Cosmological two tream instability

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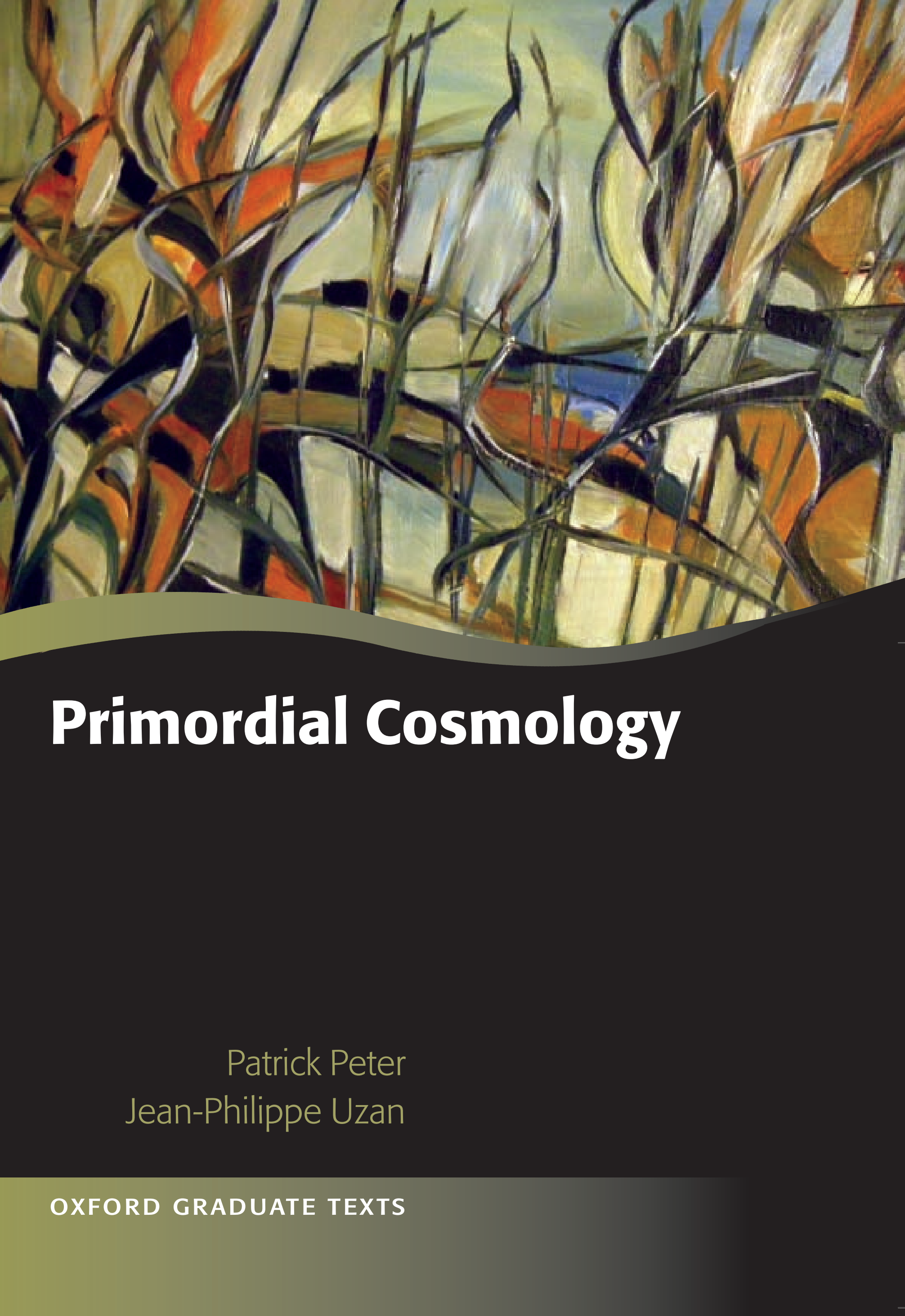


Standard paradigm: single-field inflation + (p)reheating → single fluid dominated FLRW Universe

FLRW metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\mathcal{K}r^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

Matter component: perfect fluid

$$T_{\mu\nu} = p g_{\mu\nu} + (\rho + p)u_\mu u_\nu$$

$$p = \omega\rho \begin{cases} \omega = \frac{1}{3} & \text{radiation} \\ \omega = 0 & \text{dust} \\ \omega = -1 & \text{cosmological constant} \end{cases}$$

