (at very high energies)!

Large non-Gaussianities

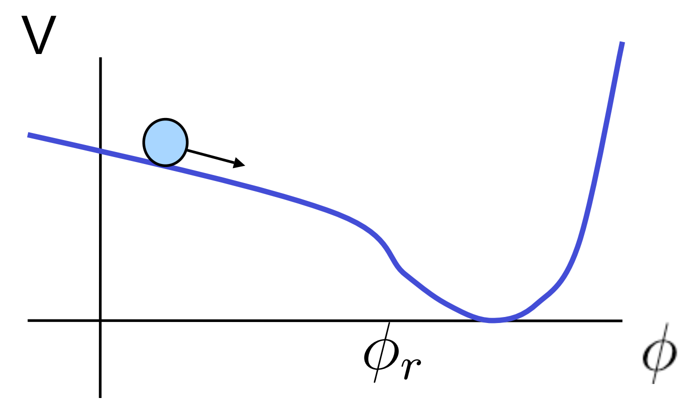
- Standard slow-roll infl.: very Gaussian

Maldacena, **JHEP 0305:013,2003**

$$\frac{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle}{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle^{3/2}} \simeq f_{\rm NL}\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle^{1/2} \sim 10^{-7}$$

$$f_{\rm NL} \sim 10^{-2}$$

So far undetectable



- DBI inflation

$$\mathcal{L} = \phi^4\sqrt{1-\lambda\frac{\dot{\phi}^2}{\phi^4}}$$

Alishahiha, Silverstein and Tong, **Phys.Rev.D70:123505,2004**

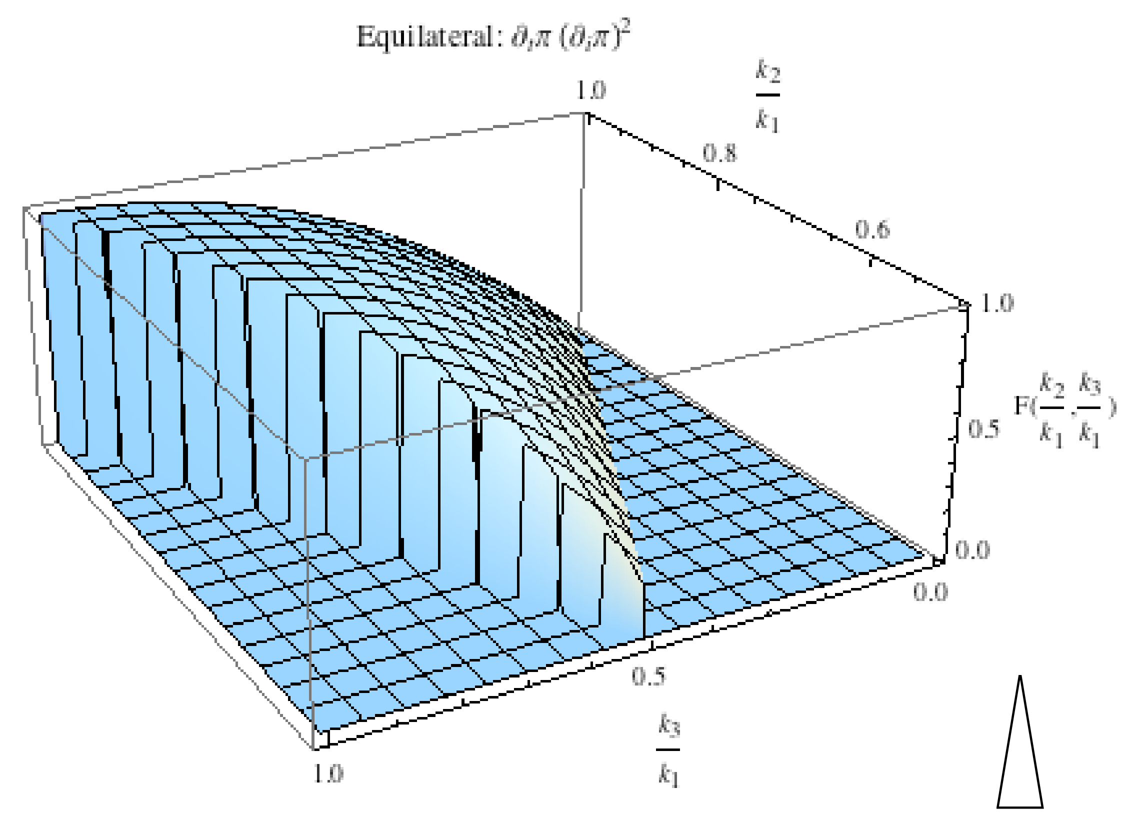
- Large non-Gaussianities

$$f_{\rm NL} \sim 10^2$$

Currently Detectable!

- Shape of non-Gaussianities

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\sum_i \vec{k}_i)F\left(\frac{k_2}{k_1},\frac{k_3}{k_1}\right)$$



- What are the generic signatures?

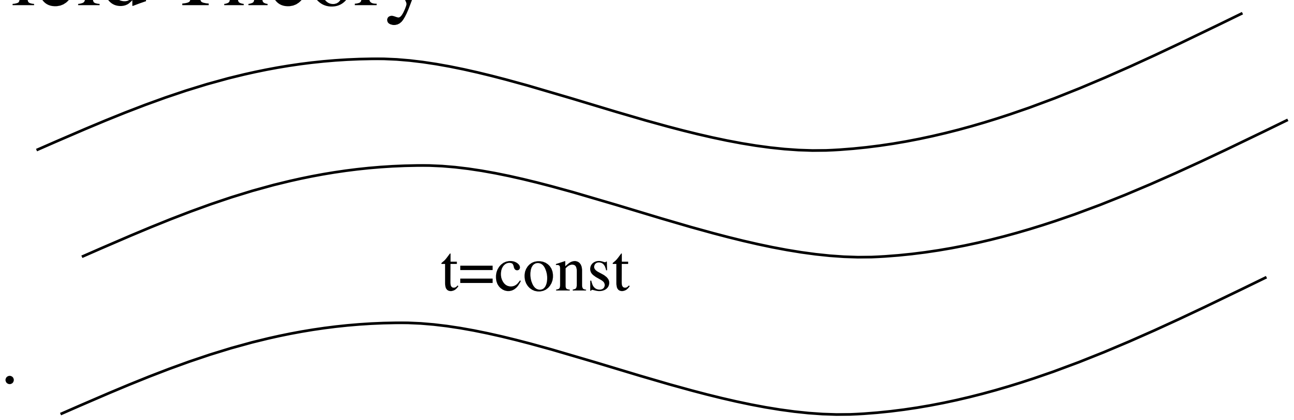
The Effective Field Theory

What is Inflation?

with C. Cheung, P. Creminelli,
L. Fitzpatrick, J. Kaplan
JHEP 0803:014,2008

The Effective Field Theory

Inflation. Quasi dS phase with a privileged spacial slicing



Unitary gauge. This slicing coincide with time.

$$\delta\phi(\vec{x},t) = 0 \quad \left(\delta\phi(\vec{x},t) \to \delta\phi(\vec{x},t) - \dot{\phi}(t)\, \delta t(\vec{x},t) \right)$$

Most generic Lagrangian built by metric operators invariant only under $x^i \to x^i + \xi^i(t,\vec{x})$

- Generic functions of time
- Upper 0 indices are ok. E.g. g^{00} R^{00}
- Geometric objects of the 3d spatial slices: e.g. extrinsic curvature K_{ij} and covariant derivatives

$$S = \int d^4x \sqrt{-g} \Big[M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H}(-1+\delta g^{00}) - M_{\rm Pl}^2 (H^2 + \dot{H}) + M_2^4(t)(\delta g^{00})^2 + M_3^4(t)(\delta g^{00})^3$$
$$- \bar{M}_1^3(t)\delta g^{00}\delta K_i^i - \bar{M}_2^2(t)\delta K_i^{i\;2} + \dots \Big]$$

All single field models are unified

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- Slow Roll Inflation: $\int d^4x \, \sqrt{-g} \left[-\frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \rightarrow \int d^4x \, \sqrt{-g} \left[-\frac{\dot{\phi}_0(t)^2}{2} g^{00} - V(\phi_0(t)) \right]$

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- k-inflation, DBI inflation $\mathcal{L} = \phi^4\sqrt{1-\lambda\frac{\dot{\phi}^2}{\phi^4}}$

Alishahiha, Silverstein and Tong,
Phys.Rev.D70:123505,2004

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Alishahiha, Silverstein and Tong,
Phys.Rev.D70:123505,2004

- Ghost Inflation $-(\partial\phi)^2+\frac{1}{M^4}(\partial\phi)^4+\dots$

WRONG SIGN

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga,
JCAP 0404:001,2004

Leonardo Senatore
Phys. Rev. D71:043512,2005

All single field models are unified

$$S=\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R + M_{\rm Pl}^2\dot H(-1+\delta g^{00}) - M_{\rm Pl}^2(H^2+\dot H) + M_2^4(t)(\delta g^{00})^2 + M_3^4(t)(\delta g^{00})^3\right.$$
$$\left. - \bar M_1^3(t)\delta g^{00}\delta K_i^i - \bar M_2^2(t)\delta K_i^{i\,2} + \dots\right]$$

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WRONG SIGN

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga,
JCAP 0404:001,2004

Leonardo Senatore
Phys. Rev. D71:043512,2005

- Something else

A simplifying limit

$$S = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R + M_{\rm Pl}^2\dot{H}(-1+\delta g^{00}) - M_{\rm Pl}^2(H^2+\dot{H}) + M_2^4(t)(\delta g^{00})^2 + M_3^4(t)(\delta g^{00})^3 \right.$$
$$\left. -\bar{M}_1^3(t)\delta g^{00}\delta K_i^i - \bar{M}_2^2(t)\delta K_i^{i\ 2} + \ldots\right]$$

Spontaneously Broken Gauge Symmetry

Reintroduce the Goldstone boson

Reintroduce the Goldstone boson

Reintroduce the Goldstone: $g^{00} \quad \to \quad g^{\mu\nu}\partial_\mu(t+\pi)\partial_\nu(t+\pi)$

Decoupling limit:

At high energy, no mixing with gravity.

$\pi \longrightarrow \pi + \delta t$

Cosmological perturbations probe the theory at E ~ H

$$S_\pi = \int d^4x \, \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H} (\dot{\pi}^2 - (\partial_i \pi)^2) + M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}(\partial_i\pi)^2 \right) - M_3^4 \dot{\pi}^3 + \ldots \right]$$

- All single field models are unified (Ghost Inflation, DBI inflation, ...); prove theorems on signal.

