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Cosmological implications of gravity at a Lifshitz point

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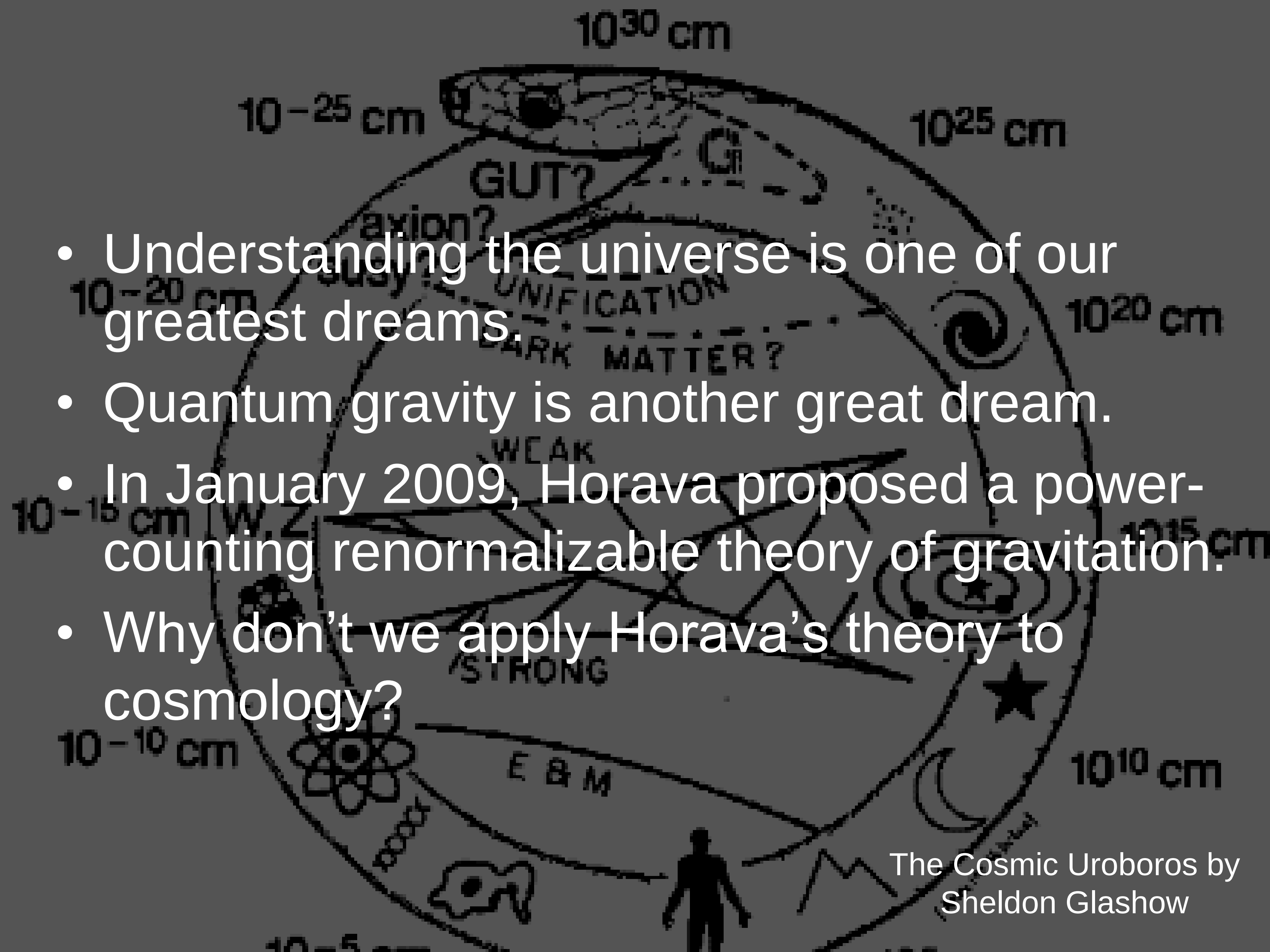
ref. Horava-Lifshitz Cosmology: A Review
arXiv: 1007.5199 [hep-th]



- Understanding the universe is one of our greatest dreams.
- Quantum gravity is another great dream.
- In January 2009, Horava proposed a power-counting renormalizable theory of gravitation.
- Why don't we apply Horava's theory to cosmology?

The Cosmic Uroboros by Sheldon Glashow

Power counting

$$I \supset \int dt\,dx^3 \dot{\phi}^2$$

$$\int dt\,dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

  $$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  $t \rightarrow b^z t$ ($E \rightarrow b^{-z} E$)

  $x \rightarrow b\, x$

  $\phi \rightarrow b^s \phi$

  $z+3-2z+2s = 0$

  $s = -(3-z)/2$

- $s = 0$ if $z = 3$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For $z = 3$, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.

