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Primordial Cosmology

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OXFORD GRADUATE TEXTS



IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Speakers include  
S. Asai, F. Bernardreau, E. Brandenberger, W. Buchmüller, P. Chen, L. Covi, M. Dine(\*), J. Dunkley, K. Hamaguchi, M. Hazumi, P. Horava, N. Kaloper, A. E. Kim, E. Komatsu, K. Kohno, B. Karofyllis, S. Miyazaki, M. Murphy, S. Profumo, L. Roszkowski, E. Roulet, I. Sautera, P. Siklos, E. Silverstein, A. Starobinsky, G. Steigman(\*), E. Stewart, Y. Suzuki, T. Takahashi, K. Umetsu, E. Verlinde(\*), Z. Z. Xing (\*) to be confirmed

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**Standard paradigm: single-field inflation + (p)reheating**  
 **single fluid dominated FLRW Universe**

FLRW metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - \mathcal{K}r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

Matter component: perfect fluid

$$T_{\mu\nu} = pg_{\mu\nu} + (\rho + p)u_\mu u_\nu$$

$$p = \omega\rho \begin{cases} \omega = \frac{1}{3} & \text{radiation} \\ \omega = 0 & \text{dust} \\ \omega = -1 & \text{cosmological constant} \end{cases}$$

+ cosmological constant = Einstein equation

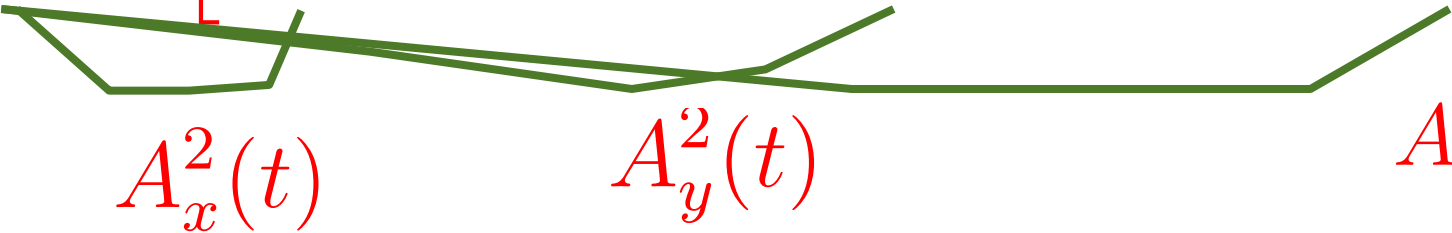
$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{1}{3} (8\pi G_N \rho + \Lambda)$$

$$\frac{\ddot{a}}{a} = \frac{1}{3} [\Lambda - 4\pi G_N (\rho + p)]$$

# More complicated models: hybrid (GUT), vector modes, ...

 **anisotropic (locally or globally)**

Bianchi I metric:

$$ds^2 = -dt^2 + a^2(t) \left[ e^{2\beta_x(t)} dx^2 + e^{2\beta_y(t)} dy^2 + e^{2\beta_z(t)} dz^2 \right]$$


$$\sum_{i \in \{x,y,z\}} \beta_i = 0$$

Matter: 2 perfect fluids

Master function  $\Lambda(n_{xy}^2)$

conserved currents

$$n_{xy}^2 = -g_{\mu\nu} n_x^\mu n_y^\nu$$

$$\nabla_\mu n_x^\mu = 0$$

Generalized pressure

$$\Psi = \Lambda + \sum_{x,y} \frac{\partial \Lambda}{\partial n_{xy}^2} n_{xy}^2$$

Stress energy tensor

$$T^\mu_\nu = \Psi \delta^\mu_\nu + \frac{\partial \Lambda}{\partial n_{xy}^2} n_x^\mu n_{y\nu} = \sum_x [P_x \delta^\mu_\nu + (\rho_x + P_x) u_x^\mu u_{x\nu}]$$

Vorticity

$$\omega_{\mu\nu}^x = 2\nabla_{[\mu} \left( \frac{\partial \Lambda}{\partial n_{xy}^2} n_{y\nu]} \right)$$