+ cosmological constant = Einstein equation

$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{1}{3}\left(8\pi G_N \rho + \Lambda\right)$$

$$\frac{\ddot{a}}{a} = \frac{1}{3}\left[\Lambda - 4\pi G_N (\rho + p)\right]$$

More complicated models: hybrid (GUT), vector modes, ...
→ anisotropic (locally or globally)

Bianchi I metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\mathrm{e}^{2\beta_x(t)}\mathrm{d}x^2 + \mathrm{e}^{2\beta_y(t)}\mathrm{d}y^2 + \mathrm{e}^{2\beta_z(t)}\mathrm{d}z^2\right] \qquad \sum_{i\in\{x,y,z\}} \beta_i = 0$$

$A_x^2(t)$ $A_y^2(t)$ $A_z^2(t)$

Matter: 2 perfect fluids

Master function $\Lambda\left(n_{\mathrm{xy}}^2\right)$ $\qquad n_{\mathrm{xy}}^2 = -g_{\mu\nu} n_{\mathrm{x}}^\mu n_{\mathrm{y}}^\nu$

conserved currents $\qquad \nabla_\mu n_{\mathrm{x}}^\mu = 0$

Generalized pressure $\qquad \Psi = \Lambda + \sum_{\mathrm{x,y}} \frac{\partial \Lambda}{\partial n_{\mathrm{xy}}^2} n_{\mathrm{xy}}^2$

Stress energy tensor $\qquad T^\mu_{\ \nu} = \Psi \delta^\mu_{\ \nu} + \frac{\partial \Lambda}{\partial n_{\mathrm{xy}}^2} n_{\mathrm{x}}^\mu n_{\mathrm{y}\nu} = \sum_{\mathrm{x}} \left[P_{\mathrm{x}} \delta^\mu_{\ \nu} + (\rho_{\mathrm{x}} + P_{\mathrm{x}})\, u_{\mathrm{x}}^\mu u_{\mathrm{x}\nu}\right]$

Vorticity $\qquad \omega_{\mu\nu}^{\mathrm{x}} = 2\nabla_{[\mu}\left(\frac{\partial \Lambda}{\partial n_{\mathrm{xy}}^2} n_{\mathrm{y}\nu]}\right)$

Integrability condition
(geometric) $\quad n_{\mathrm{x}}^\mu \omega_{\mu\nu}^{\mathrm{x}} = 0$

A simple example: the radiation/matter transition

Fluids: massive particles, number density n - *entropy* s

Master function $$\Lambda = -\left(mn + \kappa s^{4/3}\right)$$

mn → *usual* a^{-3} *behaviour*

$\kappa s^{4/3}$ → a^{-4}

Shear from the metric $$\sigma_i = \dot{\beta}_i \mathrm{e}^{2\beta_i}$$

Initial condition = radiation dominated FLRW $\rho_{\rm rad} \gg \rho_{\rm mat}$ $\beta_i(t) = C_i^{\rm in} \quad \forall i$

Initial condition = matter dominated FLRW $\rho_{\rm rad} \ll \rho_{\rm mat}$ $\beta_i(t) = C_i^{\rm out} \quad \forall i$

$\sigma_i(t) \to 0$

A simple example: the radiation/matter transition

Fluids: massive particles, number density n - entropy s

Master function $$\Lambda = -\left(m + \tau n^{\sigma_\mathrm{n}-1} s^{\sigma_\mathrm{s}}\right) n - \kappa s^{4/3}$$

New coupling constant — $\tau n^{\sigma_\mathrm{n}-1} s^{\sigma_\mathrm{s}}$

usual a^{-3} behaviour — m, n

a^{-4} — $\kappa s^{4/3}$

Shear from the metric $$\sigma_i = \dot{\beta}_i \mathrm{e}^{2\beta_i}$$

Initial condition = radiation dominated FLRW $\rho_\mathrm{rad} \gg \rho_\mathrm{mat}$ $\beta_i(t) = C_i^\mathrm{in} \quad \forall i$

Initial condition = matter dominated FLRW $\rho_\mathrm{rad} \ll \rho_\mathrm{mat}$ $\beta_i(t) = C_i^\mathrm{out} \quad \forall i$

