Non-Gaussianity and finite length inflation

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Abstract

In the present paper, certain inflation models are shown to have large non-Gaussianity in special cases. Namely, finite length inflation models with an effective higher derivative interaction, in which slow-roll inflation is adopted as inflation and a scalar-matter-dominated period or power inflation is adopted as pre-inflation, are considered. Using Holman and Tolley's formula of the nonlinearity parameter $f_{NL}^{flattened}$, we calculate the value of $f_{NL}^{flattened}$. A large value of $f_{NL}^{flattened}$ ($f_{NL}^{flattened} > 100$) can be obtained for all of the models considered herein when the length of inflation is 60-63 *e*-folds and f_{NL} has strong dependence on the length of inflation. Interestingly, this length is similar to that for the case in which the suppression of the CMB angular power spectrum of $\ell = 2$ was derived using the inflation models described in our previous papers.

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1. Introduction

Non-Gaussianity of primordial perturbations is one of the most interesting problems implied by the WMAP data [1, 2]. The observational limits on the nonlinearity parameter from WMAP seven-year data [2] are $-10 < f_{NL}^{local} < 74$ (95% CL), $-214 < f_{NL}^{equil} < 266$ (95% CL) and $-410 < f_{NL}^{orthog} < 6$ (95% CL). However, the standard simple inflation model predicts approximately Gaussian fluctuation, the deviation from Gaussian of which is very small. Several studies have attempted to achieve such large non-Gaussianity. Holman and Tolley [3] showed that if the effective action for the inflaton contains a higher-derivative interaction, which is derived, for example, from k-inflation [4] or DBI inflation [5], and the initial state of inflation is not the Bunch-Davies vacuum, then enhanced non-Gaussianity is derived in the "flattened" triangle configurations, the contribution of which is also discussed in [6]. In their paper, the initial state of the curvature perturbation in inflation was assumed not to be the Bunch-Davies vacuum, i.e., squeezed states, but they did not report a concrete value or the physical mechanism that generates the initial state in inflation, although the value of the coefficient of the initial state in inflation has a very important effect on the non-Gaussianity.

On the other hand, the effect of the initial condition in inflation on the power spectrum of curvature perturbations has been considered [7] and the effect of the length of inflation and pre-inflation physics on the power spectrum and the angular power spectrum of scalar and tensor perturbations has been examined by the present authors. [8-9]. The suppression of the spectrum at $l = 2$ as indicated by Wilkinson Microwave Anisotropy Probe (WMAP) data [1] may be explained to a certain extent by the finite length of inflation for an inflation of 50–60 *e*-folds [9]. Of course, there are many attempts [10] to derive this suppression. Based on the physical conditions before inflation, we have shown that the initial state of scalar perturbations in inflation is not simply the Bunch-Davies state, but rather a more general state (a squeezed state), where a scalar-matter-dominated period, a radiation-dominated period, or another inflation is considered as pre-inflation, and the general initial states in inflation were calculated analytically. In the present paper, we demonstrate a new property of the proposed inflation model. Using Holman and Talley's formula for the nonlinearity parameter $f_{NL}^{flattened}$, we calculate the value of $f_{NL}^{flattened}$ for the case in which the proposed finite inflation models have effective higher-derivative interactions, where slow-roll inflation is adopted as inflation and a scalar-matter-dominated period or power-law inflation period is adopted as pre-inflation. The obtained results are very interesting.

2. Scalar perturbations

We consider curvature perturbations in inflation and a scalar-matter-dominated epoch. The background spectrum considered is a spatially flat Friedman-Robertson-Walker (FRW) universe described by metric perturbations. The line element for the background and perturbations is generally expressed as [11]

$$\mathrm{d}s^2 = a^2(\eta)\{(1+2A)\mathrm{d}\eta^2 - 2\partial_i B \mathrm{d}x^i \mathrm{d}\eta - [(1-2\Psi)\delta_{ij} + 2\partial_i\partial_j E + h_{ij}]\mathrm{d}x^i \mathrm{d}x^j\}, \quad (2\text{-}1)$$