- What is forced by symmetries and large signatures are explicit:
  - The spatial kinetic term: pathologies for : $\dot{H} > 0$,
  - Speed of sound $\quad \dot{\pi}^2 - c_s^2 (\partial_i \pi)^2 \quad \dfrac{1}{c_s^2} = 1 - \dfrac{M_2^4}{M_{\rm Pl}^2 \dot{H}} \quad \langle \zeta^2 \rangle \sim \dfrac{H^2}{M_{\rm Pl}^2 \epsilon} \cdot \dfrac{1}{c_s}$
  - Connection between c_s and Non-Gaussianities: ,

  non-local NG: $f_{\rm NL}^{\rm non-loc.} \sim \dfrac{1}{c_s^2}$ (see also Chen, Huang, Kachru and Shiu **JCAP 0701:002,2007**)

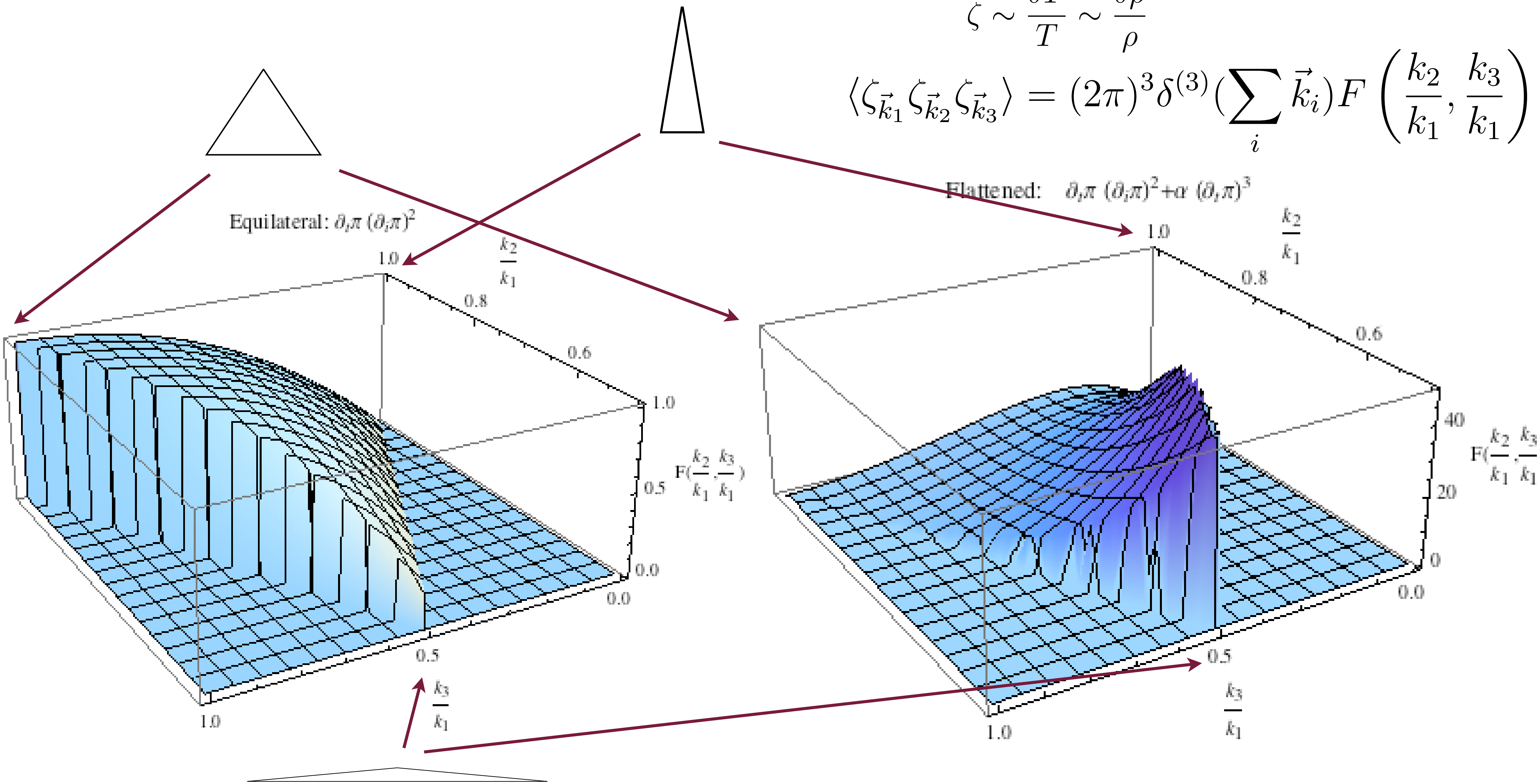
- The number of relevant operators is explicit. Large non-Gaussianities!: $\dot{\pi}(\nabla\pi)^2$ and $\dot{\pi}^3$

Large non-Gaussianites

with Smith and Zaldarriaga,
JCAP1001:028,2010

$$\zeta \sim \frac{\delta T}{T} \sim \frac{\delta \rho}{\rho}$$

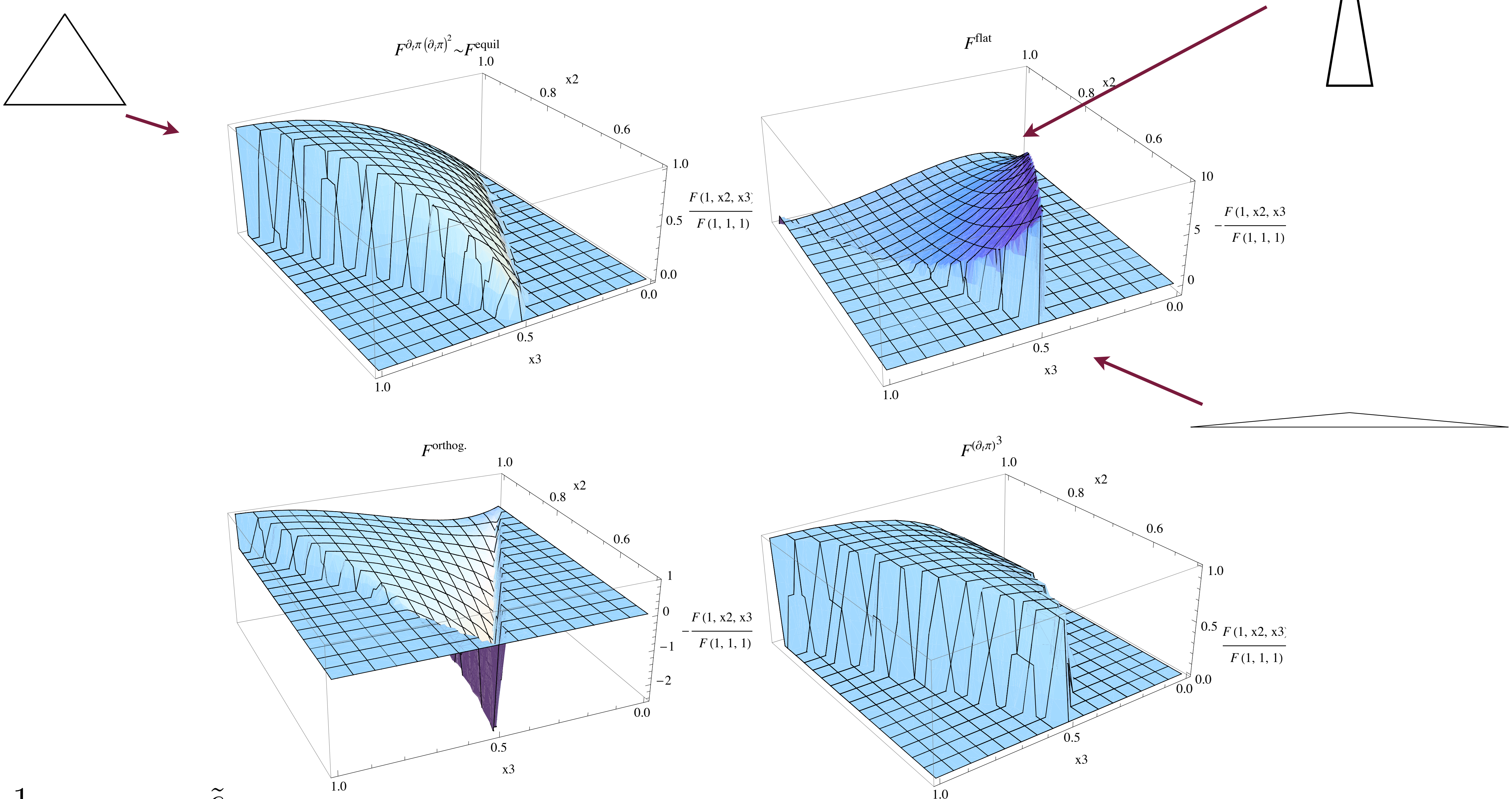
$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\sum_i \vec{k}_i) F\left(\frac{k_2}{k_1}, \frac{k_3}{k_1}\right)$$



$$\frac{1}{c_s^2}\dot{\pi}(\partial_i \pi)^2 + \frac{\tilde{c}_3}{c_s^2}\dot{\pi}^3$$

A function of two variables: we are measuring the interactions!
(and the coefficient of the Lagrangian!)

A one-parameter family of shapes



$$\frac{1}{c_s^2}\dot{\pi}(\partial_i\pi)^2 + \frac{\tilde{c}_3}{c_s^2}\dot{\pi}^3$$

- Single field consistency condition: small unless deviation from scale invariance

J. Maldacena, **JHEP 0305:013,2003**,
P. Creminelli, M. Zaldarriaga, **JCAP 0410:006, 2004,**
C. Chung L. Fitzpatrick, J.Kaplan, L.Senatore, **JCAP 0802:021, 2008.**

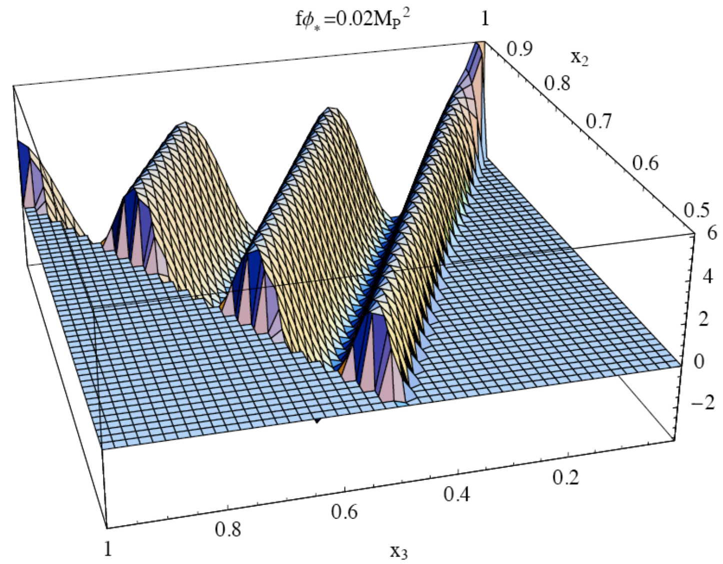
Relaxing Naturalness

- Different Shapes

See **Bartolo**'s talk

Relaxing the shift symmetry of π

- Resonance effects are possible
- Goldstone not in the vacuum state
- Oscillations become possible