$\sigma_i(t) \to 0$

Background: time dependent quantities

Einstein equations + vorticity + integrability

$$\dot{H}_x = -H_x^2 + H_y H_z - 4\pi\left[\mu n\left(1+2V_\mathrm{n}^2\right) + Ts\left(1+2V_\mathrm{s}^2\right)\right]$$

$$\dot{H}_y = -H_y^2 + H_x H_z - 4\pi\left[\mu n\left(1+2V_\mathrm{n}^2\right) + Ts\left(1+2V_\mathrm{s}^2\right)\right]$$

$$\dot{H}_z = -H_z^2 + H_x H_y - 4\pi\left(\mu n + Ts\right)$$

$$\dot{A}_\aleph = H_\aleph A_\aleph$$

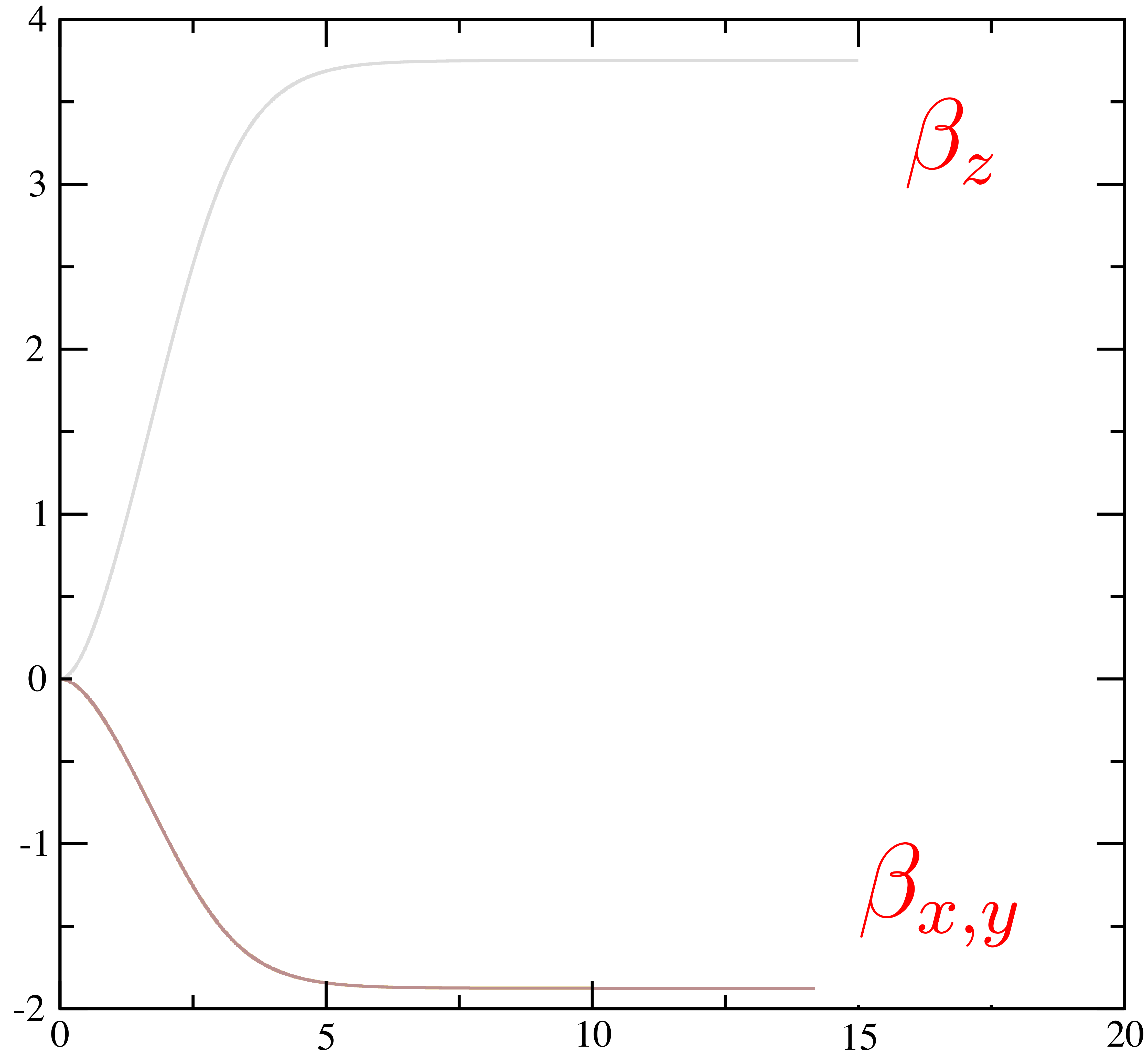
$\mu n \to \frac{\partial\Lambda}{\partial n^2}$, $Ts \to \frac{\partial\Lambda}{\partial s^2}$, $V_\mathrm{n} \to A_z u_\mathrm{n}^z$, $V_\mathrm{s} \to A_z u_\mathrm{s}^z$

