# Inflationary Infrared Divergences: Geometry of the Reheating Surface vs. $\delta N$ Formalism

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Mischa Gerstenlauer
with C. Byrnes, A. Hebecker, S. Nurmi, G. Tasinato

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# $\delta N$ Formalism

( Lyth, Rodriguez, Starobinsky, Sasaki, Steward, Wands, ...)

- *primordial curvature perturbation* $\zeta$ defined as fractional perturbation $\frac{\delta a}{a}$ ($a$: scale factor)
- equivalently: $\zeta$ is perturbation in number of e-foldings $N(\vec{x}, t)$

$$\begin{aligned}\zeta(\vec{x}) &= \delta N(\,\phi(\vec{x})\,) \\ &= N(\,\phi_0 + \delta\phi(\vec{x})\,) - N(\,\phi_0\,) \\ &= N'\,\delta\phi + \frac{1}{2!}N''\,\delta\phi^2 + \dots \qquad (\dots)' = \frac{d}{d\phi}\dots\end{aligned}$$

- in Fourier space ( $*$ : convolution):

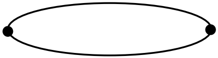
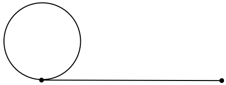
$$\delta\phi^2 \to (\delta\phi * \delta\phi)_k \qquad \delta\phi^3 \to (\delta\phi * \delta\phi * \delta\phi)_k \qquad \dots$$

# $\delta N$ Formalism - IR Divergences

(prefactors $N', N'', \ldots$ omitted)

$$\langle \zeta_k \zeta_p \rangle \sim \langle \delta\phi_k \delta\phi_p \rangle + \langle (\delta\phi*\delta\phi)_k (\delta\phi*\delta\phi)_p \rangle + \langle (\delta\phi*\delta\phi*\delta\phi)_k \delta\phi_p \rangle$$

With loops

- $\langle (\delta\phi * \delta\phi)_k (\delta\phi * \delta\phi)_p \rangle \cong$ 
- $\langle (\delta\phi * \delta\phi * \delta\phi)_k \delta\phi_p \rangle \cong$ 
- both loops IR-divergent $\sim \ln(kL)$ ($L$: IR cut-off)

# Background fluctuations in $\delta N$

- Mode $k$ leaves horizon at time $t_k$
- Locally (on scale $\sim 1/k$), modes $p \ll k$ shift homogeneous scalar field value away from classical trajectory $\phi_0$ by

$$\delta\bar{\phi} \sim \int_{p \ll k} d^3p \;\; \delta\phi_{\vec{p}}$$

Quantities at horizon exit of mode $k$ should be evaluated at $\phi_0 + \delta\bar{\phi}$, e.g. Hubble parameter

$$H \to H(\phi_0 + \delta\bar{\phi}) \sim H(\phi_0) + H' \,\delta\bar{\phi}$$

# Example: Power Spectrum $\mathcal{P}_\zeta$

$$\langle \zeta_k \zeta_p \rangle \sim \frac{\delta(\vec{k}+\vec{p})}{k^3} \mathcal{P}_\zeta(k)$$

## Result: Simple derivative structure

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta^{(0)} \left\{ 1 + \underbrace{\frac{\langle (\delta\bar{\phi})^2 \rangle}{2} \frac{1}{\mathcal{P}_\zeta^{(0)}} \frac{d^2 \mathcal{P}_\zeta^{(0)}}{d\phi^2}}_{\text{loops + effect of } \delta\bar{\phi}} \right\}$$

- $\delta\bar{\phi}$ corrections and (standard) loops are of the same order!
- $\mathcal{P}_\zeta^{(0)} \sim (N'H)^2$ : tree-level
- $\langle (\delta\bar{\phi})^2 \rangle \sim \ln(kL)$

# Background $\bar{\zeta}$ vs. Background $\delta\bar{\phi}$

- Analogously

$$\bar{\zeta}(t_k) \sim \int_{p \ll k} d^3p \;\; \zeta_{\vec{p}}$$

- $\bar{\zeta}$ changes geometry
  $\rightarrow$ *physical* wave number $\qquad k_{phys}^2 = k^2 \, e^{-2\bar{\zeta}}$

## Result

$$\mathcal{P}_\zeta(k) = \langle \, \mathcal{P}_\zeta^{(0)}(k_{phys}) \, \rangle_{\bar{\zeta}}$$

$$= \mathcal{P}_\zeta^{(0)} - \langle \, \bar{\zeta} \, \rangle \, \frac{d\mathcal{P}_\zeta^{(0)}}{d \ln k} + \frac{\langle \, \bar{\zeta}^2 \, \rangle}{2} \, \frac{d^2\mathcal{P}_\zeta^{(0)}}{d(\ln k)^2}$$

!!! Only in agreement with $\delta N$ result when including our modification !!!

( see also Giddings & Sloth ’10 )

# More comments on this result

$$\mathcal{P}_\zeta(k) = \langle\, \mathcal{P}_\zeta^{(0)}(k_{phys})\,\rangle_{\bar\zeta}$$
$$= \mathcal{P}_\zeta^{(0)} - \langle\, \bar\zeta\,\rangle \, \frac{d\mathcal{P}_\zeta^{(0)}}{d\ln k} + \frac{\langle\, \bar\zeta^2\,\rangle}{2} \, \frac{d^2\mathcal{P}_\zeta^{(0)}}{d(\ln k)^2}$$

- divergences occur due to misidentification of *physical* wavelength on large scales
- using $k_{phys}$ instead of $k$ : spectrum $\mathcal{P}_\zeta$ is (IR) divergent-free (see also Urakawa & Tanaka ’10)

# Including tensors

Analogously ( $\bar{\gamma}$ : tensor background) :

- $$k_{phys}^2 = k_i \, e^{-2\bar{\zeta}} \left(e^{-\bar{\gamma}}\right)_{ij} \; k_j$$
- $$\ln k_{phys} = \ln k - \bar{\zeta} + (\ldots)\, \bar{\gamma} + (\ldots)\, \bar{\gamma}^2 + \mathcal{O}(\bar{\gamma}^3)$$

Result (setting $\langle \bar{\zeta} \rangle = 0 = \langle \bar{\gamma} \rangle$) :

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta^{(0)} \; \left\{ 1 + \langle \bar{\gamma}^2 \rangle \, \frac{1}{\mathcal{P}_\zeta^{(0)}} \, \frac{d\mathcal{P}_\zeta^{(0)}}{d \ln k} + \frac{\langle \bar{\zeta}^2 \rangle}{2} \, \frac{1}{\mathcal{P}_\zeta^{(0)}} \, \frac{d^2\mathcal{P}_\zeta^{(0)}}{d\phi^2} \right\}$$

# Summary

1. We included effects of long-wavelength (background) modes in $\delta N$ Formalism.

2. We found: loops + effect of background modes allow for simple derivative structure.

3. Geometrical interpretation of IR divergences: IR log-divergences occur due to misidentification of *physical* wave-length.

4. Tensors are included easily in the formalism.

Open issues: multiple fields, ...