Integrability condition

(geometric)  $n_x^\mu \omega_{\mu\nu}^x = 0$

# A simple example: the radiation/matter transition

*Fluids: massive particles, number density  $n$  - entropy  $s$*

*Master function*

$$\Lambda = - (mn + \kappa s^{4/3})$$

*usual  $\alpha^{-3}$  behaviour*                       $\alpha^{-4}$

*Shear from the metric*       $\sigma_i = \dot{\beta}_i e^{2\beta_i}$

<i>Initial condition = radiation dominated FLRW</i>	$\rho_{\text{rad}} \gg \rho_{\text{mat}}$	$\beta_i(t) = C_i^{\text{in}} \quad \forall i$	$\sigma_i(t) \rightarrow 0$
<i>Initial condition = matter dominated FLRW</i>	$\rho_{\text{rad}} \ll \rho_{\text{mat}}$	$\beta_i(t) = C_i^{\text{out}} \quad \forall i$	

# A simple example: the radiation/matter transition

Fluids: massive particles, number density  $n$  - entropy  $s$

Master function

$$\Lambda = - \left( m + \tau n^{\sigma_n - 1} s^{\sigma_s} \right) n - \kappa s^{4/3}$$

New coupling constant

usual  $a^{-3}$  behaviour

$a^{-4}$

Shear from the metric

