Implications of genuine gauge-invariant perturbations

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in collaboration with Takahiro Tanaka (YITP)

Y.U. and T. Tanaka 0902.3209[hep-th], P.T.P.122:779-803,2009

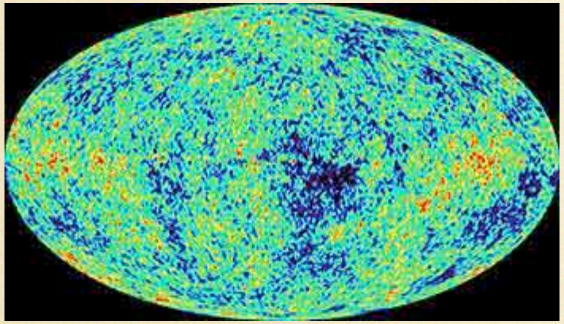
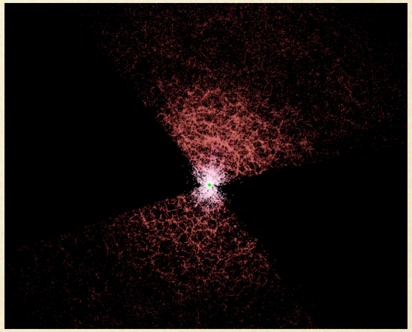
Y.U. and T. Tanaka 1007.0468[hep-th],

Y.U. and T. Tanaka 1009.2947[hep-th],

Importance of gauge-invariance

- Comparison to observations

Precise measurements of cosmological fluctuations

→ Probe of early universe ⊃ models of inflation

Origin of large scale structures

Observed fluctuations: Gauge-invariant

- Solution to IR divergence problem

Wait for the later discussion

Gauge-inv. in non-linear theory

- Notes on the preservation of gauge invariance
- How to calculate the genuine gauge-invariant quantities

Single field inflation

$$S_\phi = -\frac{1}{2}\int \sqrt{-g}\,\left[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)\right]\mathrm{d}^4x$$

ADM metric

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)$$

Comoving gauge

Maldacena (2002)

$$\delta\phi = 0 \qquad h_{ij} = e^{2(\rho+\zeta)}\left[e^{\delta\gamma}\right]_{ij}$$

e^ρ: scale factor

$$\delta\gamma_{ii} = 0 \qquad \partial_j\delta\gamma_{ij} = 0$$

Gauge-invariant perturbation

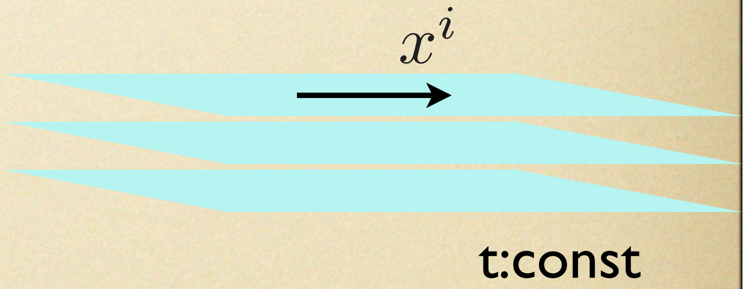
Gauge-invariant perturbation ⟷ Complete gauge fixing

- **Gauge-fixing condition**

  Gauge transformation $(t,\, x^i) \to (t + \delta t,\, x^i + \delta x^i)$

  - Time coordinate

    $\delta\phi = 0$ Fixed



  - Spatial coordinates

    $\delta\gamma_{ii} = 0 \quad \partial_j \delta\gamma_{ij} = 0$

    → $\partial^2 \delta x^i = \ldots$ Poisson eq.