See **Flauger**'s talk

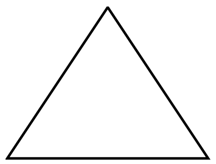
Effects from late-time non-linearities

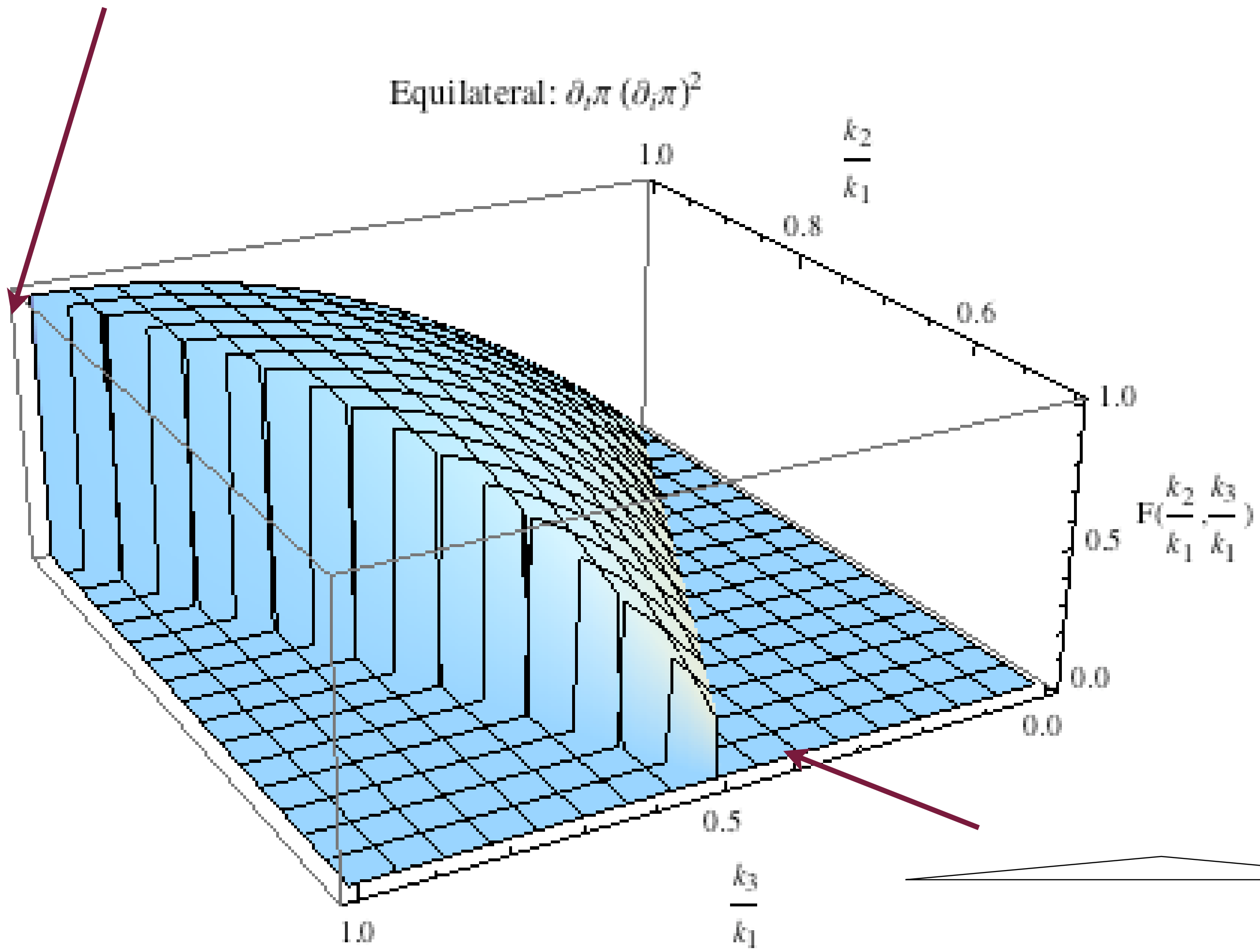
- Non-linearities in plasma physics and GR induce non-linearities
- Very large industry
- Final upshots: $f_{NL} \sim \text{few}$

See **Pritrou**'s talk

Shape of NG

Single field

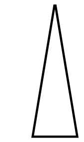




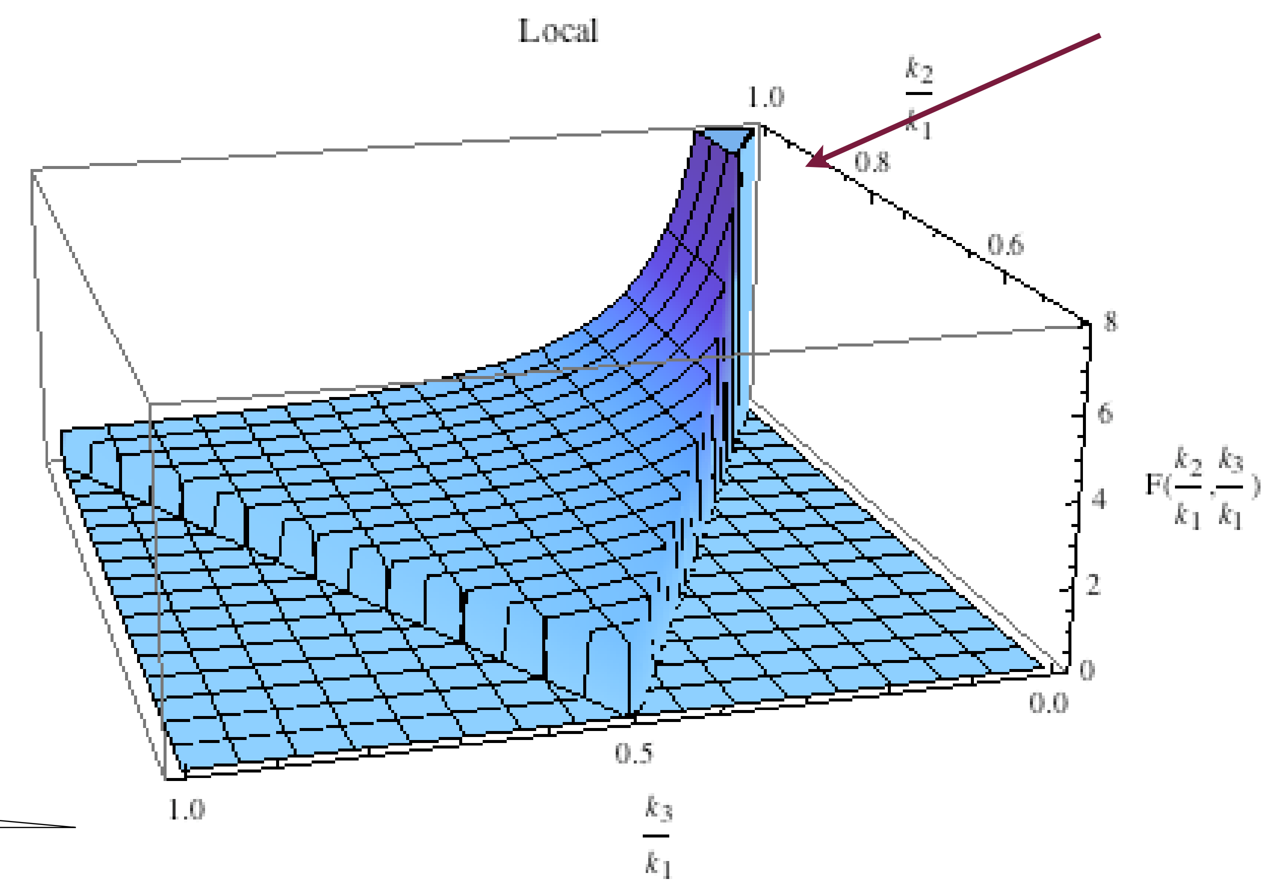
Multi field

Dvali, Gruzinov and Zaldarriaga **Phys.Rev.D69:023505,2004**

with Creminelli **JCAP 0711:010,2007**



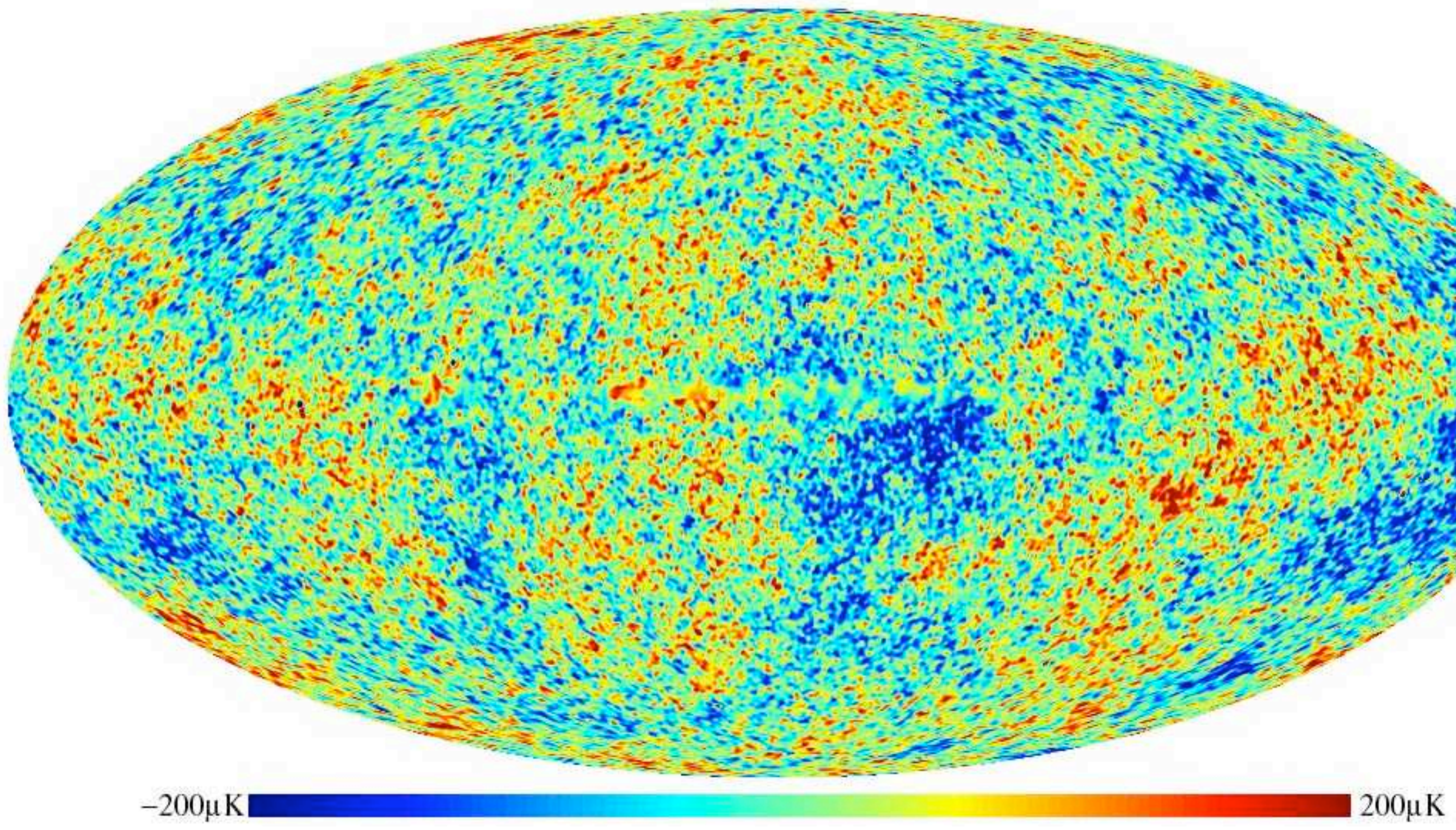
$$\zeta(x) = \zeta_g(x) + f_{\rm NL}^{\rm loc.}\zeta_g(x)^2$$



- Theoretically motivated
- Wealth of information
- They need to be analyzed in the data

- Possible generalizations   see **Rajante**'s talk

Is there some non-Gaussianity now?
We are ready!