$$\sigma_i = \dot{\beta}_i e^{2\beta_i}$$

Initial condition = radiation dominated FLRW

$$\rho_{\text{rad}} \gg \rho_{\text{mat}}$$

$$\beta_i(t) = C_i^{\text{in}} \quad \forall i$$

$$\sigma_i(t) \rightarrow 0$$

Initial condition = matter dominated FLRW

$$\rho_{\text{rad}} \ll \rho_{\text{mat}}$$

$$\beta_i(t) = C_i^{\text{out}} \quad \forall i$$

# Background: time dependent quantities

*Einstein equations + vorticity + integrability*

$$\dot{H}_x = -H_x^2 + H_y H_z - 4\pi [\mu n (1 + 2V_n^2) + Ts (1 + 2V_s^2)]$$

$$\dot{H}_y = -H_y^2 + H_x H_z - 4\pi [\mu n (1 + 2V_n^2) + Ts (1 + 2V_s^2)]$$

$$\dot{H}_z = -H_z^2 + H_x H_y - 4\pi (\mu n + Ts)$$

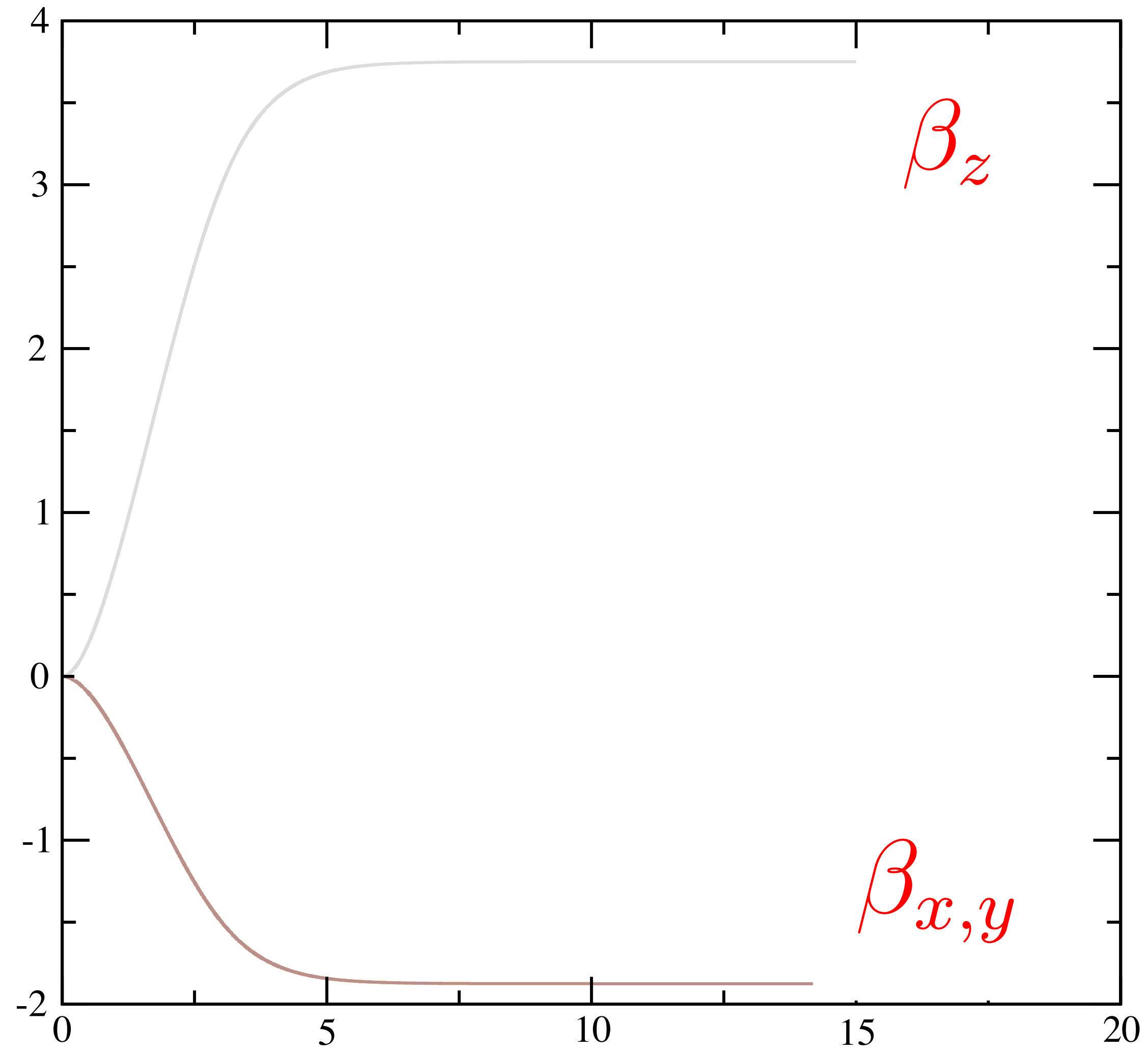
$$\dot{A}_N = H_N A_N$$

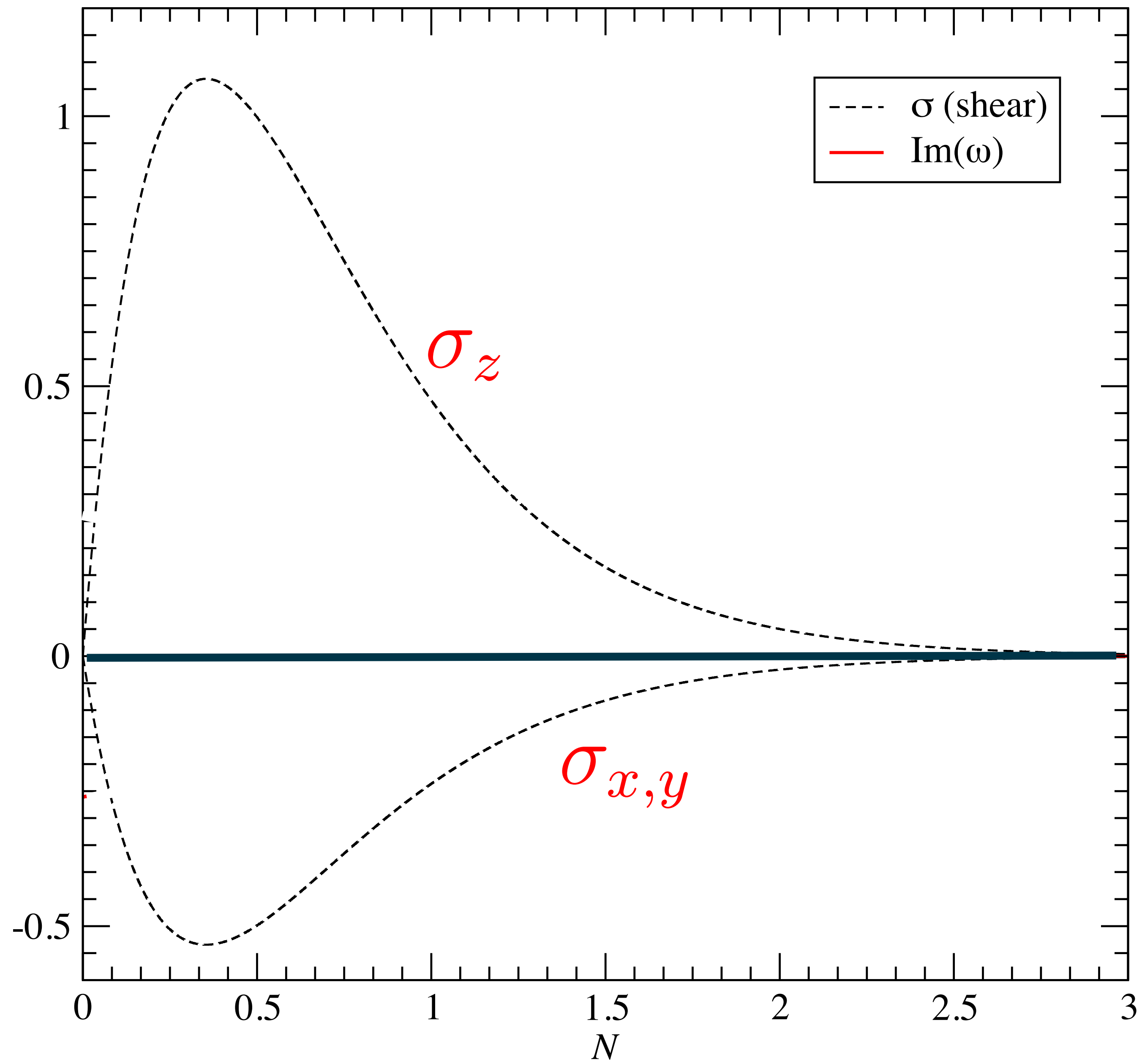
$$\begin{pmatrix} 1 - \frac{c_n^2 V_n^2}{1 + V_n^2} & -\frac{C_{ns} V_n^2}{1 + V_n^2} \\ -\frac{C_{sn} V_s^2}{1 + V_s^2} & 1 - \frac{c_s^2 V_s^2}{1 + V_s^2} \end{pmatrix} \begin{pmatrix} \frac{\dot{n}}{n} \\ \frac{\dot{s}}{s} \end{pmatrix} = - \begin{pmatrix} H_x + H_y + \frac{H_z}{1 + V_n^2} \\ H_x + H_y + \frac{H_z}{1 + V_s^2} \end{pmatrix}$$

