Large Field Inflation, Ignobly

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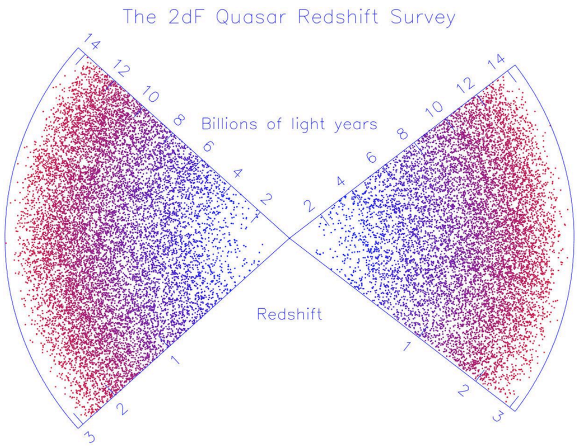
with A. Lawrence (Brandeis) & L. Sorbo (UMass),
to appear, and also
arXiv:0810.5346 (PRD), arXiv:0811.1989 (PRL)

Outline

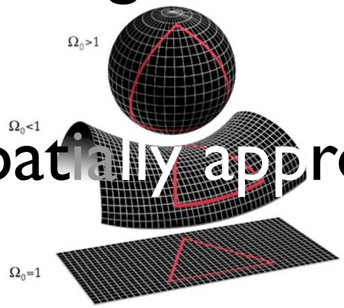
- Inflation: intro
- Interactions, radiative corrections & symmetries
- Field theory picture (in progress): 4-forms and inflation with (weakly) broken (local!) shift symmetry: *a (topological) Higgs effect*
- Summary

INTERESTING FACTS ABOUT THE UNIVERSE

- old and very large

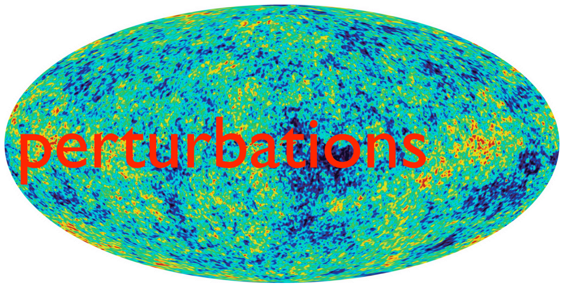


- homogeneous and isotropic

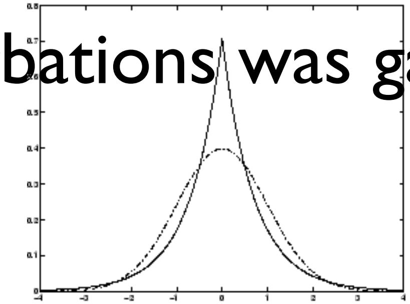


- spatially approximately flat

- structure seeded by small, scale invariant perturbations

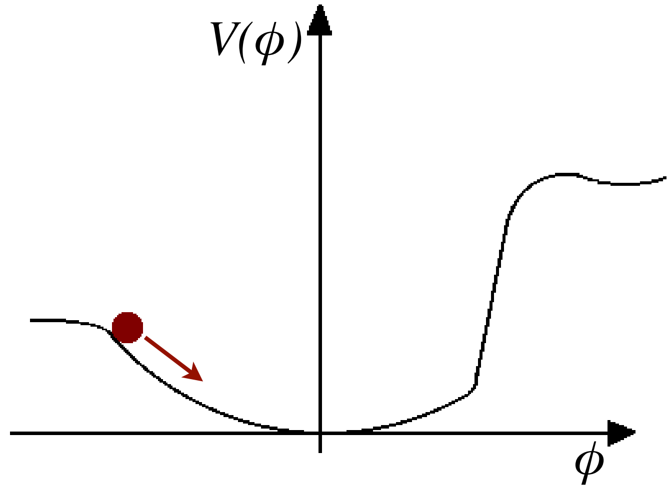


- spectrum of perturbations was gaussian



It all points to inflation...