$$\begin{pmatrix} 1-\frac{c_\mathrm{n}^2 V_\mathrm{n}^2}{1+V_\mathrm{n}^2} & -\frac{\mathcal{C}_\mathrm{ns} V_\mathrm{n}^2}{1+V_\mathrm{n}^2} \\ -\frac{\mathcal{C}_\mathrm{sn} V_\mathrm{s}^2}{1+V_\mathrm{s}^2} & 1-\frac{c_\mathrm{s}^2 V_\mathrm{s}^2}{1+V_\mathrm{s}^2} \end{pmatrix} \begin{pmatrix} \frac{\dot{n}}{n} \\ \frac{\dot{s}}{s} \end{pmatrix} = -\begin{pmatrix} H_x + H_y + \frac{H_z}{1+V_\mathrm{n}^2} \\ H_x + H_y + \frac{H_z}{1+V_\mathrm{s}^2} \end{pmatrix}$$

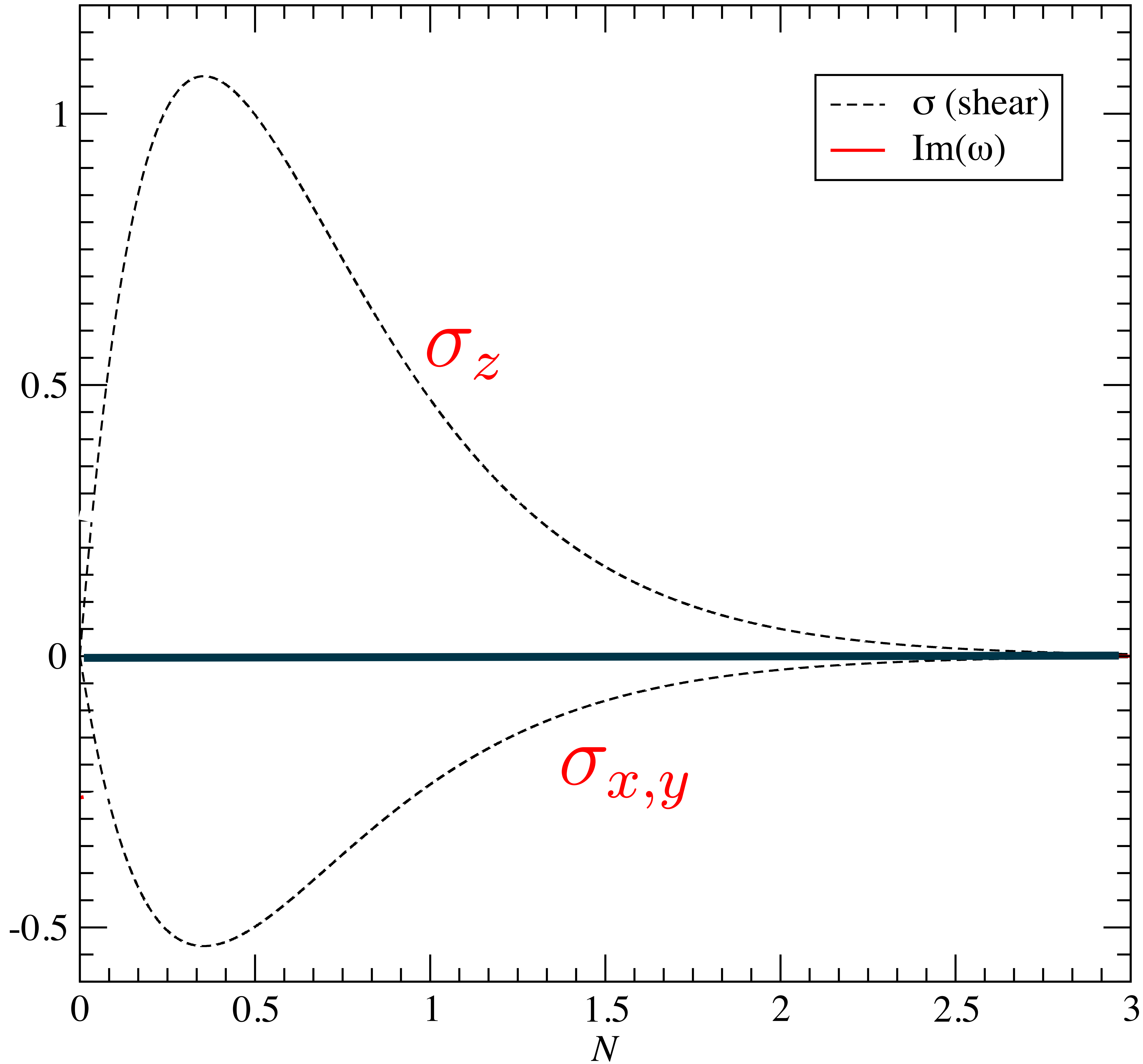
$$c_\mathrm{x}^2 \equiv \frac{\partial \ln \mu^\mathrm{x}}{\partial \ln n_\mathrm{x}}$$

$$\mathcal{C}_\mathrm{xy} \equiv \frac{\partial \ln \mu^\mathrm{x}}{\partial \ln n_\mathrm{y}}$$