$$c_x^2 \equiv \frac{\partial \ln \mu^x}{\partial \ln n_x}$$

$$c_{xy} \equiv \frac{\partial \ln \mu^x}{\partial \ln n_y}$$

*Constraint*  $H_x H_y + H_x H_z + H_y H_z = 8\pi (-\Lambda + \mu n V_n^2 + Ts V_s^2)$







# Linear perturbations around the Radiation/Matter background

*Perturbative expansion*

$$f(\mathbf{x}, t) = f_{\text{back}}(t) + \delta f(z, t) + \dots$$

$$\delta\Lambda = -\mu^x \delta n_x - \mu^y \delta n_y$$

$$\begin{aligned} \delta\Psi = & n_x^2 \delta\mathcal{B}^x + n_y^2 \delta\mathcal{B}^y - n_x u_x^t \mu_t^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^t}{u_x^t} \right) - n_x u_x^z \mu_z^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^z}{u_x^z} + \frac{\delta A_z}{A_z} \right) \\ & - n_y u_y^t \mu_t^y \left( \frac{\delta n_y}{n_y} + \frac{\delta u_y^t}{u_y^t} \right) - n_y u_y^z \mu_z^y \left( \frac{\delta n_y}{n_y} + \frac{\delta u_y^z}{u_y^z} + \frac{\delta A_z}{A_z} \right) \end{aligned}$$

