Off-center CMB polarization anisotropy in the local void model

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Introduction

Type Ia supernova (SNIa) observations imply an acceleration of the cosmic expansion if we assume the Cosmological Principle. If we abandon this assumption, then other explanations become possible. The most interesting model of such a nature is the local void model, which was first proposed by Kenji Tomita in 2000. This model assumes that we are around the center of a low density spherically symmetric void and the spacetime is well described by the Lemaitre-Tolman-Bondi (LTB) model.

It is of crucial importance to find observational tests that enable us to discriminate this void model from the FLRW-based models, in order to establish the necessity of dark energy or a modification of gravity. One possible such test is to observe effects of the inhomogeneity on the cosmic microwave background (CMB) temperature and polarization.

In the present work, we calculate the gravitational lensing effect on the CMB polarization for an off-center observer in the local void model. As a result, we discover that the B-mode and the EB correlation are generated. With these remarkable observables, we are able to verify or falsify this model by future CMB experiments.

Gravitational lensing in the LTB model

Lemaitre-Tolman-Bondi (LTB) metric

$$ds^2 = -dt^2 + S^2 d\chi^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$S = \frac{r'}{\xi} \qquad r = r(t,\chi) \qquad \xi = \sqrt{1-k(\chi)\chi^2} \qquad k(\chi)\text{: curvature function}$$

geodesic equation $dp^\mu/d\lambda = -\Gamma^\mu_{\nu\rho}p^\nu p^\rho$

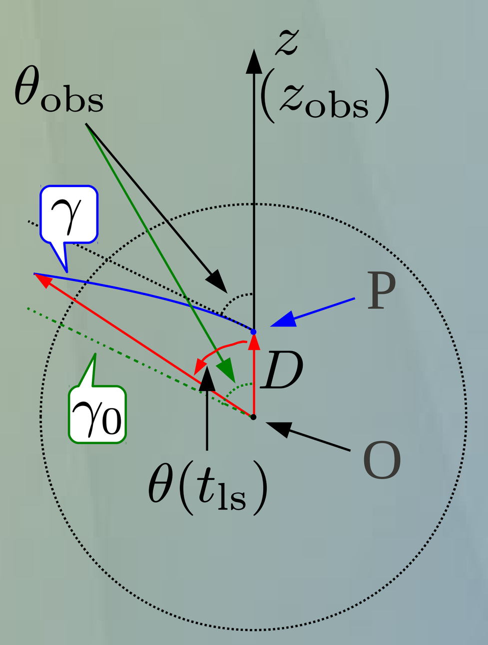
↓ spherical symmetry

ordinary differential equations (ODEs) of $\omega, \mu, p_\perp$

$$p^t = \omega,\ p^\chi = \mu\omega/S,\ p_\perp := r\sqrt{(p^\theta)^2 + (p^\varphi)^2 \sin^2\theta}$$

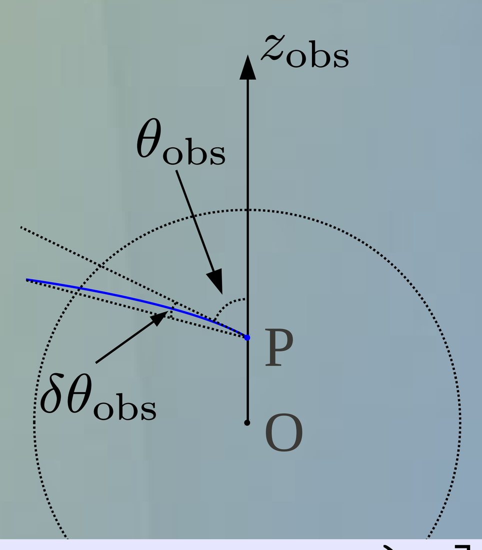
↓ 2-dim. problem → use the plane of $\varphi = 0, \pi$

ODEs of $\omega(t),\ \chi(t),\ \theta(t),\ \mu(t)$



If the distance D between the center of the spherically symmetric spacetime O and the observer P is small, γ (the null ray passing through O with the angle $\theta_{\rm obs}$ relative to OP) stays close to γ_0 (that through P) until the last scattering surface at $t = t_{\rm ls}$

→ Solve the linear perturbation equation obtained from the above ODEs



Finally, $\delta\theta_{\rm obs} = D\Gamma \sin\theta_{\rm obs}$

$$\Gamma := -\int_{t_{\rm ls}}^{t_0} \frac{dt}{\chi^2}\left[\frac{\chi}{r}(1-\xi(\chi)) + \frac{1}{S} - \frac{\chi}{r}\exp\int_t^{t_0} dt_1 \left\{\frac{\xi(\chi)}{\chi}\left(\frac{1}{r'} - \frac{\chi}{r}\right) + \frac{\dot r}{r} - \frac{\dot S}{S}\right\}_{t_1}\right]$$