DOFs in boundary conditions

Gauge DOFs & IR divergence

- Complete gauge fixing

Y.U.&T.Tanaka(09,10)

Gauge fixing at each point

+ Boundary conditions $\partial^2 \delta x^i = \ldots$

Need to fix

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Need to fix

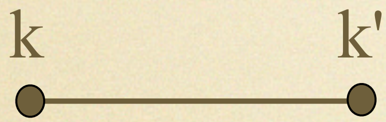
- Solve IR divergence problem

IR divergence problem

- **Correlation fn.** $\langle \zeta_k \, \zeta_{k'} \rangle$

$\mathcal{L}_{\rm int} \propto \zeta^4$ ζ: Curvature perturbation

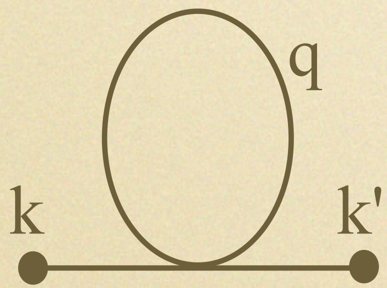
- Leading order



$$\langle \zeta_k \, \zeta_{k'} \rangle = |\zeta_k|^2 \propto k^{-3}$$

Scale-invariant

- Next to leading order



Momentum (Loop)integral

$$\int d^3q \, |\zeta_q|^2 = \int d^3q/q^3$$

Logarithmic divergence

IR divergence problem

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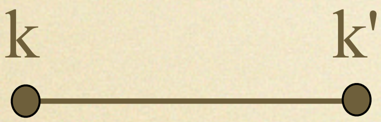
■ Leading order



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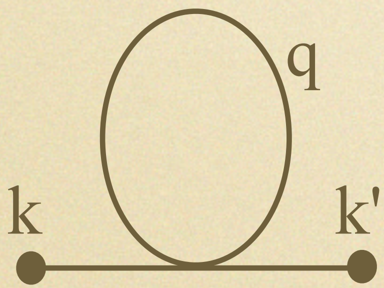

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Logarithmic divergence

Break down of perturbation theory??

Gauge DOFs & IR divergence

- Complete gauge fixing

Y.U.&T.Tanaka(09,10)

Gauge fixing at each point

+ Boundary conditions $\partial^2 \delta x^i = \ldots$

Need to fix

- Solution to IR divergence problem Y.U.&T.Tanaka(09)

Fix boundary conditions to remove gauge modes

→ Loop corrections no longer diverge

Presence of non-local gauge DOFs

$\rightarrow$ Influence on fluctuations at large scales

$\rightarrow$ Origin of IR divergence

Gauge DOFs & IR divergence

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Y.U.&T.Tanaka(09,10)

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Y.U.&T.Tanaka(09)

Fix boundary conditions to remove gauge modes (a part of ... a part of)

→ Loop corrections no longer diverge

Non-local gauge DOFs
→ Influence on fluctuations at large scales
→ Origin of IR divergence

Genuine gauge-inv. quantities

Complete gauge fixing → Technically difficult

Genuine gauge-inv. quantities

~~Complete gauge fixing~~ → Technically difficult

- Genuine gauge-invariant quantities

Spatial coordinates $x^i \to \tilde{x}^i = x^i + \delta x^i$

${}^sR$: 3D scalar curvature ${}^sR \propto \partial^2 \zeta$ Y.U.&T.Tanaka(10)

Scalar quantity ${}^sR(x) \to {}^sR(x) - \delta x^i \partial_i {}^sR(x)$

Due to change of the argument

Genuine gauge-inv. quantities

~~Complete gauge fixing~~ → Technically difficult

- Genuine gauge-invariant quantities

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Due to change of the argument

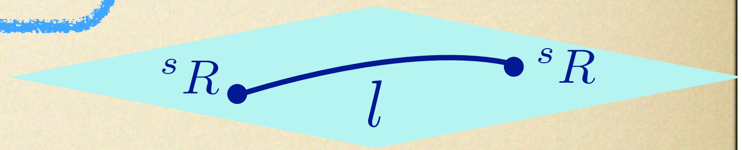
Not appear if we specify the arguments of ${}^sR$ by gauge-invariant quantity