*Matter quantities*  $\frac{\partial \delta \mu_t^x}{\partial z} = \frac{\partial \delta \mu_z^x}{\partial t}$

$$\delta \mu_t^x = \mu_t^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^t}{u_x^t} + \frac{\delta \mathcal{B}^x}{\mathcal{B}^x} \right)$$

$$\delta \mu_z^x = \mu_z^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^z}{u_x^z} + \frac{\delta \mathcal{B}^x}{\mathcal{B}^x} + 2 \frac{\delta A_z}{A_z} \right)$$

$$\delta \mathcal{B}^x = \mathcal{B}^x \left[ (c_x^2 - 1) \frac{\delta n_x}{n_x} + c_{xy} \frac{\delta n_y}{n_y} \right]$$

$$\delta T^t_t = \delta\Psi + n_x u_x^t \mu_t^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^t}{u_x^t} + \frac{\delta \mu_t^x}{\mu_t^x} \right) + n_y u_y^t \mu_t^y \left( \frac{\delta n_y}{n_y} + \frac{\delta u_y^t}{u_y^t} + \frac{\delta \mu_t^y}{\mu_t^y} \right)$$

$$\delta T^x_x = \delta T^y_y = \delta\Psi$$

$$\delta T^z_t = n_x u_x^z \mu_t^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^z}{u_x^z} + \frac{\delta \mu_t^x}{\mu_t^x} \right) + n_y u_y^z \mu_t^y \left( \frac{\delta n_y}{n_y} + \frac{\delta u_y^z}{u_y^z} + \frac{\delta \mu_t^y}{\mu_t^y} \right)$$

$$\delta T^z_z = \delta\Psi + n_x u_x^z \mu_z^x \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^z}{u_x^z} + \frac{\delta \mu_z^x}{\mu_z^x} \right) + n_y u_y^z \mu_z^y \left( \frac{\delta n_y}{n_y} + \frac{\delta u_y^z}{u_y^z} + \frac{\delta \mu_z^y}{\mu_z^y} \right)$$

*Stress energy tensor*

## Geometric quantities

$$\delta H = \frac{\delta \dot{A}}{A} - H \frac{\delta A}{A}$$

$$\delta G^t_t = -2(H + H_z) \delta H - 2H \delta H_z + \frac{2}{A_z^2} \delta I'$$

$$\delta G^x_x = \delta G^y_y = -\delta \dot{H} - \delta \dot{H}_z - (2H + H_z) \delta H - (H + 2H_z) \delta H_z + \frac{1}{A_z^2} \delta I'$$

$$\delta G^z_t = -\frac{2}{A_z^2} \left[ \delta \dot{I} + (H - H_z) \delta I \right]$$

$$\delta G^z_z = -2 \left( \delta \dot{H} + 3H \delta H \right)$$

## Plane wave analysis

$$\delta n_x = \mathcal{N}_x e^{ik_\mu x^\mu} \quad \delta u_x^\mu = A_x^\mu e^{ik_\nu x^\nu}$$



$$0 = -ik_z \mathcal{B}^x n_x u_x^t \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^t}{u_x^t} + \frac{\delta \mathcal{B}^x}{\mathcal{B}^x} \right) + ik_t \mathcal{B}^x n_x u_x^t \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^t}{u_x^t} + \frac{\delta \mathcal{B}^x}{\mathcal{B}^x} \right)$$

$$0 = ik_t u_x^t \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^t}{u_x^t} \right) + ik_z u_x^z \left( \frac{\delta n_x}{n_x} + \frac{\delta u_x^z}{u_x^z} \right)$$


## Matrix equation




$$\begin{bmatrix} (u_x \sigma_z - 1)^2 c_x^2 - (\sigma_z - u_x)^2 & \mathcal{C}_{xy} (u_x \sigma_z - 1)^2 \\ \mathcal{C}_{yx} (u_y \sigma_z - 1)^2 & (u_y \sigma_z - 1)^2 c_y^2 - (\sigma_z - u_y)^2 \end{bmatrix} \begin{pmatrix} \frac{\delta n_x}{n_x} \\ \frac{\delta n_y}{n_y} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

*Vanishing determinant*  *quartic equation*

$$\left[ (u_x \sigma_z - 1)^2 c_x^2 - (\sigma_z - u_x)^2 \right] \left[ (u_y \sigma_z - 1)^2 c_y^2 - (\sigma_z - u_y)^2 \right] - C_{xy} C_{yx} (u_x \sigma_z - 1)^2 (u_y \sigma_z - 1)^2 = 0$$