But: what inflated?



\`Standard Models': very early Universe controlled by scalar field ϕ, whose potential $V(\phi)>0$ dominates over kinetic energy

to induce acceleration, $V(\phi)$ must be *flat* $\boxed{|V'(\phi)| \ll V(\phi)/M_P}$

to have long inflation, $V(\phi)$ must *stay* flat for long enough $\boxed{|V''(\phi)| \ll V(\phi)/M_P^2}$

Radiative corrections could deform the inflationary potential

Even if we write a theory with a classically flat potential for some scalar inflaton, this field cannot ignore the rest of the world: inflation must end, the universe must be repopulated: the field driving inflation MUST couple to other stuff!!!

Due to quantum corrections these couplings are NOT inert: they

1- affect the functional form of $V(\phi)$

2- affect the value of the parameters that appear in $V(\phi)$

But: do they really do it?...

Oftentimes NOT! We know several explicit examples:

1) Self-interacting scalars: no, even though the daisy diagrams look dangerous: they seem to yield corrections which individually look terrible, like

$$(-1)^n \lambda^n \phi^4 (\frac{\phi}{M})^{2n-4}$$

BUT: they **alternate** and resum to log corrections:

$$\lambda \phi^4 (1 + c \ln(\frac{\phi}{M}))$$

as in Coleman-Weinberg

2) Graviton loops: no, since they - as in induced gravity - yield finite potential and Planck mass renormalizations that go like

$$\Big(\frac{\partial_\phi^2 V}{M_{Pl}^2} + \frac{V}{M_{Pl}^4}\Big) V \qquad\qquad \partial_\phi^2 V R$$

which are small in the inflationary regime

Why? The answer is a (weakly broken) shift symmetry!

A *shift symmetry*: invariance under $\varphi \to \varphi + c$; exact s.s. implies $V(\phi)$=const; this is not inflation: it needs variable $V(\phi)$ to end; so $V'(\phi)$ breaks it, but radiative corrections are proportional only to the breaking terms, going as some derivatives of $V'(\phi)$. Thus if potential is flat to start with, it will stay flat even with the corrections included, if the worst breaking comes from $V'(\phi)$.

Does it mean, there is no problem at all? NO! But: the problem is no worse than the usual radiative mass instability of a scalar which couples by relevant or marginal operators to some heavy physics - just like the Higgs mass instability.