where η is the conformal time, the functions A, B, Ψ, and E represent the scalar perturbations, and h_{ij} represents tensor perturbations. The density perturbation in terms of the intrinsic curvature perturbation of comoving hypersurfaces is given by $\Re = -\Psi - (H/\dot{\phi})\delta\phi$, where ϕ is the inflaton field, $\delta\phi$ is the fluctuation of the inflaton field, H is the Hubble expansion parameter, and $\Re$ is the curvature perturbation. Overdots represent derivatives with respect to time t, and primes represent derivatives with respect to the conformal time η. Introducing the gauge-invariant potential $u \equiv a(\eta)(\delta\phi + (\dot{\phi}/H)\Psi)$ allows the action for scalar perturbations to be written as [12]

$$S=\frac{1}{2}\int \mathrm{d}\eta \mathrm{d}^3x\,\{(\frac{\partial u}{\partial \eta})^2-c_s^2\ (\nabla u)^2+\frac{Z''}{Z}u^2\ \}, \tag{2-2}$$

where c_s is the velocity of sound, and in inflation $Z=a\dot{\phi}/H$, and $u=-Z\mathfrak{R}$. The field $u(\eta,\boldsymbol{x})$ is expressed using annihilation and creation operators as follows:

$$u(\eta,\ \boldsymbol{x})\ =\frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3k\ \{\,u_k(\eta)\ \boldsymbol{a_k}+\ u_k\,{*}(\eta)\ \boldsymbol{a}_{-\boldsymbol{k}}{}^{\dagger}\ \}\ e^{-i\,\boldsymbol{kx}}\ . \tag{2-3}$$

The field equation for $u_k(\eta)$ is derived as

$$\frac{\mathrm{d}^2u_k}{\mathrm{d}\eta^2}+(c_s^2\,k^2-\frac{1}{Z}\frac{\mathrm{d}^2Z}{\mathrm{d}\eta^2})\ u_k=0\,, \tag{2-4}$$

where $c_s{}^2=1$ is assumed in inflation. The solution of u_k satisfies the normalization condition $u_k\mathrm{d}u_k^*/\mathrm{d}\eta-u_k^*\mathrm{d}u_k/\mathrm{d}\eta=i$.

First, slow-roll inflation is considered. The slow-roll parameters are defined as [13, 14]:

$$\varepsilon=3\frac{\dot{\phi}^2}{2}(\frac{\dot{\phi}^2}{2}+V)^{-1}=2M_P^2\left(\frac{H'(\phi)}{H(\phi)}\right)^2, \tag{2-5}$$

$$\delta=2M_P^2\frac{H''(\phi)}{H(\phi)}, \tag{2-6}$$

$$\xi=4M_P^4\frac{H'(\phi)H'''(\phi)}{(H(\phi))^2}. \tag{2-7}$$

The quantity $V(\phi)$ is the inflation potential, and M_P is the reduced Plank mass. Other slow-roll parameters $(\varepsilon_V,\eta_V,\xi_V)$ can be written in terms of the slow-roll parameters ε, δ, and ξ for first-order slow roll, i.e., $\varepsilon=\varepsilon_V$, $\delta=\eta_V-\varepsilon_V$, and $\xi=\xi_V-3\varepsilon_V\eta_V+3\varepsilon_V^2$, where $\varepsilon_V=M_P^2/2(V'/V)^2$, $\eta_V=M_P^2(V''/V)$, and $\xi_V=M_P^4(V'V'''/V^2)$. Using the slow-roll parameters, $(\mathrm{d}^2 Z/\mathrm{d}\eta^2)/Z$ is written exactly as

$$\frac{1}{Z}\frac{\mathrm{d}^2 Z}{\mathrm{d}\eta^2} = 2a^2 H^2\,(1+\varepsilon-\frac{3}{2}\delta+\varepsilon^2-2\varepsilon\delta+\frac{\delta^2}{2}+\frac{\xi}{2}), \tag{2-8}$$

and the scale factor is written as $a(\eta) = -((1-\varepsilon)\eta H)^{-1}$. Here, the slow-roll parameters are assumed to satisfy $\varepsilon < 1, \delta < 1$, and $\xi < 1$. As only the leading-order terms of ε and δ are adopted, ε and δ may be considered to be constant, allowing the scale factor to be written as $a(\eta) \approx (-\eta)^{-1-\varepsilon}$ [14]. Equation (2-4) can be rewritten as

$$\frac{\mathrm{d}^2 u_k}{\mathrm{d}\eta^2} + (k^2 - \frac{2+6\varepsilon-3\delta}{\eta^2})u_k = 0. \tag{2-9}$$