Analysis of the WMAP data

With Creminelli, Nicolis, Tegmark and Zaldarriaga, **JCAP 0605:004,2006**
With Smith and Zaldarriaga, **JCAP0909:006,2009**
JCAP1001:028,2010

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{lm} a_{lm} Y_{lm}(\theta,\phi) \qquad C_{l_1 m_1, l_2, m_2} = \langle a_{l_1 m_1} a_{l_2 m_2} \rangle$$

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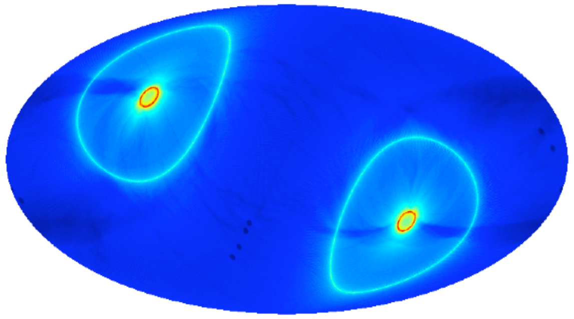
Minimum variance estimator for three shapes:

$$\mathcal{E}_{\rm lin}(a) = \frac{1}{N} \sum_{l_i m_i} \Big(\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_1 \, C^{-1}_{l_1 m_1, l_4 m_4} C^{-1}_{l_2 m_2, l_5 m_5} C^{-1}_{l_3 m_3, l_6 m_6} a_{l_4 m_4} a_{l_5 m_5} a_{l_6 m_6}$$

CMB signal diagonal in Fourier space (without NG!!). Foreground and noise in real space.



Non-diagonal error matrix + linear term in the estimator

It saturates Cramers-Rao bound. With Creminelli and Zaldarriaga
Nothing else is necessary **JCAP 0703:019,2007**

Reduces variance wrt WMAP coll. analysis (~ 60%), generalize to the other two shapes, foreground marginalization.

Technique progressively adopted by the WMAP collaboration: linear term, shapes, inverse matrix.

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$$-3 \, \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_1 \, C^{-1}_{l_1 m_1, l_2 m_2} C^{-1}_{l_3 m_3, l_4 m_4} a_{l_4 m_4} \Big)$$

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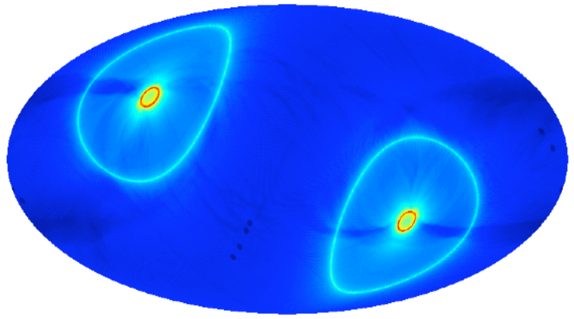

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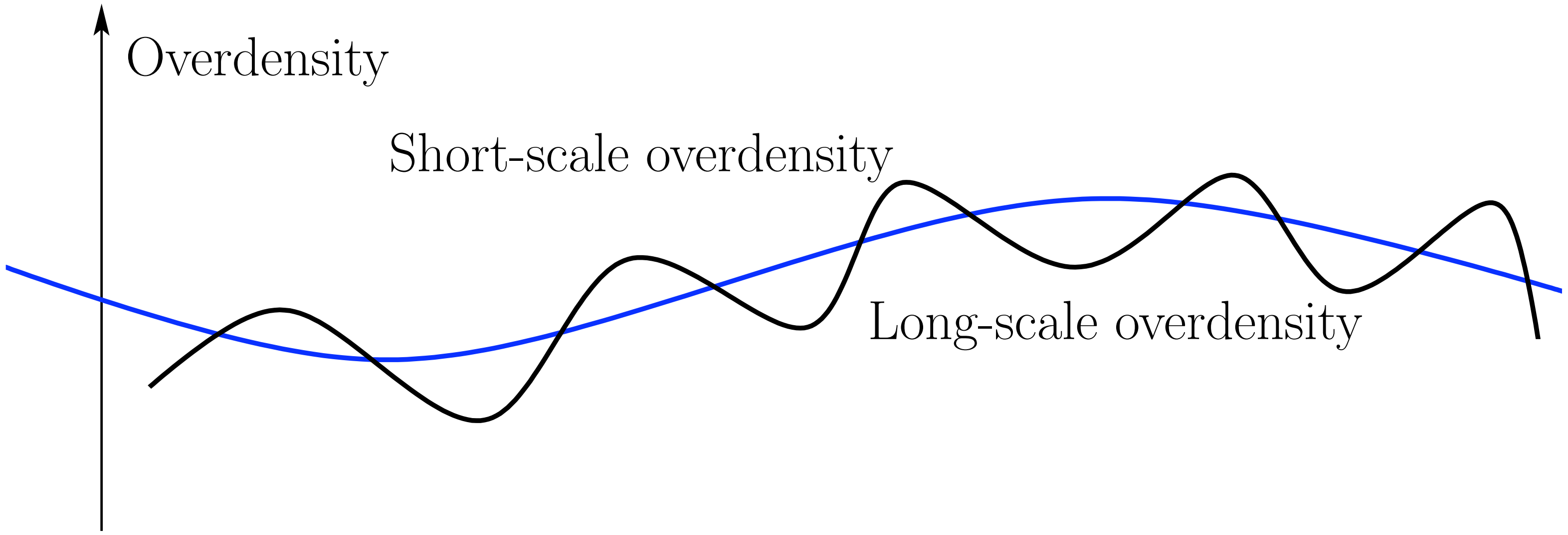
Looking in Large Scale Structures

- Bias $\delta n_g(M) = f(\delta_c/\sigma(M))$



- Gaussian: $\delta n_g(k) = b\ \delta_{\rm DM}(k)$
- Non-Gaussian: $\delta n_g(k) = b(k, f_{\rm NL})\ \delta_{\rm DM}(k)$

Dalal et al. **Phys.Rev.D77:123514,2008**

Large Activity: see
Musso, Chongchitnan, Desjacques, Sefusatti's talks

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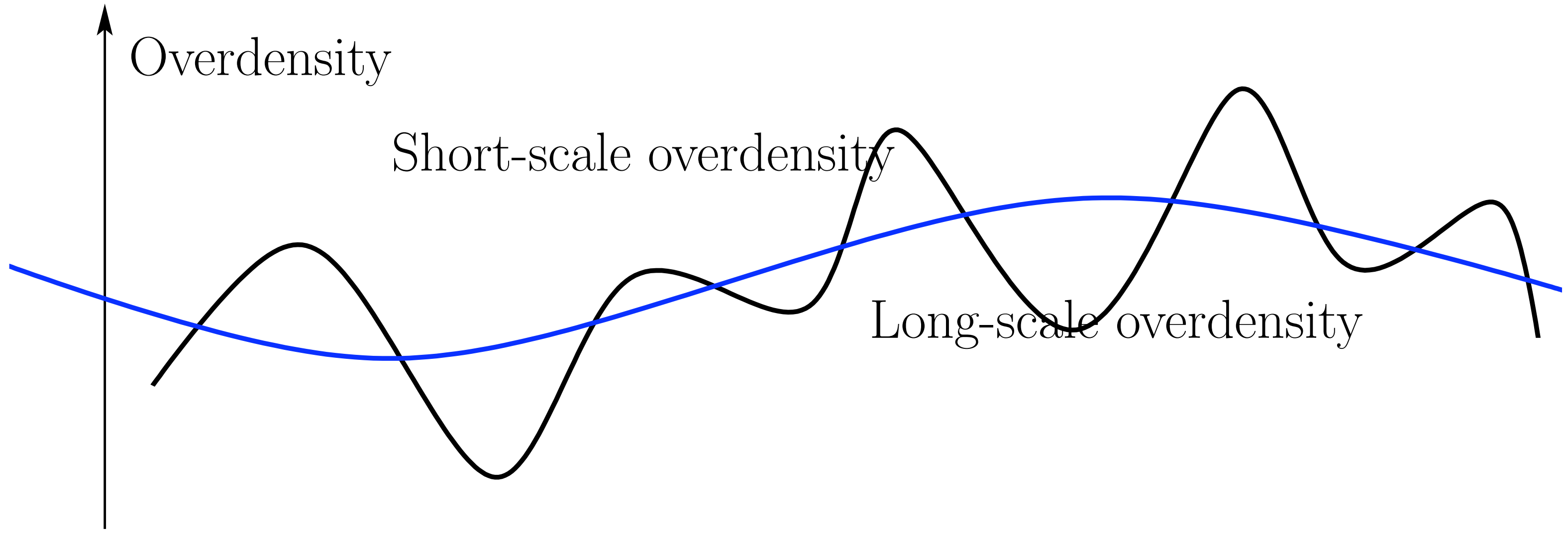

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Large Activity: see
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☹ ~No detection ☹

With Smith and Zaldarriaga,
JCAP0909:006,2009
JCAP1001:028,2010

Optimal analysis of WMAP data (foreground template corrections) are ~ compatible with Gaussianity

Optimal limits on NG

$-10 < f_{NL}^{local} < 74$ at 95% C.L.

Komatsu *et al*. **WMAP 7yr**

$(-5 < f_{NL}^{local} < 59$ at 95% C.L.)

after combining with LSS
Slosar *et al*. **JCAP 0808:031, 2008**

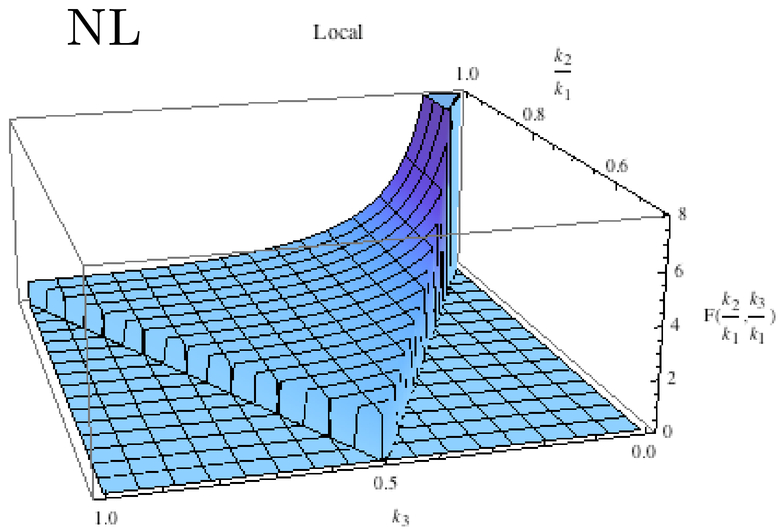
Large Activity: see
Musso, Chongchitnan, Desjacques, Sefusatti's talks

$-214 < f_{NL}^{equil.} < 266$ at 95% C.L.

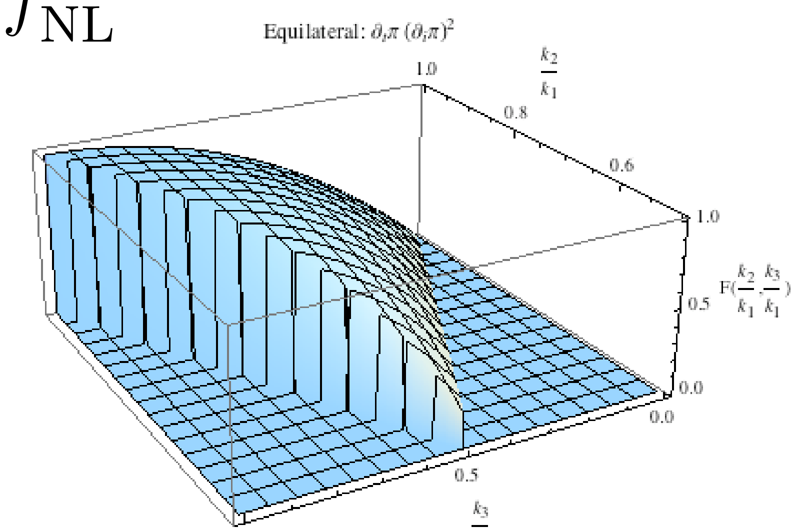
$-410 < f_{NL}^{orthog.} < 6$ at 95% C.L.