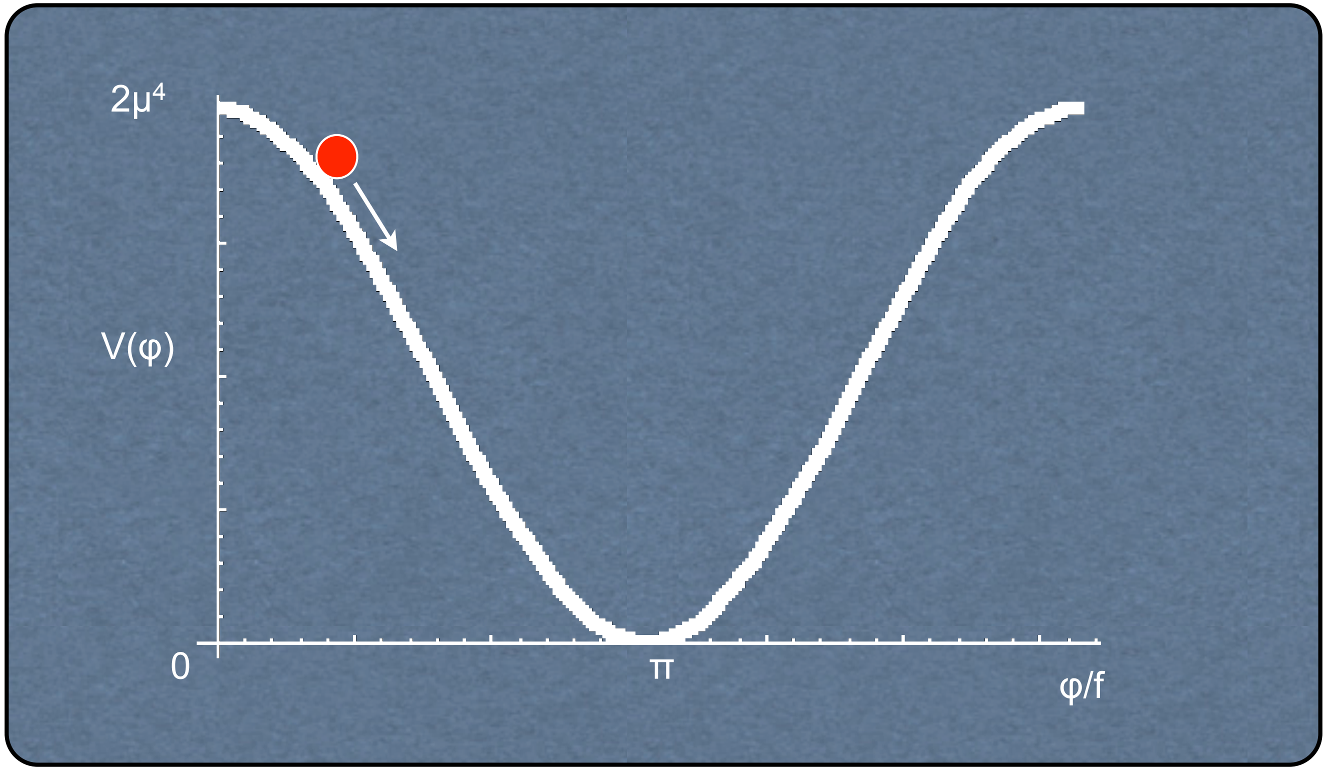
So the point is: can we generate the mass by evading strong shift symmetry breaking?

The danger is that a naive shift symmetry is a fake, when gravity is turned on: a continuous shift symmetry of a simple field theory tends to be broken in quantum gravity by nonperturbative effects.

An case in point: a pNGB as the inflaton

Idea: natural inflation

$$V(\varphi) = \mu^4\Big(\cos(\varphi/f) + 1\Big)$$



Adams, Bond, Freese, Friemann, Olinto 1990

To have inflation (ie get 60 or more efolds) we need

$$M_{Pl} \ll \phi \ll f$$

so, just take a very large pNGB decay constant f; easy in field theory...
However:

String Theory seems to require **f<M$_P$**

Banks, Dine, Fox and Gorbatov;
Adams, Arkani-Hamed, Motl, Vafa;

The field φ still needs to be large; this is **bad**, because higher harmonics in the nonperturbative potential win over the leading order term:

n-instanton actions contribute $\propto e^{-(n\, M_P/f)} \cos(n\, \phi/f)$ to pNGB potential

$$\Downarrow$$

subleading f/M_P harmonics in $V(\phi)$ get big and make it wiggly

A novel approach: use 4-forms!

$$\boxed{S_{4form} = -\ \frac{1}{48} \int F_{\mu\nu\varrho\lambda}\, F^{\mu\nu\varrho\lambda}\; d^4x}$$

$$\boxed{F_{\mu\nu\varrho\lambda} = \partial_{[\mu} A_{\nu\varrho\lambda]}}$$

tensor structure in 4d $\Rightarrow F_{\mu\nu\varrho\lambda} = q(x^\alpha)\, \varepsilon_{\mu\nu\varrho\lambda}$

equations of motion $D^\mu F_{\mu\nu\varrho\lambda} = 0 \Rightarrow q(x^\alpha)$ = constant

(this is why particle physicists used to ignore 4-forms:
trivial *LOCAL* dynamics)

Sources for the 4-form: membranes

$$S_{brane} \ni \frac{e}{6} \int d^3\xi \sqrt{\gamma}\, e^{abc}\, \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda A_{\mu\nu\lambda}$$

Enter the 4-form/pseudoscalar mixing...

$$S_{bulk} = \int d^4x\sqrt{g}\Big(\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma} + \ldots\Big)$$

Di Vecchia and Veneziano; Quevedo and Trugenberger; Dvali and Vilenkin; NK & Sorbo.