The solution to Eq. (2-9) is written as [13]

$$f_k^I(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu+1/2)\pi/2}\,(-\eta)^{1/2}\,H_\nu^{(1)}\,(-k\eta), \tag{2-10}$$

where $\nu = 3/2 + 2\varepsilon - \delta$, and $H_\nu^{(1)}$ is the Hankel function of the first kind of order ν. The mode functions $u_k(\eta)$ of a general initial state in inflation are written as

$$u_k(\eta) = c_1\; f_k^I(\eta) + c_2\; f_k^{I*}(\eta), \tag{2-11}$$

where the coefficients c_1 and c_2 obey the relation $|c_1|^2 - |c_2|^2 = 1$. The important point here is that the coefficients c_1 and c_2 do not change during inflation. In ordinary cases, the field $u_k(\eta)$ is considered to be in the Bunch-Davies state, i.e., $c_1 = 1$ and $c_2 = 0$, because as $\eta \to -\infty$, the field $u_k(\eta)$ must approach plane waves ($e^{-ik\eta}/\sqrt{2k}$). Second, in the case of power-law inflation, where $a(t) \propto t^q$, a similar method can be used, and the solution is obtained as Eq. (2-10) with $\nu = 3/2 + 1/(q-1)$. Third, the curvature perturbations in the scalar matter are calculated using a method similar to that used for inflation [12, 15, 7]. The field equation u_k

can be written in a form similar to Eq. (2-4) with a value of $c_s{}^2 = 1$ and with $Z \propto a_{\mathrm{P}}(\eta)[\,(\boldsymbol{H}^2 - \boldsymbol{H}')]^{1/2}/\boldsymbol{H}$, (where $\boldsymbol{H} = a_{\mathrm{P}}{}'/a_{\mathrm{P}}$). The solution to Eq. (2-4) is then written as $f_k^S(\eta) = (1 - i/(k\eta)\exp[-ik\eta])/\sqrt{2k}$.

3. Calculation of the nonlinearity parameter

Here, an inflation model is considered. Since we consider slow-roll inflation to have a finite length, we assume a pre-inflation period to be a scalar-matter-dominated period in which the scalar field is the inflaton field, or is power-law inflation, i.e., double inflation. A simple cosmological model is assumed, as defined by

Pre-inflation: $a_{\mathrm{P}}(\eta) = b_1(-\eta - \eta_j)^r$, (3-1)

Slow-roll Inflation: $a_{\mathrm{I}}(\eta) = b_2(-\eta)^{-1-\varepsilon}$, (3-2)

where

$$\eta_j = -(\frac{r}{1+\varepsilon} + 1)\eta_1,\ b_1 = (\frac{-1-\varepsilon}{r})^r(-\eta_1)^{-1-\varepsilon-r}b_2. \qquad (3\text{-}3)$$

The scale factor $a_{\mathrm{I}}(\eta)$ represents slow-roll inflation. Here, de-Sitter inflation ($\varepsilon = 0$) is not considered. Slow-roll inflation is assumed to begin at $\eta = \eta_1$. In pre-inflation, for the case of $r = 2$, the scale factor $a_{\mathrm{P}}(\eta)$ indicates that pre-inflation is a scalar-matter-dominated period, and, for the case of $r = -q/(q-1)$, the pre-inflation is power-law inflation, where the scale factor $a_{\mathrm{P}}(t) \propto t^q$.