Komatsu *et al*. **WMAP 7yr**

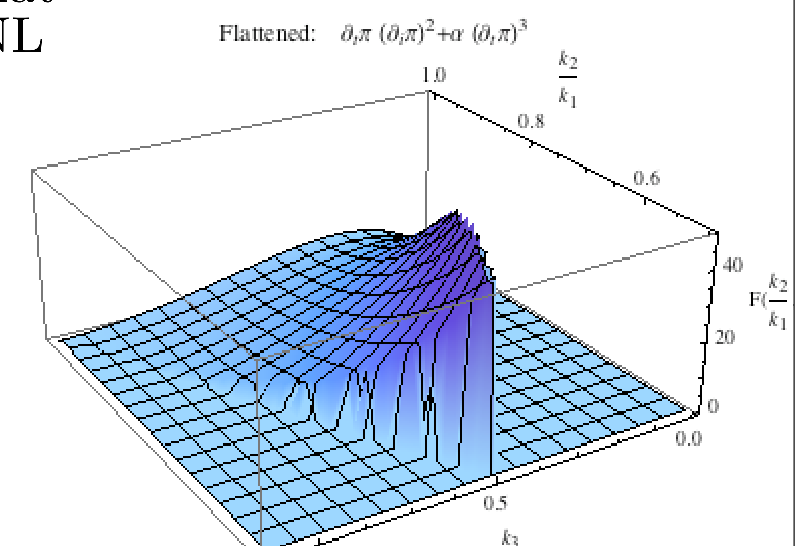
$f_{NL}^{loc.}$



$f_{NL}^{equil.}$



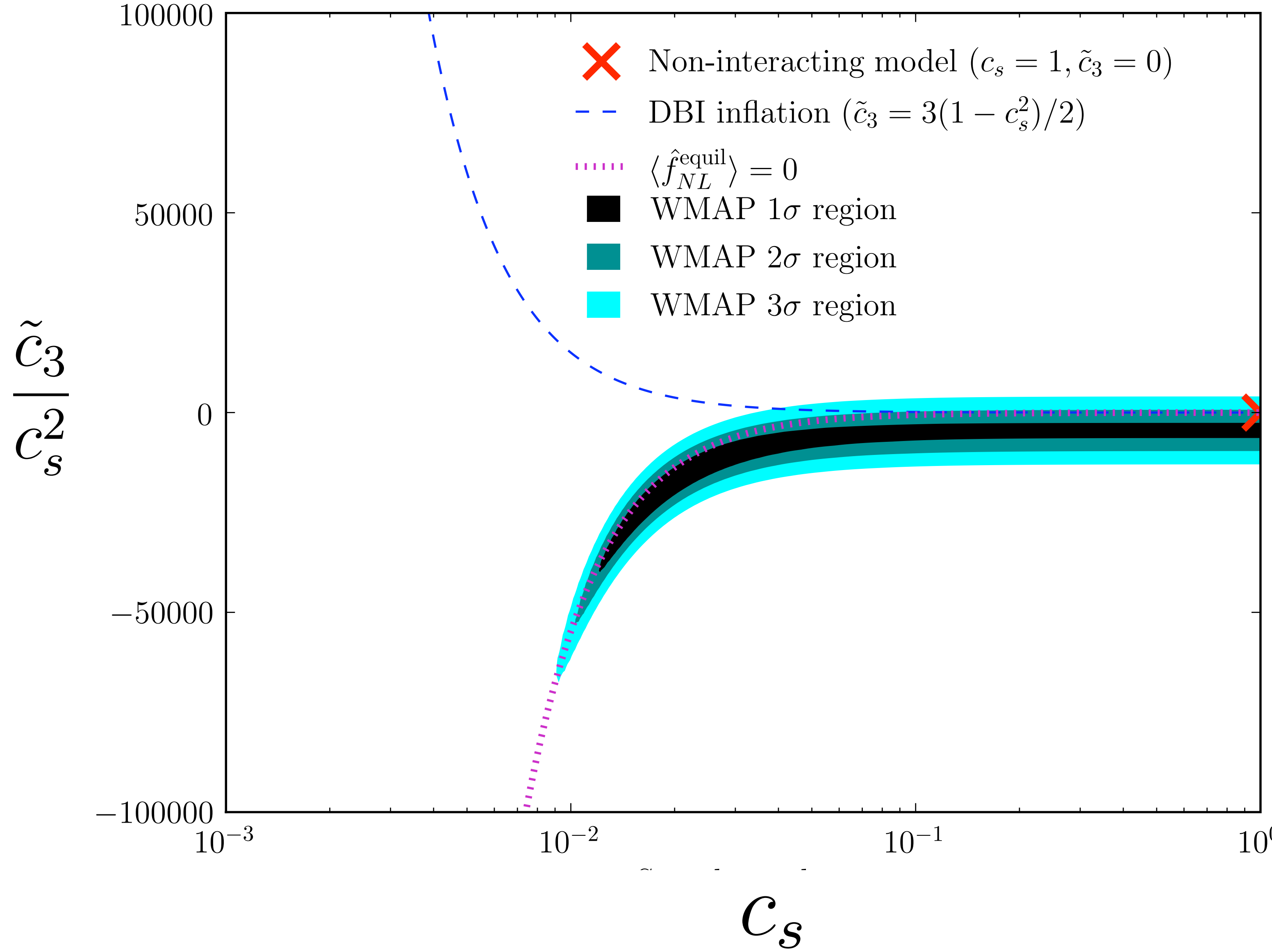
f_{NL}^{flat}



(Optimal) Limits on the parameters of the Lagrangian

$$S_\pi = \int d^4x \, \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H} (\dot{\pi}^2 - (\partial_i \pi)^2) + M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}(\partial_i \pi)^2 \right) - M_3^4 \dot{\pi}^3 + \ldots \right]$$

- Limits on f_{NL} 's get translated into limits on the parameters
- For models not-very-close to de Sitter (like DBI): c_s , $\tilde{c}_3$



With Smith and Zaldarriaga, **JCAP1001:028,2010**

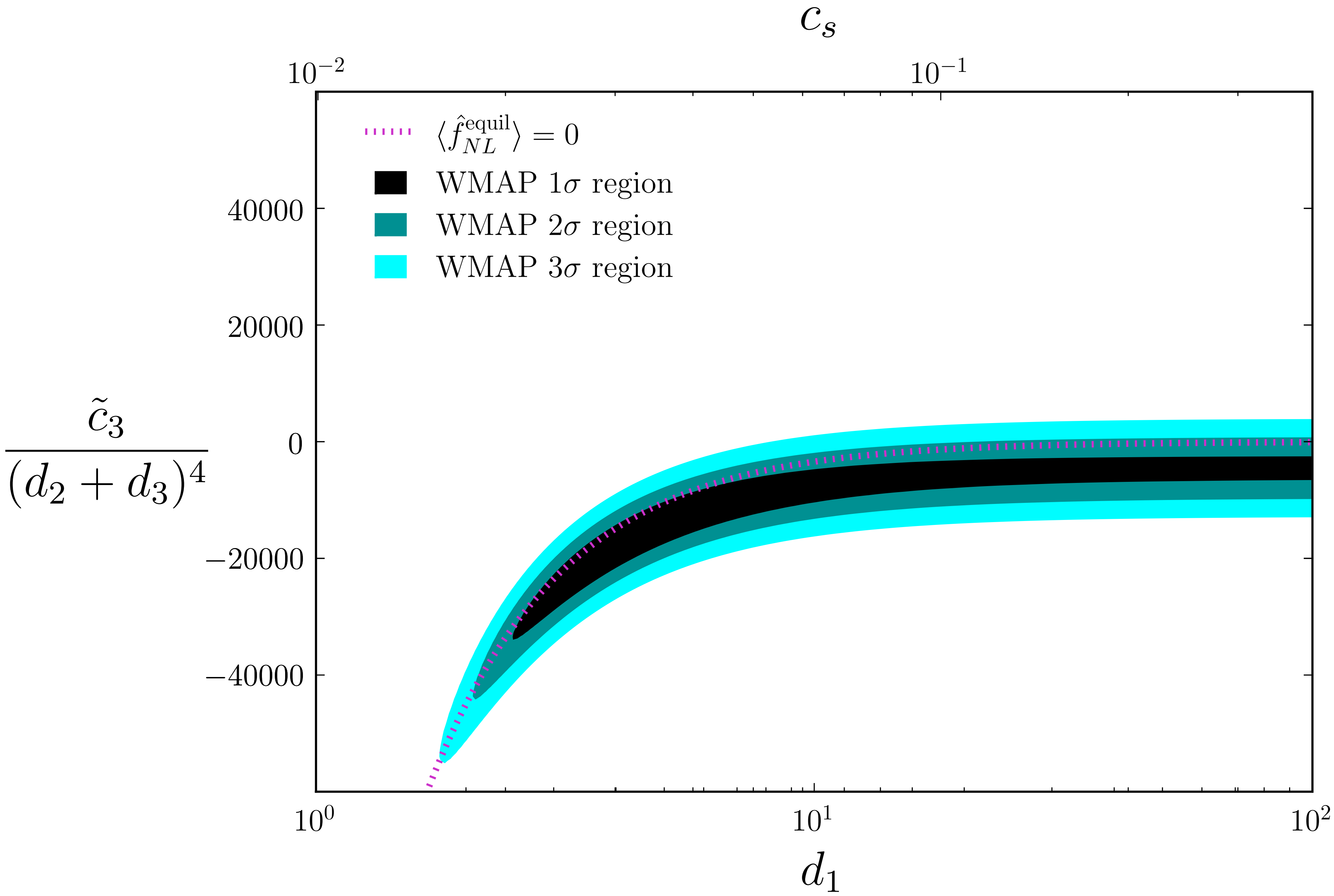
Very similar in spirit to Peskin and Takeuchi **PRD46:381,1992**

$$\frac{1}{c_s^2} \dot{\pi} (\partial_i \pi)^2 + \frac{\tilde{c}_3}{c_s^2} \dot{\pi}^3$$

- Limit on the speed of sound: $c_s \gtrsim 0.011$!

