Black Holes
in Einstein-Gauss-Bonnet-Dilaton System

I. Introduction

II. Model (action, basic equations, boundary conditions)

III. Black Holes (horizon radius, fat singularity)

V. Summary

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Introduction

GR → String theory

quantum gravity?
singularity?

BH?
Early Universe?

➢ ***Effective theory of superstring theory*** ***(heterotic string, Metsaev & Tseytlin 87)***

$$
\begin{aligned}
S \;=\; & \frac{1}{2\kappa_D^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\Big[R+4(\partial\phi)^2-\frac{1}{4}F^2-\frac{1}{12}H^2+\alpha_2\Big\{R^2_{\rm GB}-16\Big(R^{\mu\nu}-\frac{1}{2}Rg^{\mu\nu}\Big)\partial_\mu\phi\partial_\nu\phi \\
& +16\Box\phi(\partial\phi)^2-16(\partial\phi)^4-\frac{1}{2}\Big(R^{\mu\nu\sigma\rho}H_{\mu\nu\alpha}H_{\sigma\rho}{}^{\alpha}-2R^{\mu\nu}H^2_{\mu\nu}+\frac{1}{3}RH^2\Big)+2\Big(D^\mu\partial^\nu\phi H^2_{\mu\nu}-\frac{1}{3}\Box\phi H^2\Big) \\
& +\frac{2}{3}H^2(\partial\phi)^2+\frac{1}{24}H_{\mu\nu\lambda}H^{\nu}{}_{\rho\alpha}H^{\rho\sigma\lambda}H_{\sigma}{}^{\mu\alpha}-\frac{1}{8}H^2_{\mu\nu}H^{2\mu\nu}+\frac{1}{144}(H^2)^2\Big\}\Big],
\end{aligned}
$$

- ***Superstring theory is the "fundamental theory". How are the black holes and the early universe described in such theory?***
- ***Can the unsolved problem such as the singularity and dark energy are resolved in such theory?***
- ***Will the string theory be verified by the observation and/or experiments?***

Introduction

➢ *We will study ...*

- *the black hole solution in the effective string theory with*
    - *Gauss-Bonnet term + dilaton (higher order term)*
- *and compare them with the previously known solutions such as*
    - *Boulware-Deser solution* ($\phi \equiv 0$)
    - *the solution with the model without higher order term of the dilaton (Guo, Ohta & Torii, 2008)*

➢ *We find that ...*

- *The dilaton field affects the structure of the black hole solutions much.*
- *There is the lower limit for the mass (and the horizon radius) of the BH solution in D=4, 5.*
- *The fat singularity appears at non-zero radius in D=4, 5.*
- *The higher order term of the dilaton field does not affect the solutions so much except for the 5-dim case.*
- *For the charged solution, there is no extreme solution when the dilaton field is included.*

Introduction

➢ Previous works about stringy b.h.

【Stringy Black Holes with U(1) Gauge Field】

A) Spherically Symmetric Dilatonic and/or Axionic Black Hole

K. Maeda and G. W. Gibbons, NPB298, 741, (1988).

D. Garfinkle, G. T. Horowitz and A. Strominger, PRD43, 3140, (1991).
A. Shapere, S. Trived and F. Wilczek, Mod. PLA6, 2677, (1991).
S. B, Giddings and A. Strominger, PRD46, 627, (1992).
M. Cvetic and D. Youm, PRL75, 4165, (1995).
A. Chamorro and K. a. Virbhadra, ???.

B) Massive Dilaton, Massive Axion or Cosmological Constant

T. J. Allen, M. J. Bowick and A. Lahili, PLB237, 47, (1990).
R. Gregory and J. A. Harvey, PRD47, 2411, (1993).
S. J. Poletti and D. L. Wiltshire, PRD50, 7260, (1994).
D. L. Wiltshire, gr-qc/9502038, (1995).
S. J. Poletti, J. Twamley and D. L. Wiltshire, PRD51 5720, (1995).

C) Rotating Dilatonic and/or Axionic Black Hole

B. A. Campbell et al., PLB251, no.1, 34, (1990).

A. Sen, PRL. 69, no.7, 1006, (1992).
S. Mignemi and N. R. Stewart, PLB298, 299, (1993).
A. Garc'ia, D. Galtsov and O. Kechkin, PRL. 74, 1276, (1995).

