Large-Scale Suppression from Stochastic Inflation

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Overview

In this work (based on [1-4]):

- We introduce a **novel technique** to properly calculate correlation functions and in turn the **power spectrum**,
- derive a **large-scale damping**, which
- **solves the problem of infra-red divergencies** of two-point correlation functions.
- We provide a **better description of CMB data**.

Setup

One fruitful framework to study **large quantum fluctuations** is **stochastic inflation**:

- **Split** the quantum field Φ into a **short- and long-** wavelength part, $\boldsymbol{\Phi = \varphi + \phi}$, and treat φ as classical, evolving in a **bath** of the quantum modes ϕ, giving rise to a
- **Langevin-type** differential equation,

$$\big(\Box + m^2\big)\varphi(t, \boldsymbol{x}) = \mathrm{h}(t, \boldsymbol{x}) \; .$$

Technicalities

We **would like to compute** the noise-averaged

- connected **correlation functions**, and hence their **generating functional** $\ln(\mathcal{Z})$.

This is **difficult** to achieve directly. Hence,

- use the **replica trick**:

$$\overline{\ln(\mathcal{Z})} = \lim_{n \to 0} \ln\Big(\big(\overline{\mathcal{Z}^n}\big)^{1/n}\Big) \; ,$$

- i.e., **compute** only $\overline{\mathcal{Z}^n}$ (instead of the logarithm):

$$\overline{\mathcal{Z}^n} = \int \prod_{a=1}^{n} \mathcal{D}[\varphi_a] \, \exp\big(-\mathcal{S}_n[\varphi_1, \ldots, \varphi_n]\big) \; .$$

- **Approximate** the action $\mathcal{S}_n[\varphi]$ by

$$\mathcal{S}_0[\varphi] := \frac{1}{2} \sum_{a,b=1}^{n} \int \mathrm{d}t \int_k \mathrm{G}^{-1}{}_{ab}(t,k)\, \varphi_a(t,k)\, \varphi_b(t,-k)$$

- Use the **Feyman-Jensen inequality**:

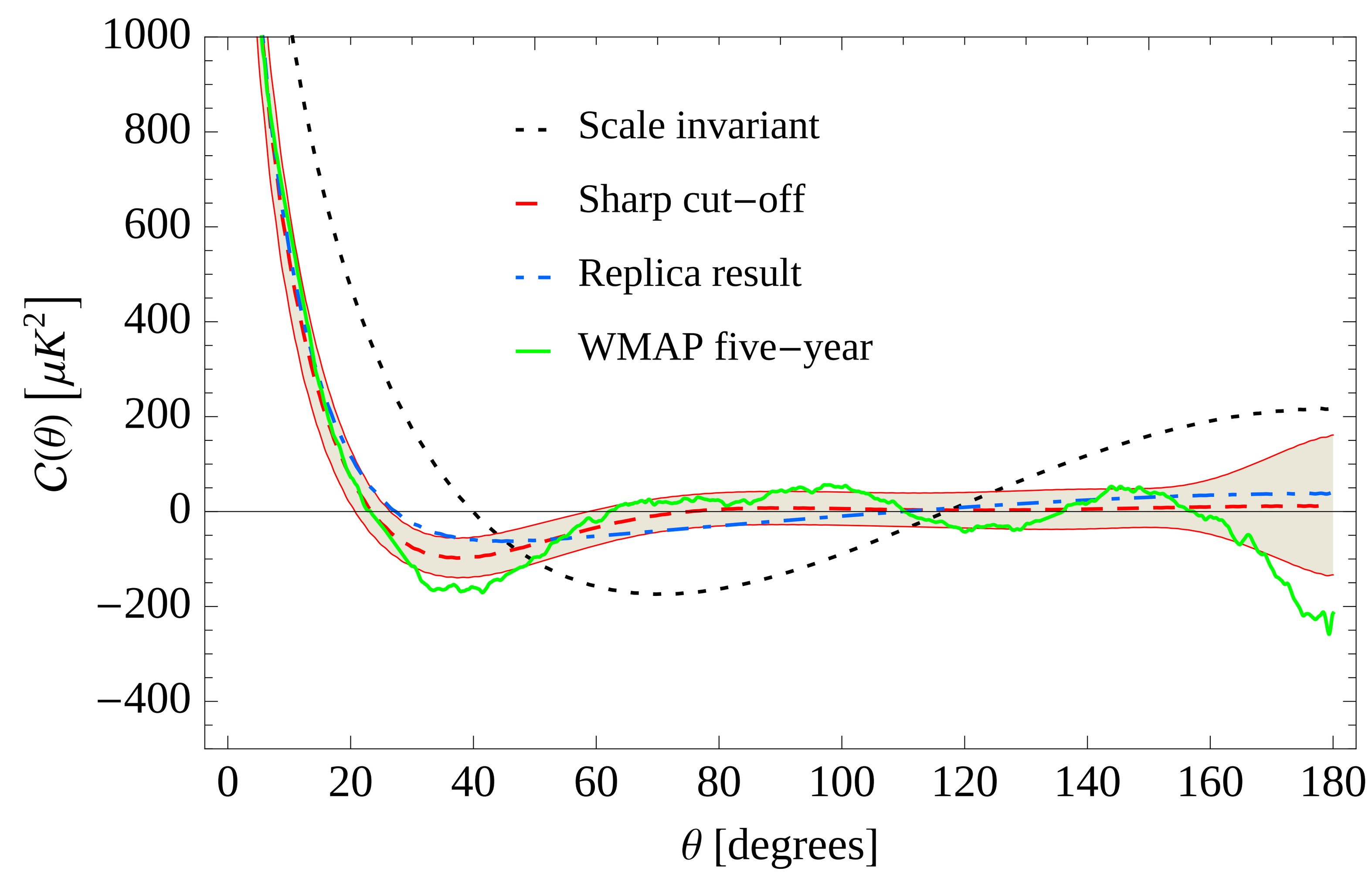
$$\ln(\mathcal{Z}) \geq \ln(\mathcal{Z}_0) + \big\langle \mathcal{S}_0 - \mathcal{S} \big\rangle_0 \; .$$

- **Maximise** the r.h.s. of this inequality, by writing

$$\mathrm{G}^{-1}{}_{ab}(t,k) := \mathrm{G}_0{}^{-1}(t,k)\, \delta_{ab} - \sigma_{ab} \; ,$$