Using above the pre-inflation model, the initial state of inflation given by Eq. (2-11) will be fixed as follows. The coefficients c_1 and c_2 are fixed using the matching condition in which the mode function and first η-derivative of the mode function are continuous at the transition time $\eta = \eta_1$. (η_1 is the time at which slow-roll inflation begins.) For simplicity pre-inflation states are assumed to be the Bunch-Davies vacuum. The coefficients c_1 and c_2 can be calculated analytically in the case of the scalar-matter-dominated period:

$$c_1 = -\frac{i}{8z^{3/2}}\sqrt{\frac{\pi}{2}}e^{i((-1+\delta-2\varepsilon)\pi/2-2z/(1+\varepsilon))}$$

$$\{2z(-1-2iz-\varepsilon)H^{(2)}_{5/2+2\varepsilon-\delta}(z)+(4z^2+(3-2\delta+3\varepsilon)(1+\varepsilon+2iz)H^{(2)}_{3/2+2\varepsilon-\delta}(z)\}, \quad (3\text{-}4)$$

$$c_2 = -\frac{i}{8z^{3/2}}\sqrt{\frac{\pi}{2}}e^{-i((-1+\delta-2\varepsilon)\pi/2+2z/(1+\varepsilon))}$$

$$\{2z(-1-2iz-\varepsilon)H^{(1)}_{5/2+2\varepsilon-\delta}(z)+(4z^2+(3-2\delta+3\varepsilon)(1+\varepsilon+2iz)H^{(1)}_{3/2+2\varepsilon-\delta}(z)\}, \quad (3\text{-}5)$$

and in the case of the double inflation model:

$$c_1 = -\frac{\pi}{4\sqrt{q(q-1)(1+\varepsilon)}}\{e^{i\pi(6+2/(q-1)-2\delta+4\varepsilon)/4}(-q\,z\,H^{(1)}_{5/2+1/(q-1)}(zz)\,H^{(2)}_{3/2+2\varepsilon-\delta}(z)-$$

$$H^{(1)}_{3/2+1/(q-1)}(zz)\,(-q\,z\,H^{(2)}_{5/2+2\varepsilon-\delta}(z)+(1+q(-2+\delta-4\varepsilon)+\varepsilon)\,H^{(2)}_{3/2+2\varepsilon-\delta}(z)))\}, \quad (3\text{-}6)$$

$$c_2 = -\frac{\pi}{4\sqrt{q(q-1)(1+\varepsilon)}}\{e^{i\pi(-\delta+q(1+\delta-2\varepsilon)+2\varepsilon)/2(q-1)}(-q\,z\,H^{(1)}_{5/2+1/(q-1)}(zz)\,H^{(1)}_{3/2+2\varepsilon-\delta}(z)-$$

$$H^{(1)}_{3/2+1/(q-1)}(zz)\,(-q\,z\,H^{(1)}_{5/2+2\varepsilon-\delta}(z)+(1+q(-2+\delta-4\varepsilon)+\varepsilon)\,H^{(1)}_{3/2+2\varepsilon-\delta}(z)))\}, \quad (3\text{-}7)$$

where $z = -k\eta_1$ and $zz = q\,z/((q-1)(1+\varepsilon))$. The initial states of inflation can be written in terms of the slow-roll parameters, the start time of slow-roll inflation η_1, and the double inflation parameter q. Here, three slow-roll inflation models are adopted: the new inflation model with the potential term given by $V(\phi)=\lambda^2 v^4(1-2(\phi/v)^p)$ ($p=3,4$, $v\approx M_P$), the chaotic inflation model with the potential term given by $V(\phi)=M^4/2(\phi/m)^a$ ($a=2,4,6$, $m\approx M_P$), and the hybrid model $V(\phi)=\alpha\{(v^2-\sigma^2)^2+m^2/2\phi^2+g^2\phi^2\sigma^4\}$

$\approx \alpha(v^4 + m^2/2\phi^2)$, ($v \approx 10^{-2} M_P$, $m \approx 2\times 10^{-5} M_P$) [16]. Using the normalization value from the WMAP five-year data, we obtain the values of the spectral index and the slow-roll parameters, such as

New inflation: $n_s = 0.935$, $\varepsilon = 1.027\times 10^{-9}$, $\delta = -0.03228$

Hybrid inflation: $n_s = 0.9816$, $\varepsilon = 0.00504$, $\delta = 0.000878$

Chaotic inflation model:

ϕ^2 model: $n_s = 0.967$, $\varepsilon = 0.00828$, $\delta = 0.000022$

ϕ^4 model: $n_s = 0.950$, $\varepsilon = 0.01655$, $\delta = 0.008298$

ϕ^6 model: $n_s = 0.9334$, $\varepsilon = 0.0248$, $\delta = 0.01657$.