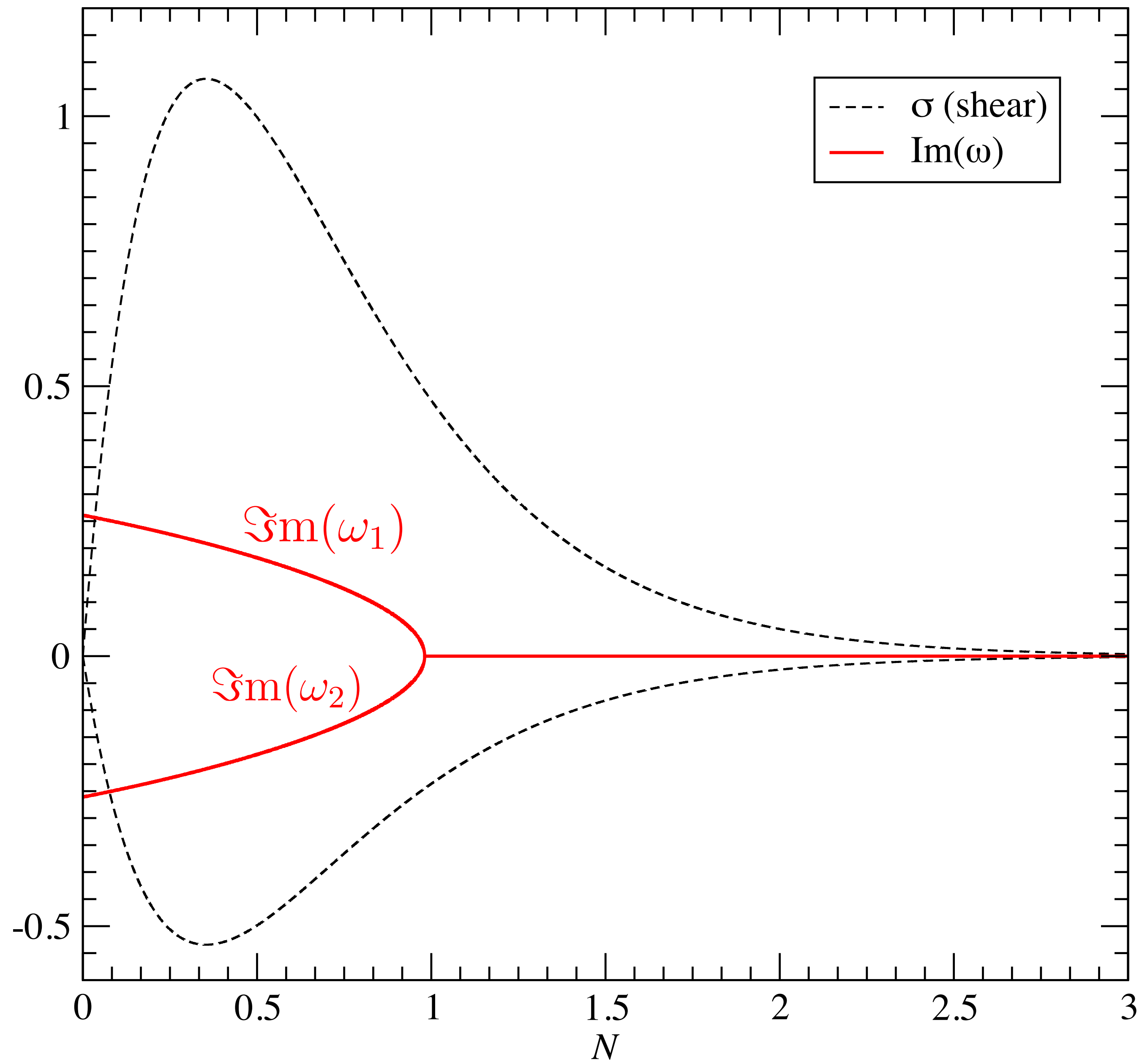
  $A_z u_x^z / u_x^t$

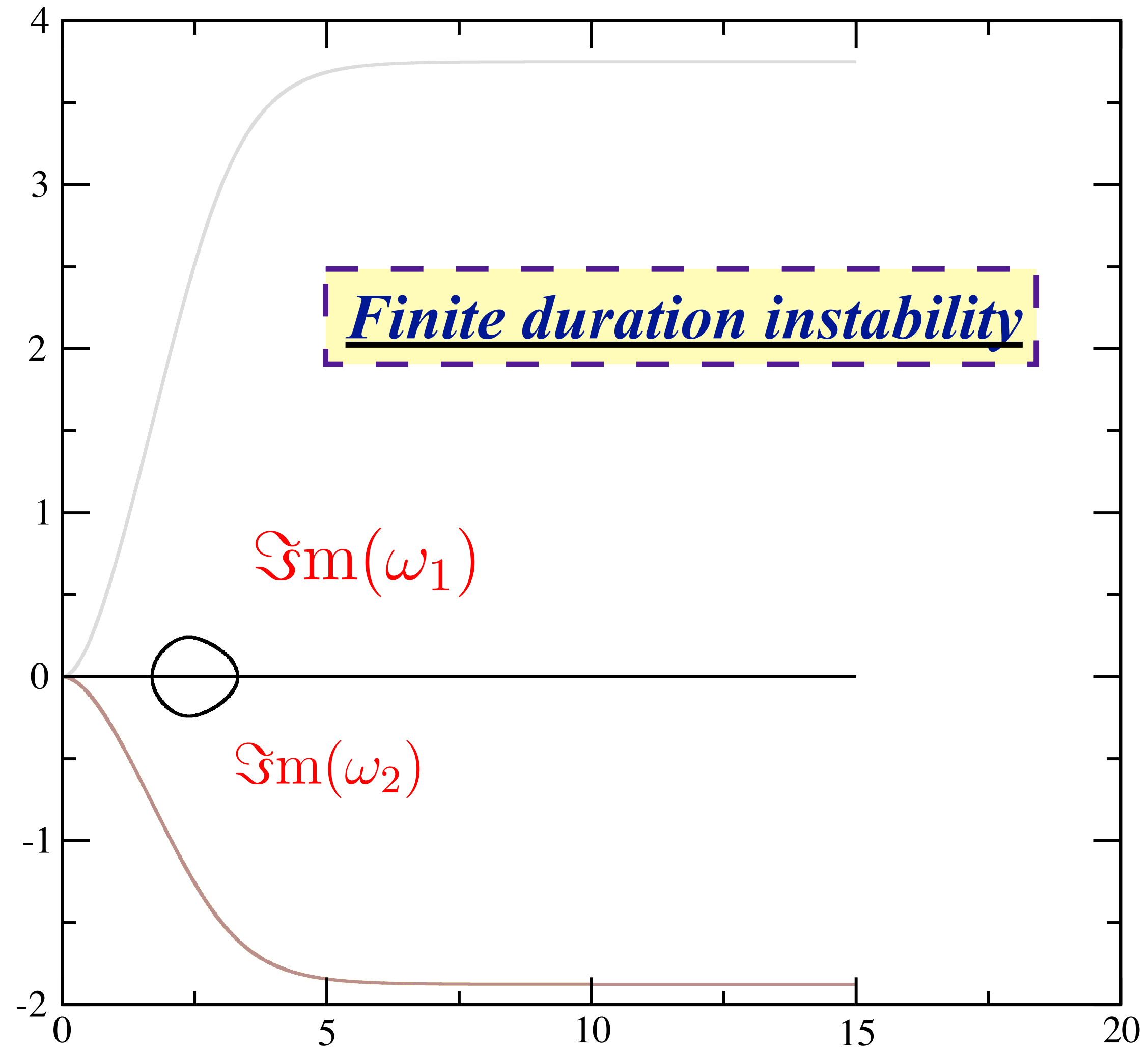
  $\sigma_z = -A_z k_t / k_z$

$k \equiv k_z \in \mathbb{R}$  &  $\omega \equiv k_t$    $P_4(\omega) = \omega^4 + \#_3 \omega^3 + \#_2 \omega^2 + \#_1 \omega + \#_0 = 0$

*Complex solution*  *Unstable mode*

$\frac{\delta \rho}{\rho} \propto e^{\Im m(\omega)t}$  *can grow non linear very rapidly*





# Possible observational consequences

*Start non linear growth of structure earlier and faster*

*Privileged spatial direction in matter structures*

*correlation functions*

*low CMB multipoles*

$$\frac{\delta\alpha}{\alpha}$$

*Magnetic fields ...*

*Constraints*

*Work in progress, any idea welcome!*



Thank you!

ありがとう!