Genuine gauge-inv. quantities 2

1. Geodesic distance

$$\langle {}^sR\,{}^sR\rangle(l),\ \langle {}^sR\,{}^sR\,{}^sR\rangle(l_1,\, l_2), \ldots$$

l_m : Geodesic distance



t:const

c.f. C. Byrnes et al. (2010)

⟶ Talk by G. Mischa

2. Gauge-invariant initial state

Quantization { Physical DOFs / Gauge DOFs

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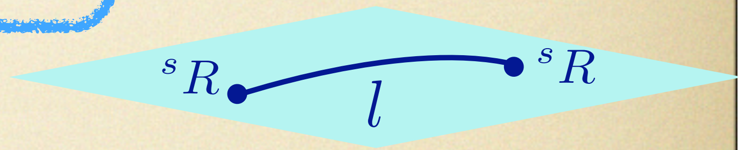

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2. Gauge-invariant initial state

Quantization { Physical DOFs / ~~Gauge DOFs~~

Initial state $|\Psi\rangle$: Need to restrict to the physical state

Gauge-invariant initial state

- Initial condition in interaction picture

ζ_H : Heisenberg picture field

Y.U.&T.Tanaka(10)

ζ_I : Interaction picture field

1. $\zeta_H(t_i) = \zeta_H[\zeta_I(t_i)]$
2. Positive frequency fn. for ζ_I

- One-loop corrections

Total derivatives

$$\langle {}^s\!R\, {}^s\!R \rangle(l) \simeq \int \mathrm{d}(\log k)\partial_{\log k}(\ldots)$$

+ (Divergent terms) + (Regular terms)

Gauge-invariant initial state

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■ One-loop corrections

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$$\langle {}^sR\,{}^sR\rangle(l) \simeq \int \mathrm{d}(\log k)\partial_{\log k}(\ldots)$$

+ (Divergent terms) + (Regular terms)

=0

Choose initial conditions 1&2

Necessary condition for gauge-invariance

Gauge-invariant initial state 2

- Relation between $\zeta_H(t_i)$ and $\zeta_I(t_i)$

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1. $\zeta_H(t_i) = \zeta_H[\zeta_I(t_i)]$

Heisenberg eq. $\mathcal{L}\zeta_H = \mathcal{S}[\zeta_H]$ $\mathcal{L}$: Derivative op.

$$\zeta_H = \Sigma_i a_i F[\zeta_I] + \mathcal{L}^{-1}\mathcal{S}$$

homogeneous solution $\mathcal{L}F[\zeta_I] = 0$

Conditions on a_i → (C1)

2. Mode function for ζ_I

ρ:e-folding

$$(1+\varepsilon)\,\partial_\rho\zeta_k - x^i\partial_i\zeta_k + \varepsilon\zeta_k + \ldots = -\left(\partial_{\log k} + 3/2\right)\zeta_k$$

(C2)

Remarks

- Slow-roll approximation
  - Leading order $\mathcal{O}(\varepsilon^0)$

    Bunch-Davies vacuum (C1), (C2) OK!
  - Higher orders

    Adiabatic vacuum & $\zeta_H(t_i) = \zeta_I(t_i)$

    → (C1), (C2) are not satisfied

- Canonical commutation relations

  Choosing the appropriate a_i

  ➡ Commutation relations can be compatible with the gauge-invariance condition

Conclusion

- We pointed out the presence of non-local gauge DOFs, that are left in the conventional perturbation theory.

  → One of the origins of IR divergence

  * Multi-fields models → Other origin.

- We proposed one calculable example of genuine gauge-invariant quantities.

- In the global gauge, we need to impose the gauge-invariance on initial condition.

  Leading Scale-invariant/Bunch-Davies vacuum

  Higher order Necessary to satisfy (C1)&(C2)

  → Implications for the observable fluctuations?