Now, we calculate the values of the nonlinearity parameter $f_{NL}^{flattened}$. Holman and Tolley [3] showed that if the effective action for the inflaton contains the higher-derivative interaction [17] $\pounds = \sqrt{-g}\,\dfrac{\lambda}{8M^4}((\nabla\phi)^2)^2$, which is derived, for example, from k-inflation or DBI inflation, and the initial state of inflaton is not the Bunch-Davies vacuum, then the enhanced non-Gaussianity is derived as follows:

$$f_{NL}^{flattened} \approx \frac{\dot{\phi}^2}{M^4} |c_2| \left(\frac{k}{a(\eta_1)H}\right) = \frac{2\varepsilon M_P^2}{H^2 z^3} |c_2|, \qquad \text{(3-8)}$$

where M is the cutoff scale, which is the limit of effective theory, and we assume $M \approx k/a(\eta_1)$ where η_1 is the beginning time of slow-roll inflation, and $z = -k\eta_1$. The present treatment considers the effect of the length of inflation, where $z = 1$ indicates that inflation starts at the time when the present-day size perturbation $k = 0.002$ (1/Mpc) exceeds the Hubble radius in inflation (i.e., inflation of close to 60 e-folds). Using the values of the above parameters we can calculate the values of $|c_1|$, $|c_2|$, and $f_{NL}^{flattened}$ in terms of z $(=-k\eta_1)$. The values of $|c_2|$ change only slightly among the models, but vary with the value of *z*, as 0.0063 for $z=8$, 0.004 for $z=10$, and 0.001 for $z=20$, and $|c_1|\cong 1$. From all of the models except for the ϕ^4 model, similar values of $f_{NL}^{flattened}$ are calculated, i.e., $f_{NL}^{flattened}\approx 120$ at $z=8$, and $f_{NL}^{flattened}\approx 40$ at $z=10$. Details are shown in Table 1. With respect to the other values of z, larger values of $f_{NL}^{flattened}$ can be derived at smaller z ($z<8$), and small values of $f_{NL}^{flattened}$ can be derived at larger z ($z>20$). Based on the above results, the value of $f_{NL}^{flattened}$ appears to depend strongly on the value of z, which represents the length of inflation, and the difference of the values of $f_{NL}^{flattened}$ among our three slow-roll inflation models is not large. Since the z-dependence of $f_{NL}^{flattened}$ is very steep, any value of $f_{NL}^{flattened}$ can be derived at some point of z. We next consider the case of double inflation, the value of $f_{NL}^{flattened}$ is 100 at $3<z<4$ in the chaotic inflation, at $4<z<5$ in the case of new inflation, and at $z\approx 3$ in the case of hybrid inflation. With respect to the *q*-dependence ($a(t)\propto t^q$), the values of $f_{NL}^{flattened}$ are similar at very large *q* but change at $q\approx 100$. The details are shown in Tables 2-4.