Usual story with z=1

- $\omega^2 \gg H^2$: oscillate
  $\omega^2 \ll H^2$: freeze
  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$
  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.
- Scaling law
  $t \to b\, t$ ($E \to b^{-1}E$)
  $x \to b\, x$
  $\phi \to b^{-1}\, \phi$

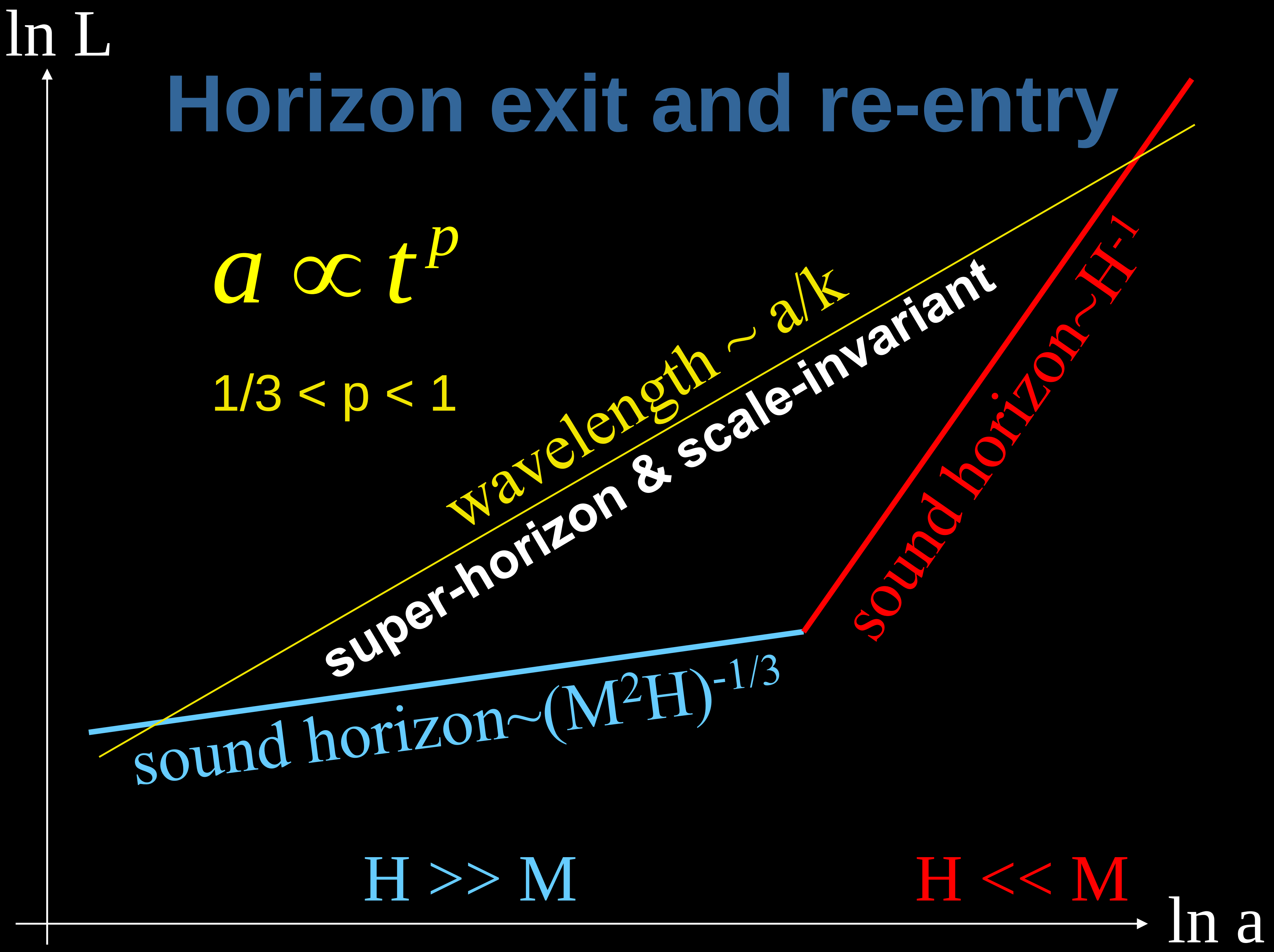
  ⇨ $$\delta\phi \propto E \sim H$$

  Scale-invariance requires almost const. H, i.e. inflation.

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3} E$)
  $x \to b\, x$
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

Horizon exit and re-entry



Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

See also arXiv:0906.5069 [hep-th]
Caustic avoidance in Horava-Lifshitz gravity

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No “local” Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  can be integrated to give
  Friedmann eq with
  “dark matter as
  integration constant”

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}{}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps (sequences of caustics & bounces) of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009]) If asymptotically free, would-be caustics does not gravitate too much.
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+CDM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.

ref. Horava-Lifshitz Cosmology: A Review
arXiv: 1007.5199 [hep-th]

Future works

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ? Does it satisfy the stability condition for the scalar graviton?
  ($|c_s| < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$ for $\mathrm{Max}[M^{-1}, 0.01\mathrm{mm}] < L < H^{-1}$)
- Is the $\lambda \rightarrow 1$ limit continuous?
  Yes, for spherically-symmetric, static, vacuum configurations. (recent review, arXiv:1007.5199)
- Can we get a common sound speed?
- Do microscopic lumps of "CDM" play the role of particles?
- Adiabatic initial condition for "CDM" from the z=3 scaling
- Spectral tilt from anomalous dimension
- Extensions of the original theory:
  Blas, et.al; Horava & Melby-Thompson …