Y.U.&T.Tanaka (in preparation)

Supplement

IR divergence is physical?

- $\mathcal{YES}$, its implications are...
  - Enhanced loop corrections $N^p = (\ln a)^p,\ \ln(kL)$
    - → New probe to models of inflation

      M. Sloth (07), D.Seery (07),....
  - Screening of the cosmological constant.

    Tsamis&Woodard (96), Polyakov (09), Kitazawa&Kitamoto (10),....
- $\mathcal{NO}$, because
  - Non-perturbative effect generates the effective mass.

    Riotto&Sloth (08), C.Burges et al. (09,10)
  - Decoherence suppresses IR corrections.

    Enqvist et al. (08), Y.U.&T.Tanaka (09)
  - In single-field model, IR divergence is originating from the gauge artifact. Y.U.&T.Tanaka (09,10)

Residual gauge modes

Lapse function δN Shift vector $N_i = e^{\rho}\check{N}_i$

- General solution of 1st order constraint eqs.

$$\delta N_1(x) = \frac{1}{\rho'}\left(\zeta_1'(x) - \frac{1}{4}\partial^i G_{i,1}(x)\right)$$

$$\partial^2 G_{i,1}(x) = 0$$

$$\check{N}_{i,1}(x) = \partial_i\left(\frac{\phi'^{\,2}}{2\rho'^{\,2}}\partial^{-2}\zeta_1'(x) - \frac{1}{\rho'}\zeta_1(x)\right) - \frac{1}{4}\left(1 + \frac{\phi'^{\,2}}{2\rho'^{\,2}}\right)\partial_i\partial^{-2}\partial^j G_{j,1}(x) + G_{i,1}(x)$$

$$(\delta N, \check{N}_i) \quad G_i = 0 \longrightarrow G_i \neq 0$$

$$x^i \to x^i + \delta x^i \qquad \delta x^i = f(\eta)x^i + \Delta x^i(x)$$

Functionals of $G_{i,1}$

Residual gauge modes 2

Comoving gauge $\delta\phi = 0$ $\quad h_{ij} = e^{2(\rho+\zeta)}\left[e^{\delta\gamma}\right]_{ij}$

$\delta\gamma_{ii} = 0 \quad \partial_j \delta\gamma_{ij} = 0$

$$\delta x_i(x) = -\int \mathrm{d}\eta\, G_i(x) + \frac{1}{4}\int \mathrm{d}\eta \partial_i \partial^{-2}\partial^j G_j(x) + \frac{1}{4}\int \frac{\mathrm{d}\eta}{\rho'}\int \mathrm{d}\eta\, \partial_i \partial^j G_j(x) + H_i(\boldsymbol{x}) + \int \frac{\mathrm{d}\eta}{\rho'}\,\partial^2 H_i(\boldsymbol{x})\,,$$

Vector fns. $\partial^2 G_i(x) = 0$

$$3\partial^2 H_i(\boldsymbol{x}) + \partial_i \partial^j H_j(\boldsymbol{x}) = 0$$

Residual gauge modes 3

- Gauge transformation: $x^i \to x^i + \delta x^i$

$$\delta x^i = f(\eta)x^i + \Delta x^i(x)$$

$$3\partial^2 \Delta x^i(x) + \partial_i \partial^j \Delta x_j = 0$$

(i) Time-dependent fn. $f_1(\eta)$

Scale transformation

$$\zeta_1(x) \to \tilde{\zeta}_1(x) = \zeta_1(x) - f_1(\eta)$$

(ii) Transverse traceless mode Δx_i

$$\delta\gamma_{ij,1}(x) \to \delta\gamma_{ij.1}(x) + \partial_j \Delta x_i + \partial_i \Delta x_j - \frac{2}{3}\partial^k \Delta x_k \delta_{ij}$$

* Change the integral region of ∂^{-2} in $N_{i,1}$