Constraint $$H_x H_y + H_x H_z + H_y H_z = 8\pi\left(-\Lambda + \mu n V_\mathrm{n}^2 + Ts V_\mathrm{s}^2\right)$$



4
3
2
1
0
-1
-2
0
5
10
15
20
β_z
$\beta_{x,y}$



σ (shear)
Im(ω)
σ_z
$\sigma_{x,y}$
1
0.5
0
-0.5
0
0.5
1
1.5
2
2.5
3
N

Linear perturbations around the Radiation/Matter background

Perturbative expansion

$$f(\boldsymbol{x},t) = f_{\text{back}}(t) + \delta f(z,t) + \cdots$$

$$\delta\Lambda = -\mu^{\rm x}\delta n_{\rm x} - \mu^{\rm y}\delta n_{\rm y}$$

$$\delta\Psi = n_{\rm x}^2\delta\mathcal{B}^{\rm x} + n_{\rm y}^2\delta\mathcal{B}^{\rm y} - n_{\rm x}u_{\rm x}^t\mu_t^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^t}{u_{\rm x}^t}\right) - n_{\rm x}u_{\rm x}^z\mu_z^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^z}{u_{\rm x}^z} + \frac{\delta A_z}{A_z}\right)$$
$$-n_{\rm y}u_{\rm y}^t\mu_t^{\rm y}\left(\frac{\delta n_{\rm y}}{n_{\rm y}} + \frac{\delta u_{\rm y}^t}{u_{\rm y}^t}\right) - n_{\rm y}u_{\rm y}^z\mu_z^{\rm y}\left(\frac{\delta n_{\rm y}}{n_{\rm y}} + \frac{\delta u_{\rm y}^z}{u_{\rm y}^z} + \frac{\delta A_z}{A_z}\right)$$

Matter quantities

$$\frac{\partial\delta\mu_t^{\rm x}}{\partial z} = \frac{\partial\delta\mu_z^{\rm x}}{\partial t}$$

$$\delta\mu_t^{\rm x} = \mu_t^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^t}{u_{\rm x}^t} + \frac{\delta\mathcal{B}^{\rm x}}{\mathcal{B}^{\rm x}}\right)$$

$$\delta\mu_z^{\rm x} = \mu_z^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^z}{u_{\rm x}^z} + \frac{\delta\mathcal{B}^{\rm x}}{\mathcal{B}^{\rm x}} + 2\frac{\delta A_z}{A_z}\right)$$

$$\delta\mathcal{B}^{\rm x} = \mathcal{B}^{\rm x}\left[\left(c_{\rm x}^2 - 1\right)\frac{\delta n_{\rm x}}{n_{\rm x}} + \mathcal{C}_{\rm xy}\frac{\delta n_{\rm y}}{n_{\rm y}}\right]$$

Stress energy tensor

$$\delta T^t{}_t = \delta\Psi + n_{\rm x}u_{\rm x}^t\mu_t^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^t}{u_{\rm x}^t} + \frac{\delta\mu_t^{\rm x}}{\mu_t^{\rm x}}\right) + n_{\rm y}u_{\rm y}^t\mu_t^{\rm y}\left(\frac{\delta n_{\rm y}}{n_{\rm y}} + \frac{\delta u_{\rm y}^t}{u_{\rm y}^t} + \frac{\delta\mu_t^{\rm y}}{\mu_t^{\rm y}}\right)$$

$$\delta T^x{}_x = \delta T^y{}_y = \delta\Psi$$

$$\delta T^z{}_t = n_{\rm x}u_{\rm x}^z\mu_t^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^z}{u_{\rm x}^z} + \frac{\delta\mu_t^{\rm x}}{\mu_t^{\rm x}}\right) + n_{\rm y}u_{\rm y}^z\mu_t^{\rm y}\left(\frac{\delta n_{\rm y}}{n_{\rm y}} + \frac{\delta u_{\rm y}^z}{u_{\rm y}^z} + \frac{\delta\mu_t^{\rm y}}{\mu_t^{\rm y}}\right)$$

$$\delta T^z{}_z = \delta\Psi + n_{\rm x}u_{\rm x}^z\mu_z^{\rm x}\left(\frac{\delta n_{\rm x}}{n_{\rm x}} + \frac{\delta u_{\rm x}^z}{u_{\rm x}^z} + \frac{\delta\mu_z^{\rm x}}{\mu_z^{\rm x}}\right) + n_{\rm y}u_{\rm y}^z\mu_z^{\rm y}\left(\frac{\delta n_{\rm y}}{n_{\rm y}} + \frac{\delta u_{\rm y}^z}{u_{\rm y}^z} + \frac{\delta\mu_z^{\rm y}}{\mu_z^{\rm y}}\right)$$