(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter. $d_1\, \delta g^{00} \delta K^i_i$
- Dispertion relation: $\omega^2 = c_s^2 k^2 \qquad c_s^2 = d_1 \frac{H}{M} \ll 1$



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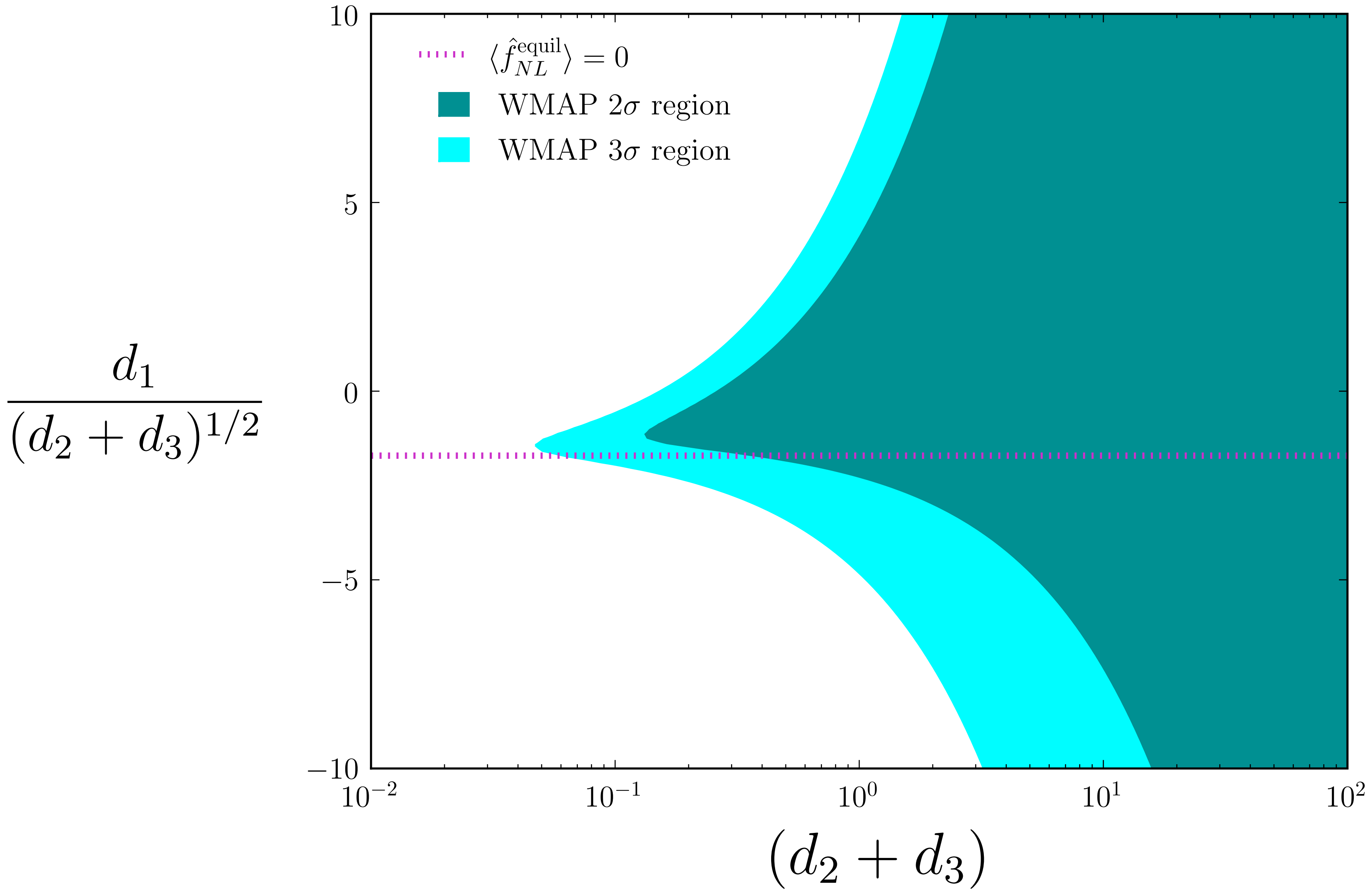
(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter. $d_2\,\delta K_i^{i\,2}$
- Dispertion relation: $\omega^2 = (d_2 + d_3)\dfrac{k^4}{M^2}$



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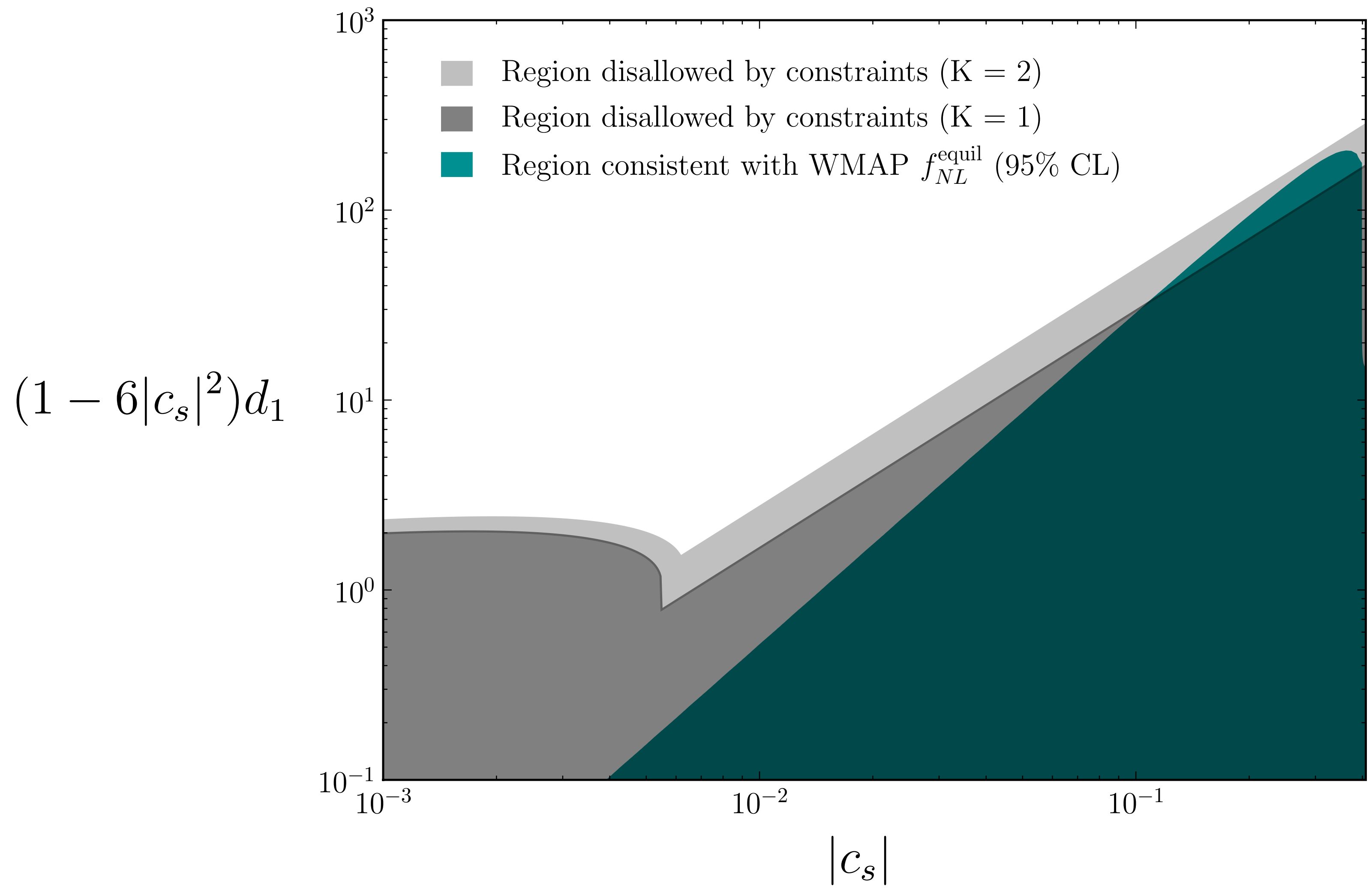
(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter.
- Negative c_s^2 due to $d_1 < 0$ $\qquad c_s^2 = d_1 \frac{H}{M} \ll 1$
- Ruled out at 95% CL.



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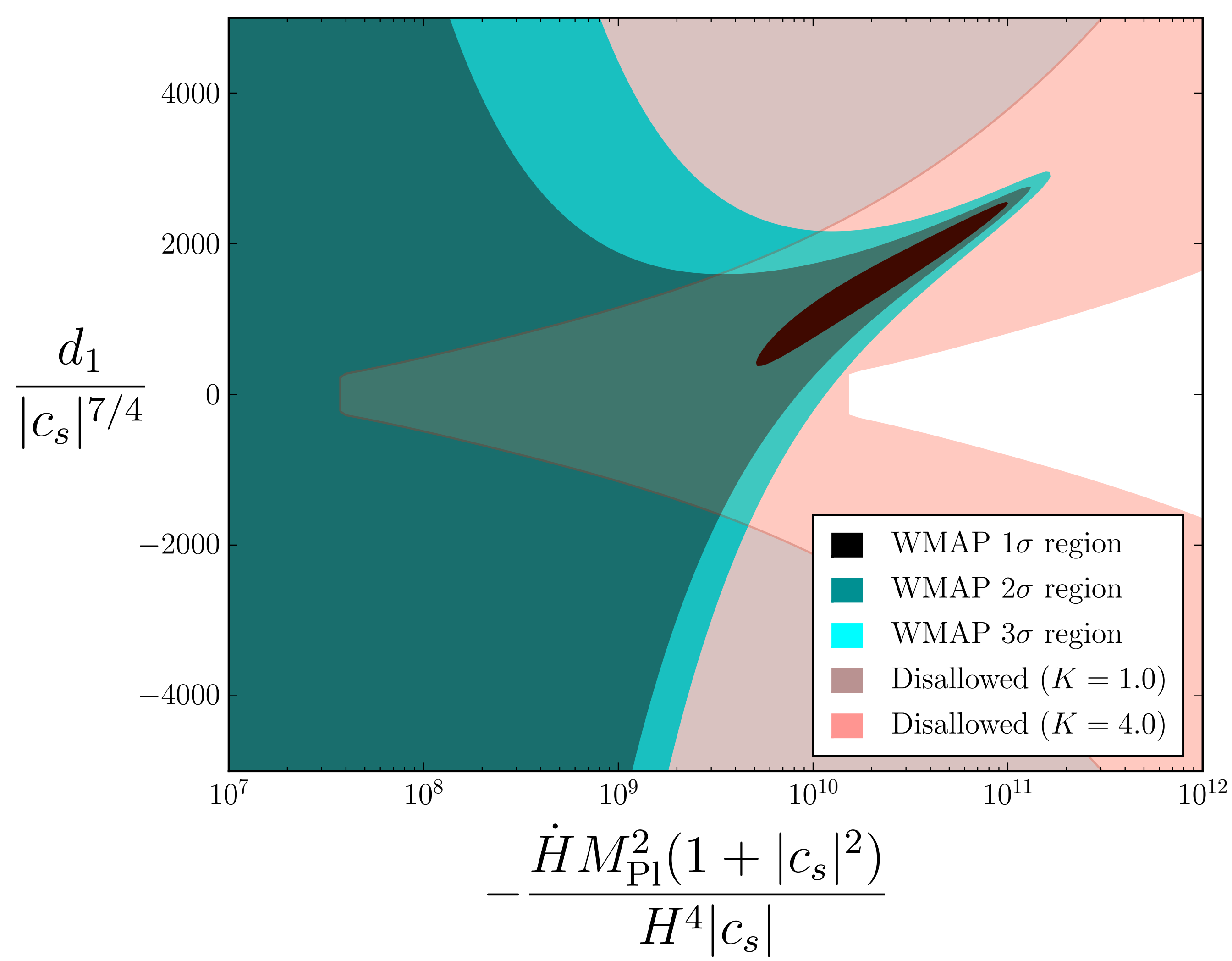
(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter.
- Negative c_s^2 due to $\dot{H} > 0$ $\qquad \dot{H} M_{\rm Pl}^2 (\partial_i \pi)^2$
- Ruled out at 95% CL.