D) Superradiance form Dilatonic Black Hole

K. Shiraishi, ???, (1992).
J. Koga and K. Maeda, WU-AP/46/95, (1995).

【Stringy Black Holes with Non-Abelian Gauge Field】

A) SU(2) Dilatonic Black Hole

G. Lavrelashvili and D. Maison, PLB295, 67, (1992).
E. E. Donets and D. V. Gal'tsov, PLB302, 411, (1993).
G. Lavrelashvili and D. Maison, NPB410, 407, (1993).
T. Torii and K. Maeda, PRD48, no.4, 1643, (1993).
Y. M. Cho et al., PLB308, no.1,2, 23, (1993).

B) SU(3) Dilatonic Black Hole

E. E. Donets and D. V. Gal'tsov, PLB312, 391, (1993).
B. Kleihaus, J, Kunz and A. Sood, PLB372, no.3,4, 204, (1996).

C) Massive Dilation and Axion

C. M. O'Neill, PRD50, no.2, 865, (1994).

【Stringy Black Holes with Higher Curvature Term】

A) analytic solutions (non-dilatonic)

D. G. Boulware and S. Deser, PRL. 55, 2656, (1985).

▪topological black holes

R. G. Cai, Phys. Rev. D 65 (2002) 084014
R. G. Cai, Y. S. Myung and Y. Z. Zhang, PRD65 (2002) 084019
R. G. Cai, PLB 582 (2004) 237

▪時空構造

T. Torii and H. Maeda, PRD71 (2005) 124002
T. Torii and H. Maeda, PRD72 (2005) 064007

B) approximated soltuions (dilatonic)

D. G. Boulware and S. Deser, PLB175, no.4, 409, (1986).
V. G. Callan, R. C. Myers and M. J. Perry, NPB311, 673,
S. Mignemi and N. R. Stewart, PRD47, no.12, 5259, (1993).

B. A. Campbell, N. Kaloper and K. A. Olive, PLB285, 199, (1992).
B. A. Campbell et al., NPB399, 137, (1993).
S. Mignemi, PRD51 no.2, 934, (1995).
M. Campanelli, C. O. Lousto and J. Audertsch, gr-qc/9412001, (1994).

C) numerical solutions (dilatonic)

E. E, Donets and D. V. Gat'tsov, PLB352, 261, (1995).
P. Kanti et al., PRD54, no.8, 5049, (1996).
S. O. Alexeyev and M. V. Pomazanov, hep-th/9605106, (1996).
T. Torii, H. Yajima and K. Maeda, PRD55, no.2, 739, (1997).
P. Kanti and K Tamvakis, PLB392, 30, (1997).

▪charged, extreme solution

C. M. Chen, D. V. Gal'tsov, D. G. Orlov, PRD75, 084030 (2007)
C. M. Chen, D. V. Gal'tsov, D. G. Orlov, PRD78, 104013 (2008)

▪higher dimensions

Z. K. Guo, N. Ohta and T. Torii, PTP. 120, 581 (2008)
Z. K. Guo, N. Ohta and T. Torii,, PTP. 121 (2009) 253

N. Ohta and T. Torii,PTP. 121 (2009) 959
N. Ohta and T. Torii, PTP. 122 (2009) 1477
N. Ohta and T. Torii, PTP. 122 (2009) 1477
N. Ohta and T. Torii, PTP. 124 (2010) 207

▪string frame

K. i. Maeda, N. Ohta and Y. Sasagawa, PRD80 (2009) 104032

▪1-Loop Effect

P. Kanti and K. Tamvakis, IOA-316/95, (1995).

Model

➢ Action

- $H = 0$, ***field redefinition***

gauge field $\qquad \gamma = 1/2 \qquad \mu = 0,\ 1$

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-\gamma\phi}F^2 + \alpha_2 e^{-\gamma\phi}\left\{R_{\rm GB}^2 + \frac{3}{16}\mu(\partial\phi)^4\right\}\right]$$