Geometric quantities

$$\delta H = \frac{\delta \dot{A}}{A} - H\frac{\delta A}{A}$$

$$\delta G^t{}_t = -2\left(H + H_z\right)\delta H - 2H\delta H_z + \frac{2}{A_z^2}\delta I'$$

$$\delta G^x{}_x = \delta G^y{}_y = -\delta \dot{H} - \delta \dot{H}_z - \left(2H + H_z\right)\delta H - \left(H + 2H_z\right)\delta H_z + \frac{1}{A_z^2}\delta I'$$

$$\delta G^z{}_t = -\frac{2}{A_z^2}\left[\delta \dot{I} + \left(H - H_z\right)\delta I\right]$$

$$\delta G^z{}_z = -2\left(\delta \dot{H} + 3H\delta H\right)$$

Plane wave analysis

$$\delta n_{\mathrm{x}} = \mathcal{N}_{\mathrm{x}} e^{ik_\mu x^\mu} \qquad \delta u^\mu_{\mathrm{x}} = A^\mu_{\mathrm{x}} e^{ik_\nu x^\nu}$$

⇒

$$0 = -ik_z \mathcal{B}^{\mathrm{x}} n_{\mathrm{x}} u^t_{\mathrm{x}}\left(\frac{\delta n_{\mathrm{x}}}{n_{\mathrm{x}}} + \frac{\delta u^t_{\mathrm{x}}}{u^t_{\mathrm{x}}} + \frac{\delta \mathcal{B}^{\mathrm{x}}}{\mathcal{B}^{\mathrm{x}}}\right) + ik_t \mathcal{B}^{\mathrm{x}} n_{\mathrm{x}} u^t_{\mathrm{x}}\left(\frac{\delta n_{\mathrm{x}}}{n_{\mathrm{x}}} + \frac{\delta u^t_{\mathrm{x}}}{u^t_{\mathrm{x}}} + \frac{\delta \mathcal{B}^{\mathrm{x}}}{\mathcal{B}^{\mathrm{x}}}\right)$$

$$0 = ik_t u^t_{\mathrm{x}}\left(\frac{\delta n_{\mathrm{x}}}{n_{\mathrm{x}}} + \frac{\delta u^t_{\mathrm{x}}}{u^t_{\mathrm{x}}}\right) + ik_z u^z_{\mathrm{x}}\left(\frac{\delta n_{\mathrm{x}}}{n_{\mathrm{x}}} + \frac{\delta u^z_{\mathrm{x}}}{u^z_{\mathrm{x}}}\right)$$

Matrix equation ⇒

$$\begin{bmatrix} \left(u_{\mathrm{x}}\sigma_z - 1\right)^2 c_{\mathrm{x}}^2 - \left(\sigma_z - u_{\mathrm{x}}\right)^2 & \mathcal{C}_{\mathrm{xy}}\left(u_{\mathrm{x}}\sigma_z - 1\right)^2 \\ \mathcal{C}_{\mathrm{yx}}\left(u_{\mathrm{y}}\sigma_z - 1\right)^2 & \left(u_{\mathrm{y}}\sigma_z - 1\right)^2 c_{\mathrm{y}}^2 - \left(\sigma_z - u_{\mathrm{y}}\right)^2 \end{bmatrix}\begin{pmatrix} \frac{\delta n_{\mathrm{x}}}{n_{\mathrm{x}}} \\ \frac{\delta n_{\mathrm{y}}}{n_{\mathrm{y}}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Vanishing determinant ⟹ *quartic equation*

$$\left[(u_{\rm x}\sigma_z-1)^2 c_{\rm x}^2-(\sigma_z-u_{\rm x})^2\right]\left[(u_{\rm y}\sigma_z-1)^2 c_{\rm y}^2-(\sigma_z-u_{\rm y})^2\right]-\mathcal{C}_{\rm xy}\mathcal{C}_{\rm yx}(u_{\rm x}\sigma_z-1)^2(u_{\rm y}\sigma_z-1)^2=0$$