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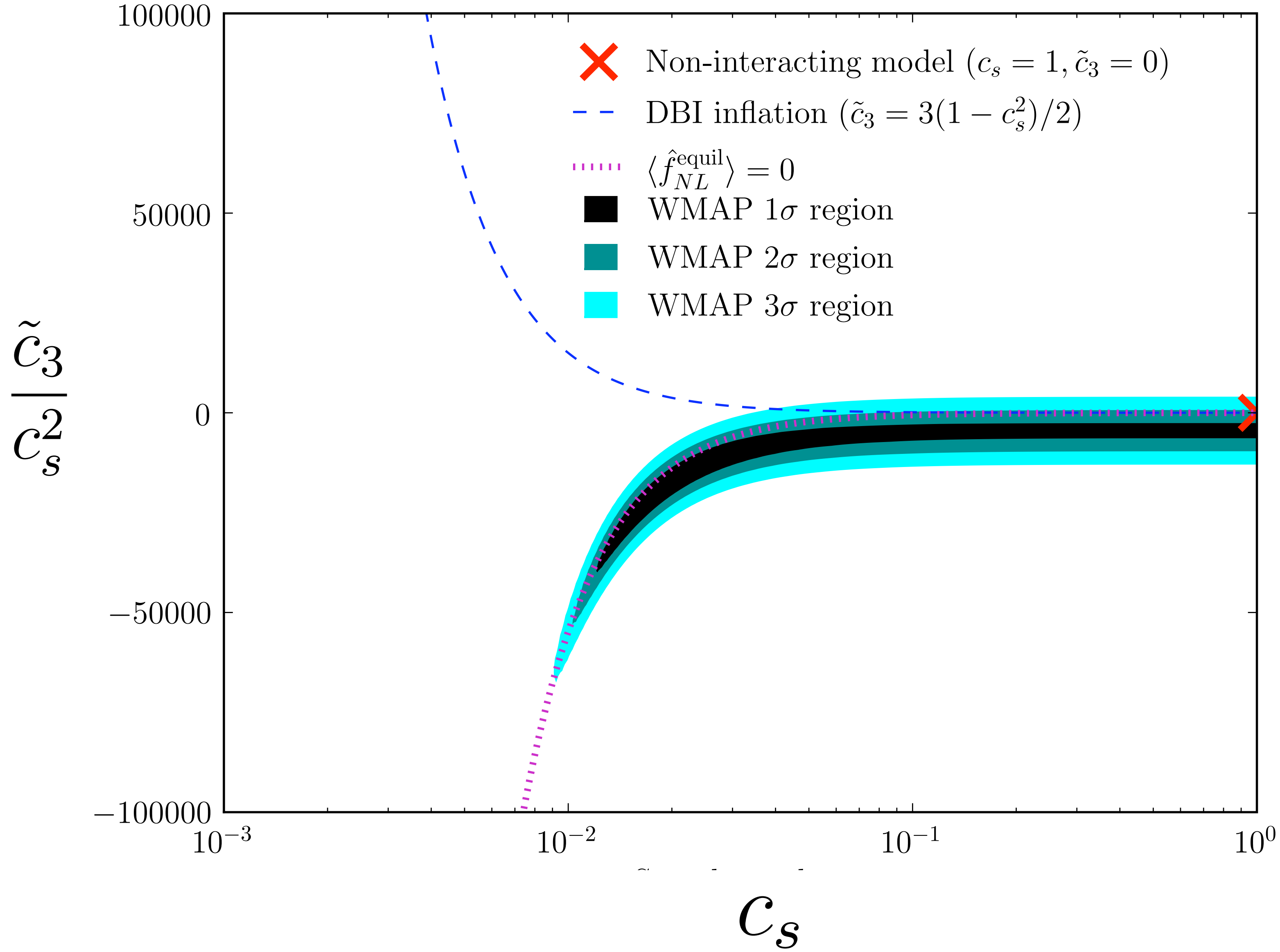
- Thanks to the EFT: A qualitatively new (and superior) way to use the cosmological data



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$$\frac{1}{c_s^2} \dot{\pi} (\partial_i \pi)^2 + \frac{\tilde{c}_3}{c_s^2} \dot{\pi}^3$$

This was about 3-point function.
What about 4-point function?

with M. Zaldarriaga
1004:1201 [hep-th]

Another New Signature:
A large 4-point function without a larger 3-point function

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1004:1201 [hep-th]

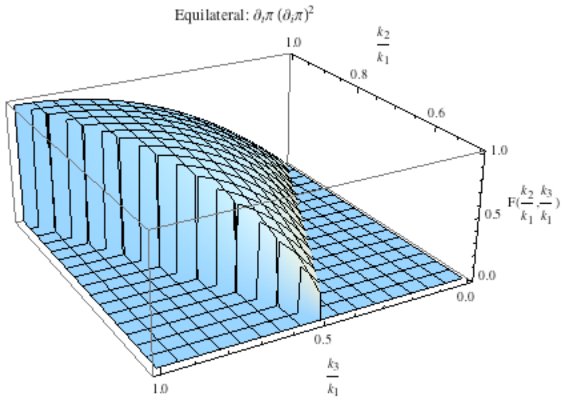
- So far 3-point: $\dot{\pi}(\partial_i\pi)^2 \quad \Rightarrow \quad \langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle$

- Large 4-point: Symmetries forces to have a leading 3-point function but for one case:

$$\dot{\pi}^4$$

- Protected by a symmetry

- Huge amount of information: function of 5 variables
- Looking it in the data



with Smith and Zaldarriaga **in progress**

Effective Field Theory of Multifield Inflation

with M. Zaldarriaga
1009.2093 hep-th

The Effective Field Theory for Multifield Inflation

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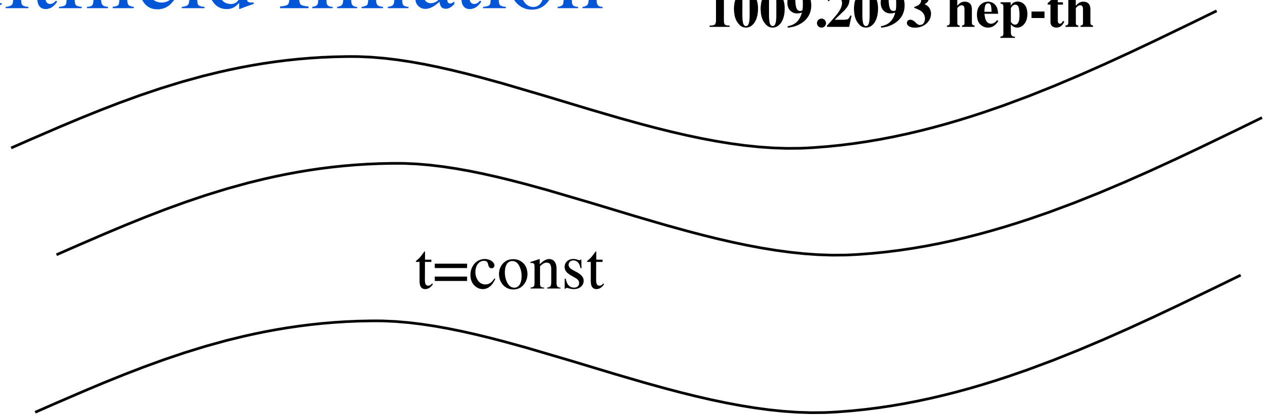

In the same Unitary Gauge,

consider another massless scalar field σ

(approximate shift symmetry, done also for non-Abelian symmetry and Supersymmetry)

Most generic Lagrangian built by metric operators and σ invariant only under $x^i \to x^i + \xi^i(t,\vec{x})$

- Generic functions of time
- Upper 0 indices are ok. E.g. g^{00} R^{00} $g^{0\mu}$

$$S_{\rm M.F.} = \int d^4x\,\sqrt{-g}\Big[\tilde M_1^2\,\delta g^{00}(g^{0\mu}\partial_\mu\sigma) + e_1\,(\partial_\mu\sigma)^2 + e_2\,(g^{0\mu}\partial_\mu\sigma)^2 + $$
$$+e_3^2\,\delta g^{00}(g^{0\mu}\partial_\mu\sigma)^2 + e_4^2\,\delta g^{00}(\partial_\mu\sigma)^2 + \tilde M_2^2\,(\delta g^{00})^2(g^{0\mu}\partial_\mu\sigma)$$
$$+\tilde M_3^{-2}\,(g^{0\mu}\partial_\mu\sigma)^3 + \tilde M_4^{-2}\,(g^{0\mu}\partial_\mu\sigma)(\partial_\mu\sigma)^2 + \ldots\Big]\,.$$

- + soft breaking terms.
- Reintroduce the Goldstone

Reintroducing the Goldstone

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- Quadratic Lagrangian

$$S^{(2)} = \int d^4x \sqrt{-g} \left[(2M_2^4 - M_{\rm Pl}^2 \dot H) \dot\pi^2 + M_{\rm Pl}^2 \dot H \frac{(\partial_i \pi)^2}{a^2} + 2\tilde M_1^2 \, \dot\pi \dot\sigma + (-e_1 + e_2)\, \dot\sigma^2 + e_1 \, \frac{(\partial_i \sigma)^2}{a^2} + \ldots \right]$$

- Cubic Lagrangian

$$S^{(3)} = \int d^4x \sqrt{-g} \quad \Big[M_2^4 \, \dot\pi \frac{(\partial_i \pi)^2}{a^2} + \left(M_2^4 - M_3^4 \right) \dot\pi^3 +$$
$$-(\tilde M_1^2 + \tilde M_2^2) \dot\pi^2 \dot\sigma - \tilde M_1^2 \frac{(\partial_i \pi)^2}{a^2} \dot\sigma - \tilde M_1^2 \dot\pi \frac{\partial_i \pi \partial_i \sigma}{a^2}$$
$$(e_2 - e_3 + e_4)\, \dot\pi \dot\sigma^2 - e_4 \dot\pi \frac{\partial_i \sigma \partial_i \sigma}{a^2} - e_2 \frac{\partial_i \pi \partial_i \sigma}{a^2} \dot\sigma$$
$$+ \left(\tilde M_4^{-2} - \tilde M_3^{-2} \right) \dot\sigma^3 - \tilde M_4^{-2} \dot\sigma \frac{\partial_i \sigma \partial_i \sigma}{a^2} + \ldots \Big] \, ,$$

- Quartic Lagrangian ....
- Notice:
  - Small π speed of sound: Large coupling $\quad M^4 \dot\pi^2 \quad \rightarrow \quad M^4 \dot\pi (\partial_i \pi)^2$
  - Small σ speed of sound: Large coupling $(-e_1 + e_2)\, \dot\sigma^2 \quad \rightarrow \quad e_2 \, (\partial_i \pi \partial_i \sigma) \dot\sigma$
  - Time-kinetic mixing σ - π .