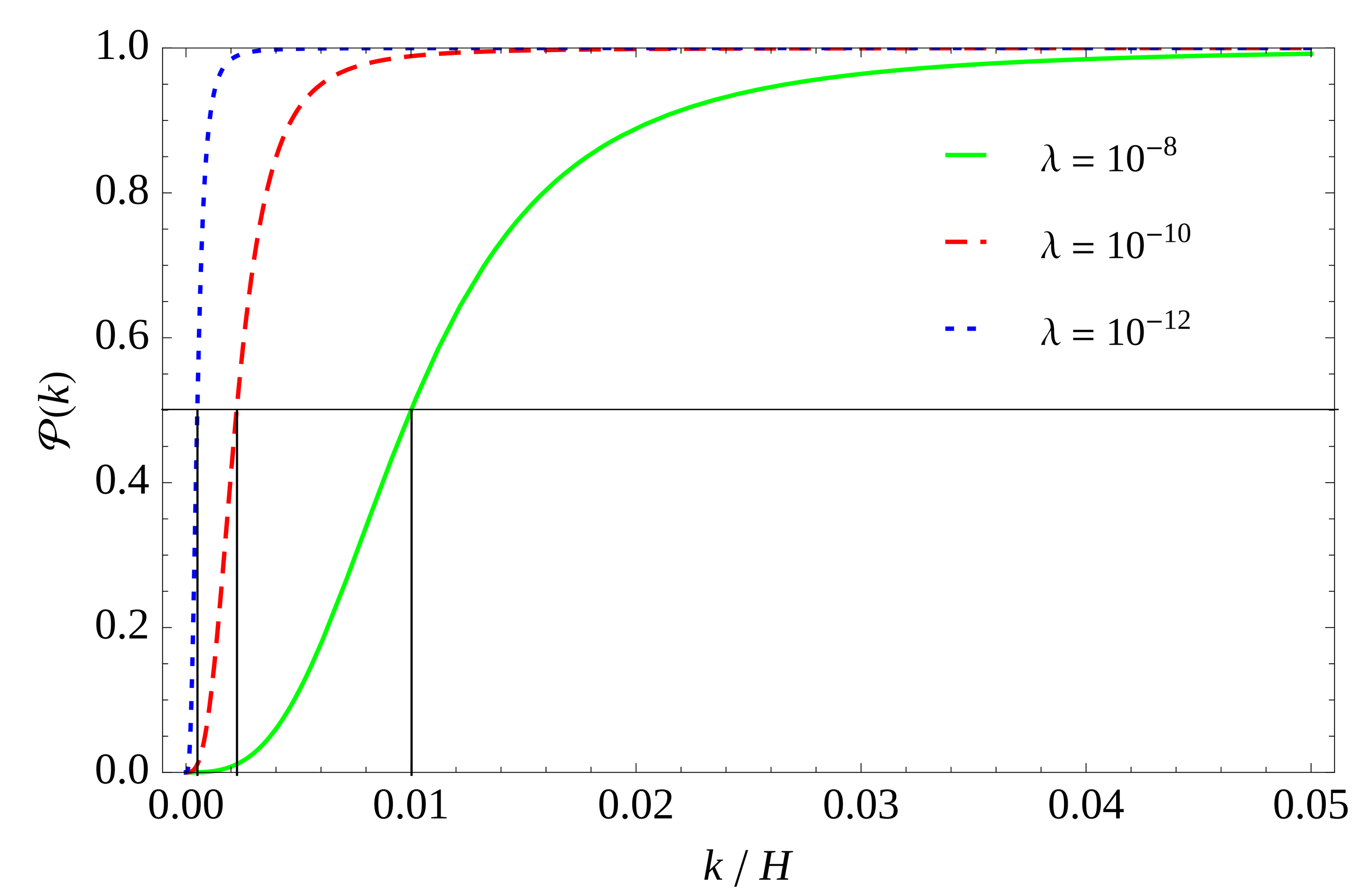
  and varying with respect to the self-energy matrix (σ_{ab}).

Application to CMB Data



Angular temperature correlation function $\mathcal{C}(\theta)$.

Large-Scale Damping



Power spectrum $\mathcal{P}(k)$ of a massless test field.

Formulæ

For a single scalar-field model with quartic self-coupling we derive:

- the **power spectrum** $\mathcal{P}(t,k)$ at late times:

$$\mathcal{P}(t,k) \simeq \frac{H^2}{(2\pi)^2} \begin{cases} \left(\dfrac{k}{k_*(t)}\right)^3 & : \; k \ll k_*(t) \; , \\ 1 & : \; k_*(t) \ll k \ll 1/|\tau(t)| \; , \end{cases}$$

- the **scale** of **deviation from scale invariance**:

$$k_*\Big|_{\text{today}} \simeq \sqrt[3]{\frac{3}{2\pi^2}}\, \sqrt[3]{\lambda N}\, \mathrm{e}^{-N} \left(\frac{H}{T_{\text{reh}}}\right) \left(\frac{T_0}{H_0}\right) H_0 \; .$$

[1] F. Kühnel and D. J. Schwarz, Phys. Rev. D **78**, 103501 (2008).
[2] F. Kühnel and D. J. Schwarz, Phys. Rev. D **79**, 44009 (2009).
[3] F. Kühnel, Ph.D. Thesis (2009).
[4] F. Kühnel and D. J. Schwarz, submitted to Phys. Rev. Lett. (2010).