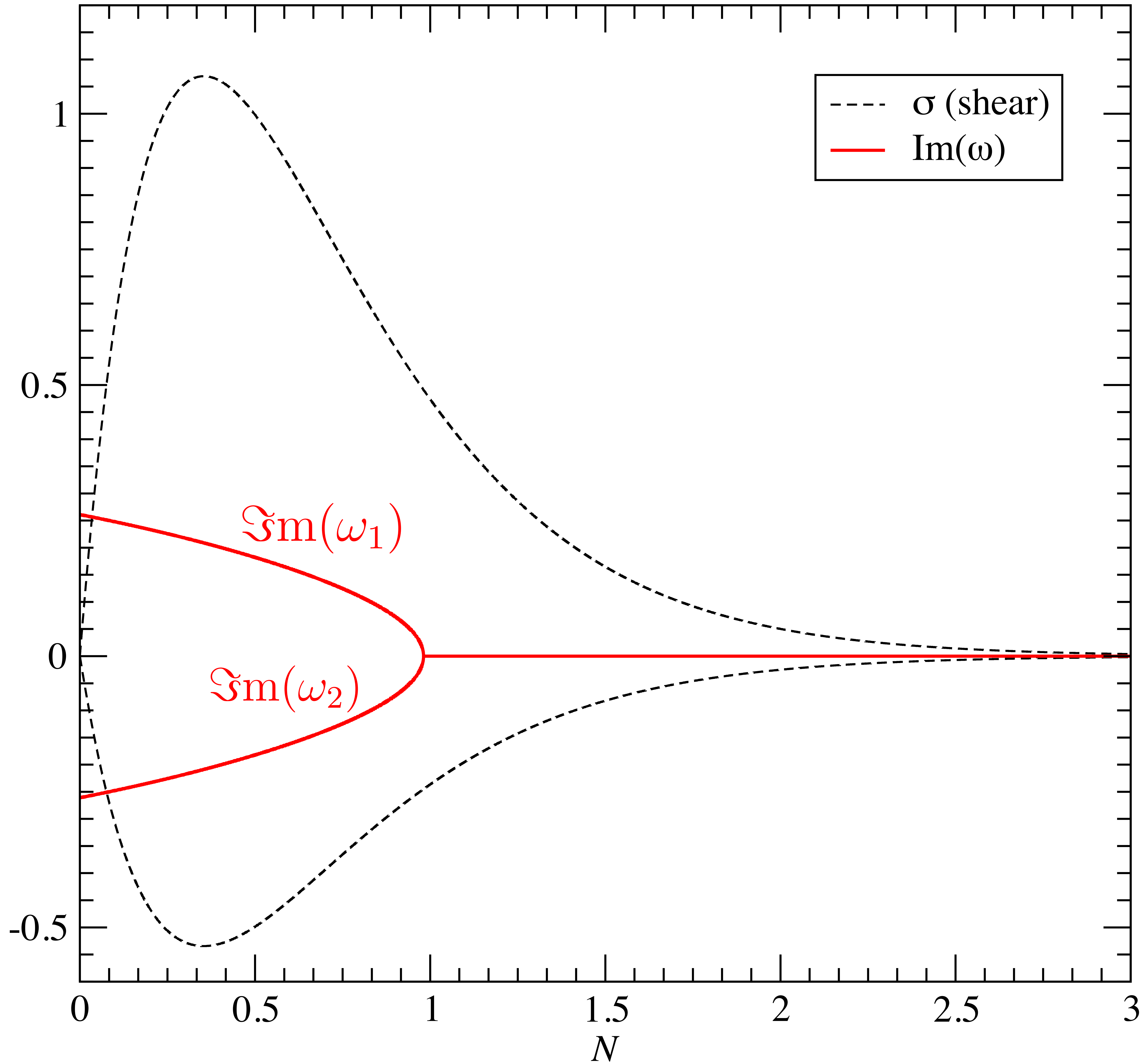
$$\sigma_z=-A_z k_t/k_z$$

$$A_z u_{\rm x}^z/u_{\rm x}^t$$

$k\equiv k_z\in\mathbb{R}$ & $\omega\equiv k_t$ ⟹ $P_4(\omega)=\omega^4+\#_3\omega^3+\#_2\omega^2+\#_1\omega+\#_0=0$

Complex solution ⟹ *Unstable mode*

$\frac{\delta\rho}{\rho}\propto {\rm e}^{\Im{\rm m}(\omega)t}$ *can grow non linear very rapidely*