`Gibbons-Hawking' boundary terms:

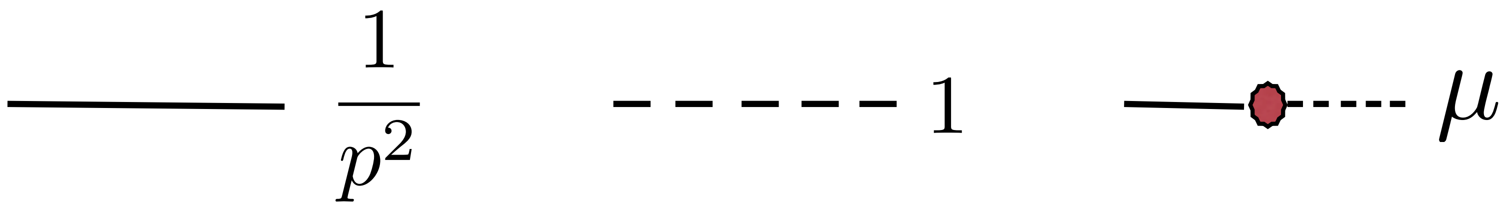
$$\int d^4x\sqrt{g}\ \tfrac{1}{6}\ \nabla_\mu\big(F^{\mu\nu\lambda\sigma}A_{\nu\lambda\sigma}\big) \ \ - \int d^4x\sqrt{g}\ \tfrac{1}{6}\ \nabla_\mu\Big(\mu\phi\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}A_{\nu\lambda\sigma}\Big)$$

Action invariant under (perturbative!) shift symmetry:

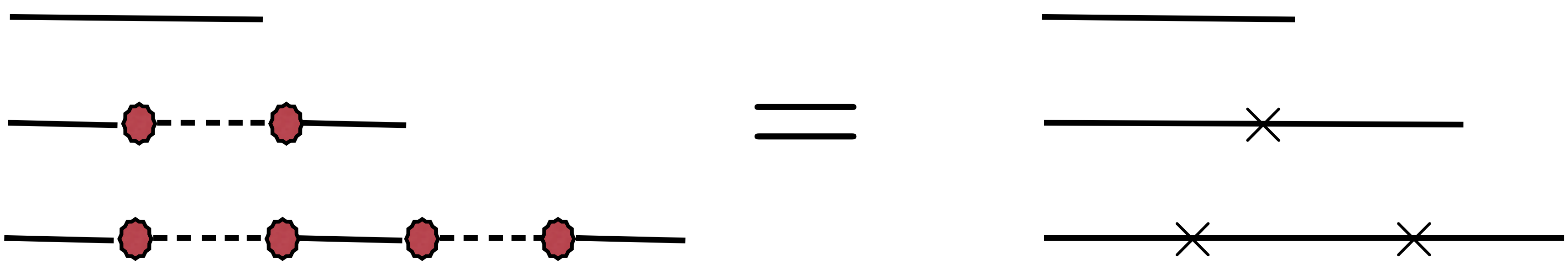
under $\phi \to \phi + c$, $\mathcal{L} \to \mathcal{L} + c\,\mu\,\varepsilon^{\mu\nu\varrho\lambda}F_{\mu\nu\varrho\lambda}/24$

A simple way to see the effect of mixing

Think of it as flavor oscillations: we have the scalar propagator, 4-form propagator and scalar-form vertex:



So we have a very simple sum of propagators



$$\frac{1}{p^2} + \frac{1}{p^2}\mu^2\frac{1}{p^2} + \frac{1}{p^2}\mu^2\frac{1}{p^2}\mu^2\frac{1}{p^2} + \ldots = \frac{1}{p^2 - \mu^2}$$

This is just a MASS TERM

Mass

- Therefore: we have a mass term!

$$\frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma}$$

- What is UNUSUAL: this RETAINS the shift symmetry

$$\phi \rightarrow \phi + \phi_0$$

- The lagrangian changes only by a total derivative:

$$\Delta\mathcal{L} = \frac{\mu\phi_0}{24}e^{\mu\nu\lambda\sigma}F_{\mu\nu\lambda\sigma}$$

- The symmetry is broken spontaneously after a solution is picked!

Making symmetry manifest

- First order formalism: enforce F = dA with a constraint

$$S_q = \int d^4x \, \frac{q}{24} \, \epsilon^{\mu\nu\lambda\sigma} \left(F_{\mu\nu\lambda\sigma} - 4\partial_\mu A_{\nu\lambda\sigma} \right)$$

NK, 1994

- Then change variables

$$\tilde{F}_{\mu\nu\lambda\sigma} = F_{\mu\nu\lambda\sigma} - \sqrt{g}\epsilon_{\mu\nu\lambda\sigma}(q + \mu\phi)$$

- This completes the square; integrate F out. What remains:

$$S_{eff} = \int d^4x \sqrt{g} \Big(\frac{M_{Pl}^2}{2} R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}(q + \mu\phi)^2 + \frac{1}{6} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} A_{\nu\lambda\sigma} \, \partial_\mu q \Big)$$

- The membrane term enforces jump on q (ie *F):

$$\Delta q|_{\vec{n}} = e$$

Mass & symmetries manifest!

- Mass term

$$V = \frac{1}{2}\left(q + \mu\phi\right)^2$$

- Shift symmetry

$$\phi \to \phi + \phi_0 \qquad q \to q - \phi_0/\mu$$

- Mass is radiatively stable; symmetry is broken spontaneously once background q is picked, as a boundary condition.
- Value of q can still change, by membrane emission

$$\Delta q|_{\vec{n}} = e$$

Note: the axion is effectively `gauging' the shift symmetry of the non-propagating field *q*; after SSB, this field `eats' the axion; *topological Higgs effect! This statement is precise, which can be seen by taking the shift function to satisfy a Klein-Gordon eq!*

Mass as charge

- 11D SUGRA (assume volume moduli stabilized as BP)

$$S_{11D\ forms} = M_{11}^9 \int *F \wedge F + M_{11}^9 \int A \wedge F \wedge F$$

- Truncate on $M_4 \times T^3 \times T^4$

$$A_{\mu\nu\lambda}(x^\mu) \qquad \phi = A_{abc}(x^\mu) \qquad A_{ijk}(y^i)$$

- This yields QUANTIZED MASS!

$$S_{4D forms} = -\int d^4x\sqrt{g}\Big(\frac{1}{2}(\partial\phi)^2 + \frac{1}{48}\sum_a (F^a_{\mu\nu\lambda\sigma})^2 + \frac{\mu\phi}{24}\frac{e^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma}\Big)$$

$$\mu = n\mu_0 \qquad \mu_0 = 2\pi V_3 M_{11}^3\left(\frac{M_{11}}{M_{Pl}}\right)^2 M_{11}$$

Quantization

- Classically q is continuous
- Quantum consistency requires that it be QUANTIZED! (Bousso, Polchinski)
- Example: 11D SUGRA

$$e_3 \int F_{\mu_1 \dots \mu_4} = 2\pi n \qquad e_6 \int {}^* F_{\mu_1 \dots \mu_7} = 2\pi n$$

- After compactification:

$$q_i = n_i \frac{2\pi M_{11}^3}{\sqrt{Z_i}} \qquad Z_e = \frac{M_{Pl}^2}{2} \qquad Z_m = \frac{M_{Pl}^2}{2 M_{11}^3 V_3^2}$$

Naturalness issues and shift breaking

So far we treated shift symmetry as a perturbative continuous symmetry, and that keeps radiative corrections under control.