CMB polarization in the LTB model

electric field polarization basis

$$\boldsymbol{E} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{2}\sum_p \{\boldsymbol{\epsilon}_p(\boldsymbol{k})a_p(\boldsymbol{k})e^{ik\cdot x} + \boldsymbol{\epsilon}_p^*(\boldsymbol{k})a_p(\boldsymbol{k})^\dagger e^{-ik\cdot x}\}$$

$\langle a_p(\boldsymbol{k})^\dagger a_q(\boldsymbol{k}')\rangle = 2(2\pi)^3\rho_{pq}(\boldsymbol{k})\delta^3(\boldsymbol{k}-\boldsymbol{k}')$ $\quad\rho_{pq}$: flux polarization matrix

flux density tensor $\rho_{ij}(\boldsymbol{k}) = \sum_{p,q}\epsilon^*_{pi}(\boldsymbol{k})\epsilon_{qj}(\boldsymbol{k})\rho_{pq}(\boldsymbol{k})$

ρ_{ij} generalized

polarized Boltzmann equation $\left(\frac{k^\mu}{k^0}\nabla_\mu + f^i\partial_{k^i}\right)\rho^{\mu\nu}(x,\boldsymbol{k}) = C^{\mu\nu}(\rho)$

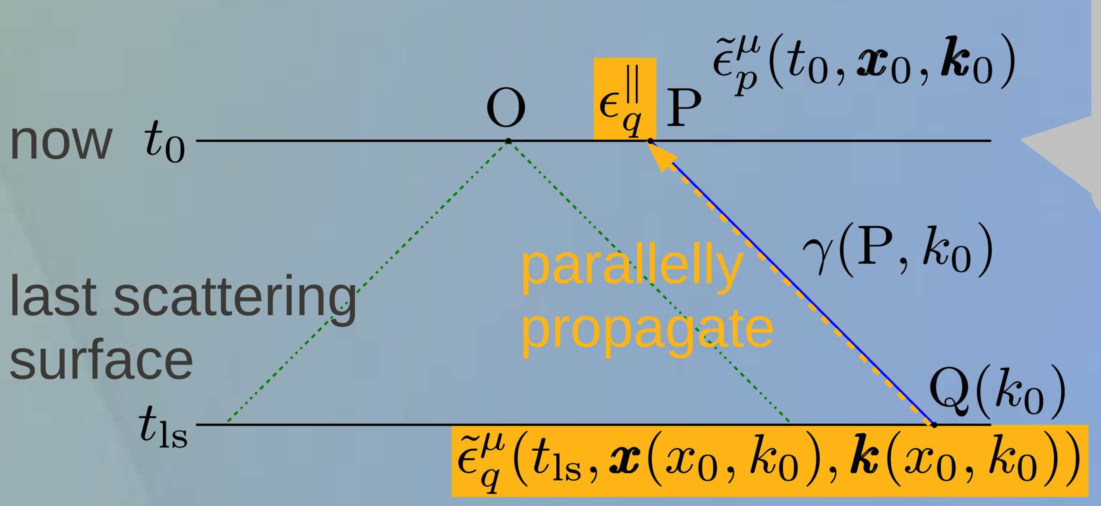
Solve Boltzmann eq. → initial condition at the last scattering surface

Propagation after the last scattering

LTB spacetime $g'_{\mu\nu}$

We work in the synchronous gauge: $ds^2 = -dt^2 + g'_{ij}(t,x)dx^i dx^j$

g'_{ij} approaches a spatially homogeneous and isotropic metric g_{ij} in the early universe → $g'_{ij} = g_{ij} + \delta_{\rm LTB}g_{ij}$



$\tilde\epsilon^\mu_p(t_0, \boldsymbol{x}, \boldsymbol{k})$: polarization basis at each point on the hypersurface $t = t_0$

$\tilde\epsilon^\mu_p(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})$: that on $t = t_{\rm ls}$