σ (shear)
Im(ω)
$\Im\mathrm{m}(\omega_1)$
$\Im\mathrm{m}(\omega_2)$
1
0.5
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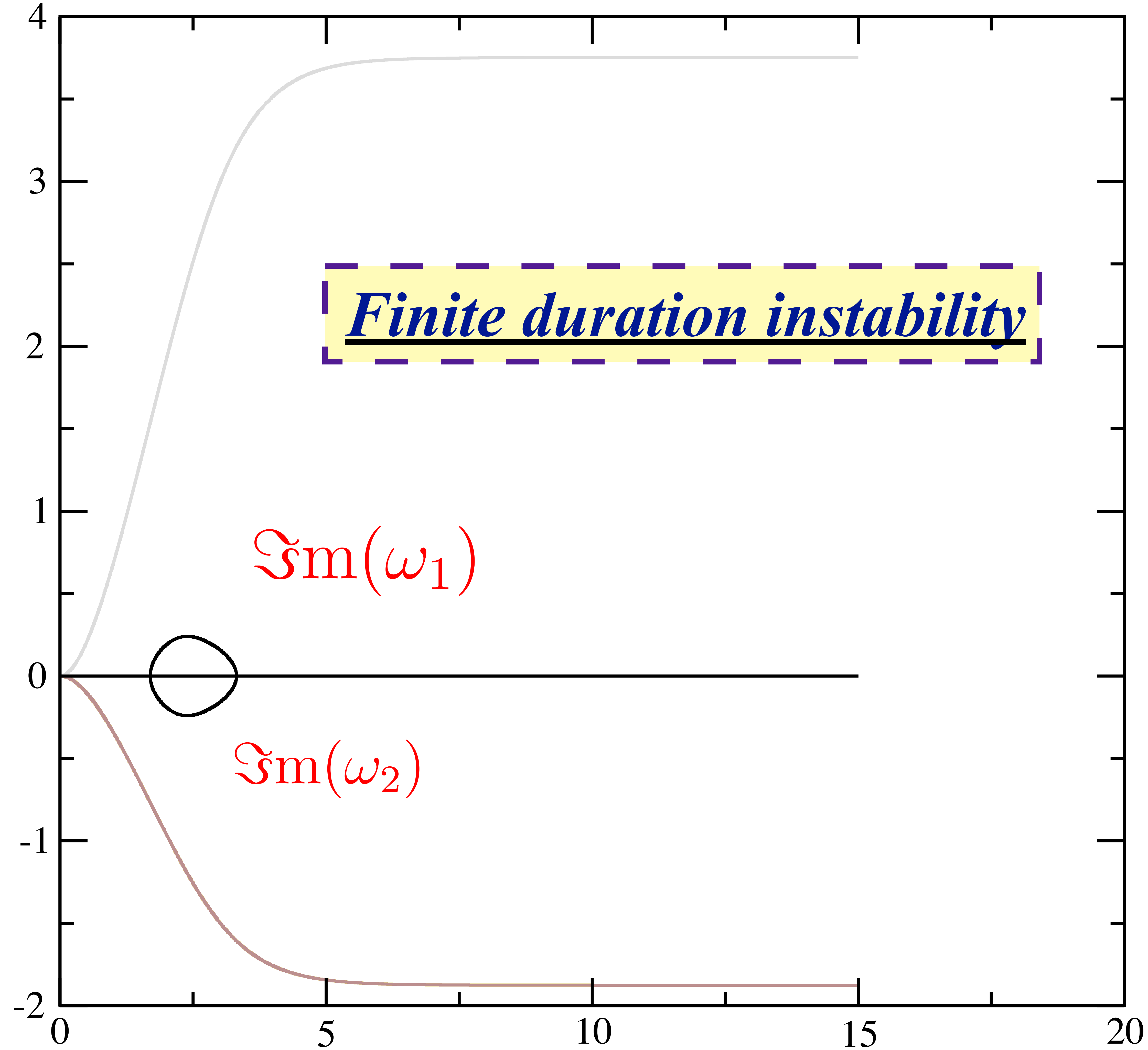

Finite duration instability
$\Im m(\omega_1)$
$\Im m(\omega_2)$
4
3
2
1
0
-1
-2
0
5
10
15
20

Possible observational consequences

Start non linear growth of structure earlier and faster

Priviledged spatial direction in matter structures

correlation functions

low CMB multipoles

$$\frac{\delta\alpha}{\alpha}$$

Magnetic fields ...

Constraints

Work in progress, any idea welcome!

Thank you!

ありがとう!