But: in UV completions of the theory the axion is a zero mode of a form on a compact space. Its shift symmetry is a *large* gauge transformation of the field ϕ. It is broken to a discrete symmetry by the periodicity of ϕ (set by boundary conditions reflecting the compactness of the internal space, and given by the period $f_\phi < M_{Pl}$).

This is still OK with the ϕ F mixing, that remans invariant.

It still helps with non-perturbative corrections, since it implies that they will be periodic in f with the same period f_ϕ.

Breaking of discrete shift symmetry = monodromy

Silverstein and Westphal 2008

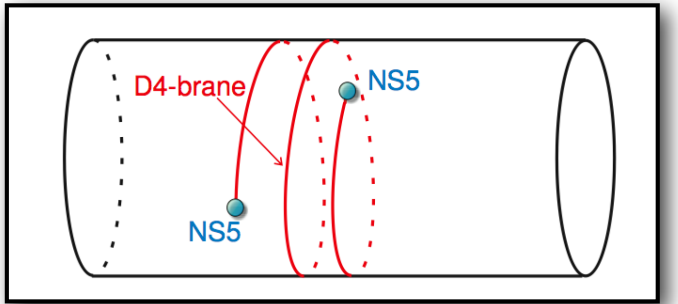
Effective potential $V(\phi) \sim (q+\mu\phi)^2$

with q, μ quantized: discrete invariance

$q \to q+n\,e, \;\; \phi \to \phi - n\,e/\mu$ Beasley and Witten 2002

at the level of action ϕ is still an angle!

Once a vev for q is chosen, the angle unwraps:



Similar ideas: Berg, Pajer and Sjors, 2009

Instanton contributions to *V*

$$V_{inst} = \lambda_{d,inst}^4 \sum_n e^{-(n-1)c'S} \cos(\frac{n\phi}{f_\phi}) \simeq \lambda_{d,inst}^4 \sum_n \Big(\frac{\lambda_{d,inst}}{\lambda_{uv,inst}}\Big)^{4(n-1)} \cos(\frac{n\phi}{f_\phi}) \ll \frac{\mu^2\phi^2}{2}$$

- Crucial for the `naturalness' of the mechanism:
  - Mass dominated by the random 4-form fluxes. In the weak coupling, the instanton potential *~cos(*ϕ*/f)* coming from a gauge theory into which the axion reheats is not needed for the mass generation. With strong couplings the potential is resummed to a polynomial (Witten, LGT), but is ***bounded***!
  - The instanton contribution must be smaller than the 4-form one!
- Pick a ϕ so it won't couple to a theory that goes strong at too high a scale; then the instantons yield small (and potentially interesting) bumps… like in chain inflation, or in multiple inflation. This can also lead to nongaussianities through coherent amplifications (Chen, Easther & Lim; parallel talk by Flauger).
- Similar suppression for gravitational instantons, with $f << M_P$

Higher order shift-symmetry invariant terms

Gauge (and shift symmetry) invariant terms:

$$S_{\phi F} = \int d^{4+d}x\sqrt{g}\Big(-F^2 + (\partial\phi)^2 + \mu\phi\epsilon F + \ldots + \frac{F^{2n}}{M^{(4+d)(n-1)}} + \ldots\Big)$$