ρ_{pq} at P is expressed in terms of that on $t = t_{\rm ls}$

$$\tilde\rho(\mathrm{P}, \boldsymbol{k}_0) = \tilde C\tilde\rho(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})\tilde C^\dagger$$

$$\tilde C_{pq} = \tilde\epsilon_p(t_0, \boldsymbol{x}_0, \boldsymbol{k}_0)\cdot\epsilon^{\|*}_q$$

$$\tilde\rho_{pq}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k}) = \frac{1}{2}\delta_{pq}I\left(\frac{\omega}{T_{\rm ls}}\right) + \delta\rho_{pq}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})$$

$$\tilde\rho_{pq}(\mathrm{P}, \boldsymbol{k}_0) = \frac{1}{2}\delta_{pq}I\left(\frac{\omega}{T_{\rm ls}}\right) + \tilde C_{pr}\tilde C^*_{qs}\delta\rho_{rs}(t_{\rm ls}, \boldsymbol{x}, \boldsymbol{k})$$

We can set $\tilde C = \mathbf{1}$ for an appropriate choice of the polarization basis

Resulting B-mode formula (See also the appendix)

ρ_{ab} : unlensed polarization anisotropy $\qquad\rho'_{ab}$: lensed that

$\hat{\boldsymbol{n}} := \hat{\boldsymbol{n}}' + \delta\boldsymbol{\theta}_{\rm obs}$ $\quad\hat{\boldsymbol{n}}'$: direction in which the observer looks

$\rho'_{ab}(\hat{\boldsymbol{n}}') = \rho_{ab}(\hat{\boldsymbol{n}})$ → calculate

Finally, the lensed B-mode turns out to be

$$B'^m_\ell = B^m_\ell - D\Gamma\sqrt{\frac{((\ell+1)^2-m^2)((\ell+1)^2-4)}{(\ell+1)^2(4(\ell+1)^2-1)}}(\ell+2)B^m_{\ell+1}$$
$$+D\Gamma\sqrt{\frac{(\ell^2-m^2)(\ell^2-4)}{\ell^2(4\ell^2-1)}}(\ell-1)B^m_{\ell-1} - iD\Gamma\frac{2m}{\ell(\ell+1)}E^m_\ell + \mathcal{O}(D^2)$$

E to B

EB correlation (non-zero part only) (TB similarly) $\quad\langle E^{m*}_\ell B'^m_\ell\rangle_{\rm CMB} \approx D\Gamma\frac{2m}{i\ell(\ell+1)}C^{\rm EE}_\ell$

Summary and Outlook

In this work, we developed a formulation to calculate the gravitational lensing effect on the CMB temperature and polarization for an off-center observer in a spherically symmetric void described by the LTB model. Next, we are going to numerically estimate this effect. In future, we will limit the distance from us to the center by the results of B-mode observations.

Collaborator

Hideo Kodama (GUAS & KEK)

Appendix: polarization distribution patterns

CMB polarization distribution ("obs" omitted)

$$\rho_{ab}(\hat{\boldsymbol{n}}) = {}_{+2}A(\hat{\boldsymbol{n}})\bar m_a\bar m_b + {}_{-2}A(\hat{\boldsymbol{n}})m_a m_b$$

$$\hat{\boldsymbol{n}} = (\theta,\ \varphi) \qquad a, b \in \{\theta \equiv 1,\ \varphi \equiv 2\}$$

$$\boldsymbol{m} := \frac{1}{\sqrt{2}}(\boldsymbol{e}_\theta + i\boldsymbol{e}_\varphi) \qquad \bar{\boldsymbol{m}} := \frac{1}{\sqrt{2}}(\boldsymbol{e}_\theta - i\boldsymbol{e}_\varphi)$$

${}_{\pm2}A(\hat{\boldsymbol{n}})$ is expanded with spin-weighted spherical harmonics:

$${}_{\pm2}A(\hat{\boldsymbol{n}}) = \sum_{\ell m}{}_{\pm2}A^m_\ell\,{}_{\pm2}Y^m_\ell(\hat{\boldsymbol{n}})$$

rotationally invariant combination of ${}_{\pm2}A^m_\ell$:

$E^m_\ell := \frac{1}{2}({}_{+2}A^m_\ell + {}_{-2}A^m_\ell)$ curl-free (E-mode)

$B^m_\ell := \frac{1}{2i}({}_{+2}A^m_\ell - {}_{-2}A^m_\ell)$ gradient-free (B-mode)

power spectra from primordial fluctuations:

$$\langle X_1{}^{m*}_\ell X_2{}^{m'}_{\ell'}\rangle = C^{\rm X_1X_2}_\ell\delta_{\ell\ell'}\delta_{mm'}$$

$$\mathrm{X}_1, \mathrm{X}_2 \in \{\mathrm{T}, \mathrm{E}, \mathrm{B}\}$$

If physics and the ensemble for averaging are invariant under a parity inversion,

$$C^{\rm TB}_\ell = C^{\rm EB}_\ell = 0$$

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