4. Discussion

We have derived a new property of the proposed finite inflation model. The possibility of large non-Gaussianity is demonstrated. The proposed inflation model is a finite length inflation model with an effective higher derivative interaction, where slow-roll inflation is adopted as inflation and a scalar-matter-dominated period or power inflation is adopted as pre-inflation. Owing to the existence of pre-inflation, the initial state in inflation is not the Bunch-Davies state, but is instead a more general state. The coefficients c_1 and c_2 can be analytically calculated. Using Holman and Tolley's formula of the nonlinearity parameter $f_{NL}^{flattened}$, we calculated the value of $f_{NL}^{flattened}$. For the case in which the scalar-matter-dominated period is considered to be pre-inflation, large values of $f_{NL}^{flattened}$ ($f_{NL}^{flattened} \approx 100$) are obtained at $8 < z < 10$ in all the models considered herein, and similar results are derived for the case of double inflation at $3 < z < 4$. These ranges can be written as 60-63 e-folds. This length is similar to that obtained when the suppression of CMB angular power spectrum of $\ell = 2$ was derived using the inflation models described in previous papers [7], but such spectral suppression is not inconsistent when considering cosmic variance. On the experimental value of $f_{NL}^{flattened}$, the orthogonal shape (f_{NL}^{orthog}) is peaked both on equilateral-triangle configurations (f_{NL}^{equil}) and on flattened-triangle configurations ($f_{NL}^{flattened}$) [18], but we think we need further consideration to drive the constraint of $f_{NL}^{flattened}$ from the constraints of f_{NL}^{orthog} and f_{NL}^{equil}. Therefore, we do not show it here. We assume such a high-derivative interaction in order to obtain non-linearity and effective interactions for slow-roll interaction. This high-derivative interaction appears to influence the parameters of slow-roll inflation. In order to clarify this problem, we must investigate a concrete inflation model such as *k*-inflation or DBI inflation. In the future, we would like to apply the proposed method to other inflation models and investigate the dependence of the length of inflation on $f_{NL}^{flattened}$.

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Table 1 Values of $f_{NL}^{flattened}$ for the case of the matter-dominated period as pre-inflation

	New inflation		Hybrid	Chaotic inflation		
	p=3	p=4		ϕ^2	ϕ^4	ϕ^6
$z=8$	123.8	123.7	122.7	123.8	187.7	126.4
$z=10$	40.5	40.4	40.1	40.4	61.3	41.3
$z=20$	1.26	1.26	1.25	1.26	1.91	1.28

Table 2 Values of $f_{NL}^{flattened}$ in the hybrid inflation for double inflation

	$q=10^5$	$q=10^4$	$q=10^3$	$q=10^2$	q=10
z=3	108.5	109.1	115.3	190.6	1096.5
z=4	23.4	23.6	25.3	45.1	266.8
z=5	7.24	7.3	7.93	14.8	88.6

Table 3 Values of $f_{NL}^{flattened}$ for the new inflation case of $n=3$ and for the new inflation case of $n=4$ for double inflation

$n=3$

	$q=10^5$	$q=10^4$	$q=10^3$	$q=10^2$	q=10
z=4	254	254.1	256	275.1	478.5
z=5	81.8	81.9	82.5	89.1	157.7
z=6	32.5	32.6	32.8	35.6	63.6

$n=4$

	$q=10^5$	$q=10^4$	$q=10^3$	$q=10^2$	q=10
z=4	194.9	195.1	197	216.5	424.2
z=5	62.8	62.9	63.5	70.2	140.1
z=6	25	25	25.3	28	56.5

Table 4 Values of $f_{NL}^{flattened}$ for the Chaotic inflation case of ϕ^2, ϕ^4, and ϕ^6 for double inflation

ϕ^2 model

	$q=10^5$	$q=10^4$	$q=10^3$	$q=10^2$	q=10
z=3	227.6	228.2	234.7	306.9	1196.5
z=3.5	100.6	100.9	104.2	140.0	561.2
z=4	49.8	50.0	51.8	71.1	291.0

ϕ^4 model

	$q=10^5$	$q=10^4$	$q=10^3$	$q=10^2$	q=10
z=3	181.1	181.5	185.6	242.9	1130.3

z=3.5	76.1	76.4	78.6	108.1	530.0
z=4	36.1	36.2	37.5	53.9	274.4

ϕ^6 model

	$q=10^5$	$q=10^4$	$q=10^3$	$q=10^2$	q=10
z=3	165.5	165.5	165.4	191.2	1061.5
z=3.5	67.1	67.1	67.1	81.4	497.6
z=4	30.6	30.6	30.6	39.0	257.6