After dimensional reduction and stabilization of moduli:

$$S_{\phi F} = \int d^4x\sqrt{g}\Big(-V_d F^2 + V_d(\partial\phi)^2 + \mu V_d\phi\epsilon F + \ldots + V_d\frac{F^{2n}}{M^{(4+d)(n-1)}} + \ldots\Big)$$
$$= \int d^4x\sqrt{g}\Big(-\tilde{F}^2 + (\partial\tilde{\phi})^2 + \mu\tilde{\phi}\epsilon\tilde{F} + \ldots + \frac{\tilde{F}^{2n}}{(V_d M^{4+d})^{n-1}} + \ldots\Big)$$

So the higher corrections are suppressed by $V_d M^{4+d} = \Lambda_F^4 \sim M_{Pl}^4$

$$V_{eff} = \frac{\mu^2\phi^2}{2}\Big(1 + \sum c_n \frac{\mu^2\phi^2}{\Lambda_F^4}\Big)!!!$$

Moduli corrections

Gauge (and shift symmetry) invariant terms, after dim red:

$$\delta\mathcal{L} = c\frac{\Psi}{M_{Pl}}F^2 + \ldots$$

Stabilizing the moduli with mass M, this shifts them by:

$$\delta\Psi \sim c\frac{F^2}{M^2 M_{Pl}} \sim c\frac{\mu^2\phi^2}{M^2 M_{Pl}}$$

This corrects the scalar potential by $M^2\delta\Psi^2$

$$V_{corr} = V(\phi)\left(1 + \frac{c^2}{2}\frac{V(\phi)}{M^2 M_{Pl}^2}\right) \simeq V(\phi)\left(1 + \frac{c^2}{2}\frac{H^2}{M^2}\right)$$

OK as long as $M > H$

Reheating

Consider the coupling

$$\delta\mathcal{L} = \frac{\phi}{f_\phi} Tr\left(G^* G\right)$$

Reheating temperature: $T_R \simeq \sqrt{\Gamma_{\phi GG} M_{Pl}}$

Decay rate: $\Gamma_{\phi GG} \simeq \frac{\mu^3}{f_\phi^2}$

Thus:

$$T_R \simeq \frac{\sqrt{\mu M_{Pl}}}{f_\phi}\mu \quad \text{this is easily} \quad \gg MeV$$

So when $f_\phi > \sqrt{\mu M_{Pl}}$ reheating temperature will be lower than the inflaton mass, and that will generically help avoid reproduction of undesired long lived relics.

Cosmological numerology

Note that for $M_{11}^3\, V_3 \sim O(1)$: $\quad \boldsymbol{\mu \propto M_{str}\, (M_{str}/M_P)^2}$

If M_{str} ~ GUT scale, and $n \sim O(1)$ then

$$\mu \sim 10^{13}\ GeV$$

as required by COBE normalization

PGW and the Lyth bound

r related to the inflaton displacement during inflation

(in single-field inflation)

$$\frac{\Delta\phi}{M_P} \sim \int H\,dt\sqrt{r/8}$$

and using $H\,\Delta t \sim 60$,

$$\boldsymbol{\Delta\phi \sim M_P\,(r/0.01)^{1/2}}$$

observable tensor modes typically related
to a planckian excursion of inflaton

Signatures

- For fixed mass and 4-form charge, predictions are identical to chaotic inflation (including gravitational waves!)
- However: emission of membranes can change q (and give a kick to ϕ) during inflation
- Emission of membranes can also change μ during inflation, producing breaks in the spectrum of perturbations
- The instanton corrections on top of the quadratic potential can also affect fluctuations, producing bumps - corrections to scale invariance and non-gaussianities (Flauger, parallel talk)

Summary

- Naturalness of inflaton potentials very nontrivial - but NOT impossible! One needs to formulate it carefully to see where the problems come from
- Shift symmetries: a key for constructing inflationary models
- String theory contains many 4-forms fields (used to generate the landscape of cosmological constants)
- We can use four forms to obtain radiatively stable, massive pseudoscalars with a "landscape" of masses and vevs thanks to only weak breaking of shift symmetry
- *Full stringy construction (as a way of proving the viability of UV complete chaotic inflation models)?*