Conversion into curvature perturbations

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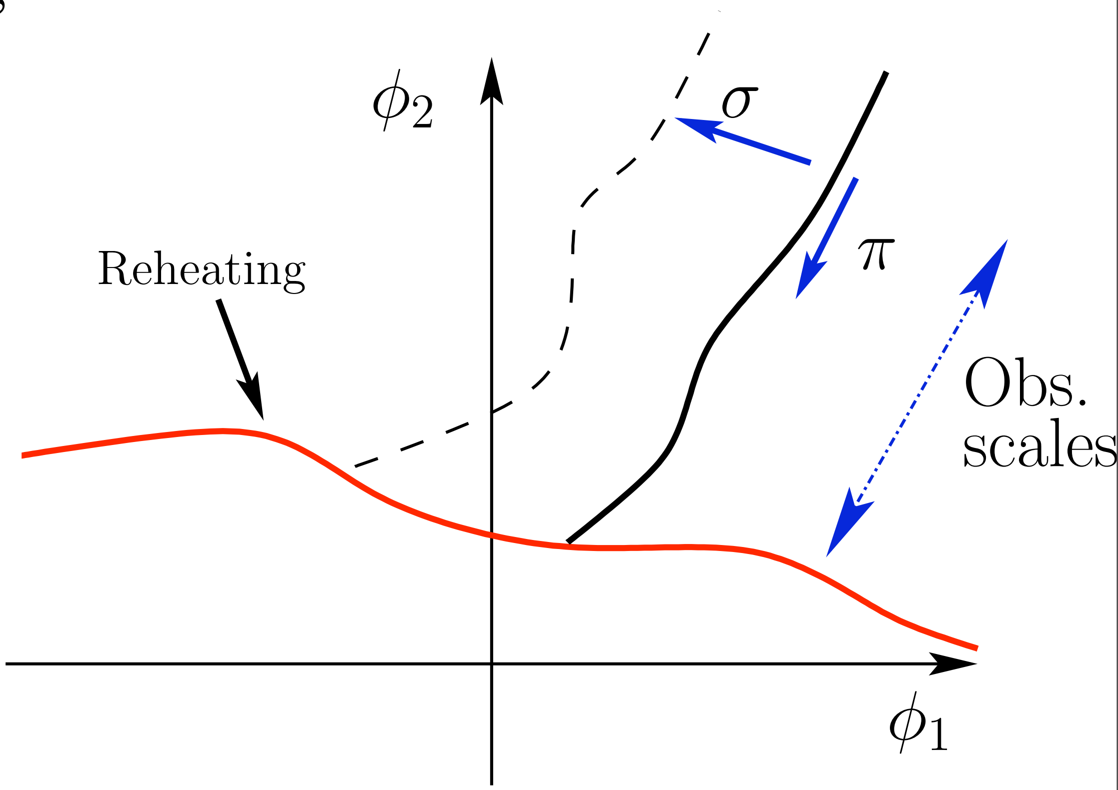
- ζ: How much the universe expanded by reheating

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta} dx_i^2$$



- the former Lagrangian is local in field space
- non-local relationship in field space
- we are interested only in some modes
- the most important effects happen when the mode is outside of the horizon
- local in real space relationship $\zeta(x^i) = f\left(\sigma(x^i)\right)$
- we can Taylor expand

$$\zeta(x) = \quad H\,\pi(x) + \left.\frac{\partial\zeta}{\partial\sigma}\right|_0 \sigma(x) +$$
$$\dot{H}\,\pi(x)^2 + \frac{1}{2!}\left.\frac{\partial^2\zeta}{\partial\pi\partial\sigma}\right|_0 \pi(x)\sigma(x) + \frac{1}{2!}\left.\frac{\partial^2\zeta}{\partial\sigma^2}\right|_0 \sigma(x)^2 + \ldots$$

- Terms in π are known or small $\quad \zeta = H\pi + \dot{H}\pi^2$
- Same for isocurvature perturbations

New Signatures: new 3-point and 4-point functions

with M. Zaldarriaga
1009.2093 hep-th

- In multifield inflation:

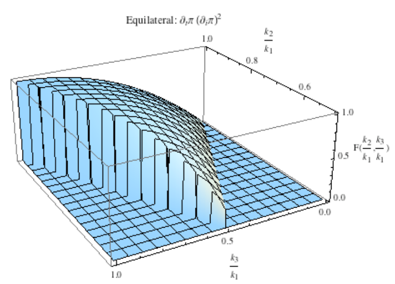
  -Impose symm. $\sigma \longrightarrow -\sigma$

  -Approximate Lorentz invariance $\Longrightarrow$ σ^3 terms

- Large 4-point function $\dot\sigma^4\ ,\quad \dot\sigma^2(\partial_i\sigma)^2\ ,\quad (\partial_i\sigma)^4\ ,\quad \sigma^2(\partial\sigma)^2\quad \sigma^4$

$$\frac{(\partial\sigma)^4}{M^4} \quad \Rightarrow \quad \text{NG} \sim \left.\frac{\mathcal{L}_4}{\mathcal{L}_2}\right|_{E\sim H} \sim \frac{H^4}{M^4} \quad \text{can be} \quad \gg 10^{-5} \quad \Rightarrow \quad \text{detectable!}$$

- and it is a function of 5 variables!



- This is observationally (and also the theory for the analysis) very unexplored.

with Smith and Zaldarriaga **in progress**

A New Signatures: new 3-point and 4-point functions

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MultiField

Operator	Dispersion		Type	Origin	Squeezed L.
	$w = c_s k$	$w \propto k^2$			
$\dot\sigma^4\ ,\ \dot\sigma^2(\partial_i\sigma)^2\ ,(\partial_i\sigma)^4$	X		Ad., Iso.	Ab., non-Ab.	
$(\partial_\mu\sigma)^4$	X		Ad., Iso.	Ab., non-Ab.	
σ^4	X	X	Ad., Iso.	Ab.$_s$, non-Ab.$_s$, S.*	X
$\dot\sigma\sigma^3$	X	X	Ad., Iso.	Ab.$_s^\dagger$, non-Ab.$_s^\dagger$.	X
$\sigma^2\dot\sigma^2\ ,\sigma^2(\partial_i\sigma)^2$	X	X†*	Ad.†*, Iso.	non-Ab, Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$,	X
$\sigma^2(\partial_\mu\sigma)^2$	X		Ad.†*, Iso.	non-Ab, Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$, S.*	X
$\sigma(\partial\sigma)^3$	X		Iso.	non-Ab.$_s^\star$.	X
$\dot\sigma^3\ ,\ \dot\sigma(\partial_i\sigma)^2$	X		Ad., Iso.	Ab., non-Ab.	
$\dot\sigma(\partial_i\sigma)^2\ ,\partial_j^2\sigma(\partial_i\sigma)^2$		X	Ad., Iso.	Ab.	
σ^3	X	X	Ad., Iso.	Ab.$_s$, non-Ab.$_s$, S, R	X
$\dot\sigma\sigma^2$	X	X	Ad., Iso.	Ab.$_s$, non-Ab.$_s$	X
$\sigma\dot\sigma^2\ ,\ \sigma(\partial_i\sigma)^2$	X	X	Ad., Iso.	Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$	X
$\sigma(\partial_\mu\sigma)^2$	X		Ad., Iso.	Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$.	X

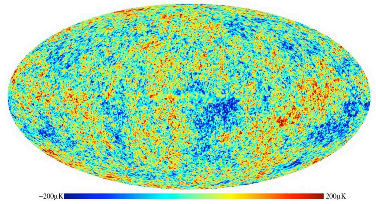
Single Field

Operator	Dispersion		Squeezed L.
	$w = c_s k$	$w \propto k^2$	
$\dot\pi^4$	X		
$(\partial_j^2\pi)^4\ ,\dot\pi(\partial_j^2\pi)^3\ ,\ldots$		X	
$\dot\pi^3\ ,\dot\pi(\partial_i\pi)^2$	X		
$\dot\pi(\partial_i\pi)^2\ ,\partial_j^2\pi(\partial_i\pi)^2$		X	

You can tell them apart!

Conclusions

Cosmology



- A data driven subject
- Huge theoretical implications (Inflation, ... , CC, Landscape, Eternal Inflation)
- We are ready for the NG!

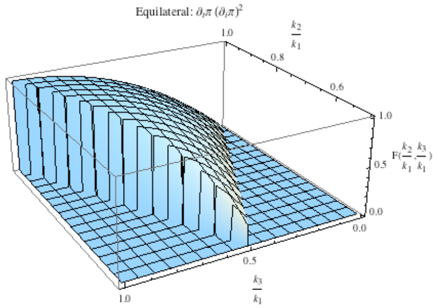
Fundamental Theory

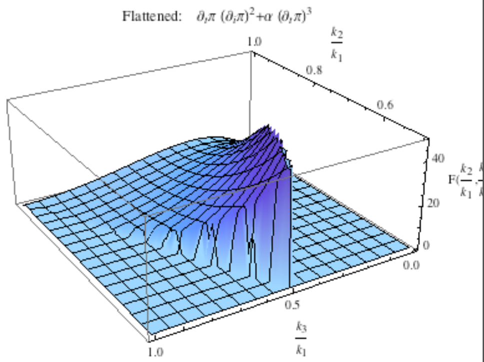
Probing Inflation

- An Effective Lagrangian to see all what is possible and what we learn from exp.
- Non-Gaussianities:
  - What they teach us
  - A full exploration

$$\frac{1}{c_s^2}\dot{\pi}(\partial_i\pi)^2 + \frac{\tilde{c}_3}{c_s^2}\dot{\pi}^3$$

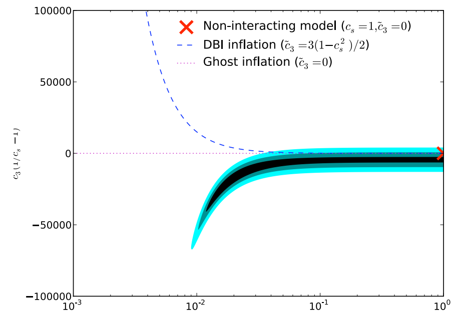
- The Effective Theory of Multifield Inflation





Data Analysis

- The optimal limits from WMAP7yr: development and application of new techniques.



$-410 < f_{NL}^{orthog.} < 6$ at 95% C.L.

Reintroducing the Goldstone

At sufficiently high energy the Goldstone mode decouples.

$$S = \int d^4x \, -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 \mathrm{Tr}\, A_\mu A^\mu \qquad \text{where } A_\mu = A_\mu^a T^a .$$

Gauge transformation:

$$A_\mu \to U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger \equiv \frac{i}{g} U D_\mu U^\dagger \, . \qquad S = \int d^4x \, -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\frac{m^2}{g^2}\mathrm{Tr} D_\mu U^\dagger D_\mu U \, .$$

Gauge invariance is "restored" introducing the Goldstones:

$$U = \exp\left[iT^a \xi^a(t,\vec{x})\right] \quad \Rightarrow \quad U = \exp\left[iT^a \pi^a(t,\vec{x})\right]$$

Under a gauge trans. Λ we impose: $e^{iT^a\tilde{\pi}^a(t,\vec{x})} = \Lambda(t,\vec{x})\, e^{iT^a\pi^a(t,\vec{x})}$ $\qquad e^{i\tilde{\pi}} = e^{i(\pi+\alpha)}$

Going to canonical normalization: $\pi_c \equiv m/g \cdot \pi$ $\qquad \pi^2(\partial\pi)^2 \quad \Rightarrow$ Cutoff: $4\pi m/g$

Mixing with transverse component: $\frac{m^2}{g}A_\mu^a \partial^\mu \pi^a = m A_\mu^a \partial^\mu \pi_c^a$ $\qquad$ Irrelevant for $E \gg m$

In the window: $m \ll E \ll 4\pi m/g$ $\qquad$ The physics of the Goldstones is perturbative and decoupled from transverse modes