$\alpha_2 = \alpha'/8\ (> 0)$ $\qquad$ $R_{\rm GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

Regge slope parameter $\qquad$ ***GB term***

- ***The model in string frame is investigated by Maeda, Ohta & Sasagawa, 2009.***

$$\mathcal{S}_{\rm S} = \frac{1}{2\kappa_D^2}\int d^D\hat{x}\sqrt{-\hat{g}}\ e^{-2\hat{\phi}}\left[\hat{R} + 4(\hat{\nabla}\hat{\phi})^2 + \alpha_2\hat{R}_{GB}^2\right]$$

- ***This model has the same properties of the Galileon modified gravity (Deffayet, Esposito-Farese & Vilman, 2009).***

EOM & Boundary conditions

➢ Basic Equations

- $$G_{\mu\nu} - \frac{1}{2}\Big[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\Big]$$
$$+ \alpha_2 e^{-\gamma\phi}\Big[H_{\mu\nu} + 4(\gamma^2\nabla^\rho\phi\nabla^\sigma\phi - \gamma\nabla^\rho\nabla^\sigma\phi)P_{\mu\rho\nu\sigma} + \frac{3}{16}\mu\nabla_\mu\phi\nabla_\nu\phi(\nabla\phi)^2 + 8(\nabla\phi)^4\Big]$$
$$= \frac{1}{2}\Big(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\Big),$$

- $$\Box\phi + \frac{1}{4}\gamma e^{-\gamma\phi}F^2 - \alpha_2\Big[\gamma e^{-\gamma\phi}R^2_{\mathrm{GB}} + \frac{3}{4}\mu e^{-\gamma\phi}\big(\nabla_\mu\nabla_\nu\phi\nabla^\mu\phi\nabla^\mu\phi - \gamma(\nabla\phi)^4\big)\Big] = 0$$

- $$\nabla_\mu\big(\sqrt{-g}e^{-\gamma\phi}F^{\mu\nu}\big) = 0$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R,$$
$$H_{\mu\nu} \equiv 2\Big[RR_{\mu\nu} - 2R_{\mu\rho}R^\rho{}_\nu - 2R^{\rho\sigma}R_{\mu\rho\nu\sigma} + R_\mu{}^{\rho\sigma\lambda}R_{\nu\rho\sigma\lambda}\Big] - \frac{1}{2}g_{\mu\nu}R^2_{\mathrm{GB}},$$
$$P_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} + 2g_{\mu[\sigma}R_{\rho]\nu} + 2g_{\nu[\rho}R_{\sigma]\mu} + Rg_{\mu[\rho}g_{\sigma]\nu}.$$

➢ Boundary condition

- ***The existence of regular black hole horizon***
- ***Regularity of the domain of outer communications***
- ***Asymptotic flatness at spatial infinity ($\phi \to 0$)***

EOM & Boundary conditions

No. 7

- ***Static & spherically symmetric spacetime:***

$$ds_D^2 = -Be^{-2\delta}dt^2 + B^{-1}dr^2 + r^2 h_{ij}dx^i dx^j,$$

metric functions

- ***Electric charge:*** $F_{0r} = f'$

charge

$$f' = \frac{e_1}{r^{D-2}} e^{\gamma\phi-\delta},$$

- ***Scaling***

The field variables can be scaled by α_2 **:** $\tilde{r} \equiv r/\sqrt{\alpha_2}$

- ***Boulware-Deser solution:*** $\gamma = 0$

non-dilatonic

$$B = 1 - \frac{r^2}{2(D-3)_4}\left[-1 \pm \sqrt{1 + \frac{8(D-3)_4\bar{M}}{r^{D-1}} - \frac{(D-4)e_1^2}{8(D-2)r^{2(D-2)}}}\right], \qquad \delta \equiv 0, \quad \phi \equiv 0$$

EOM & Boundary conditions

No. 8

- The solutions are obtained by integrating the EOM from BH horizon to infinity numerically.

- At horizon (B=0), the equation of the dilaton field becomes "singular".

$$\phi'' = \frac{1}{B} \times (\quad)$$

"singular" at the horizons

To recover the regularity, we impose the condition:

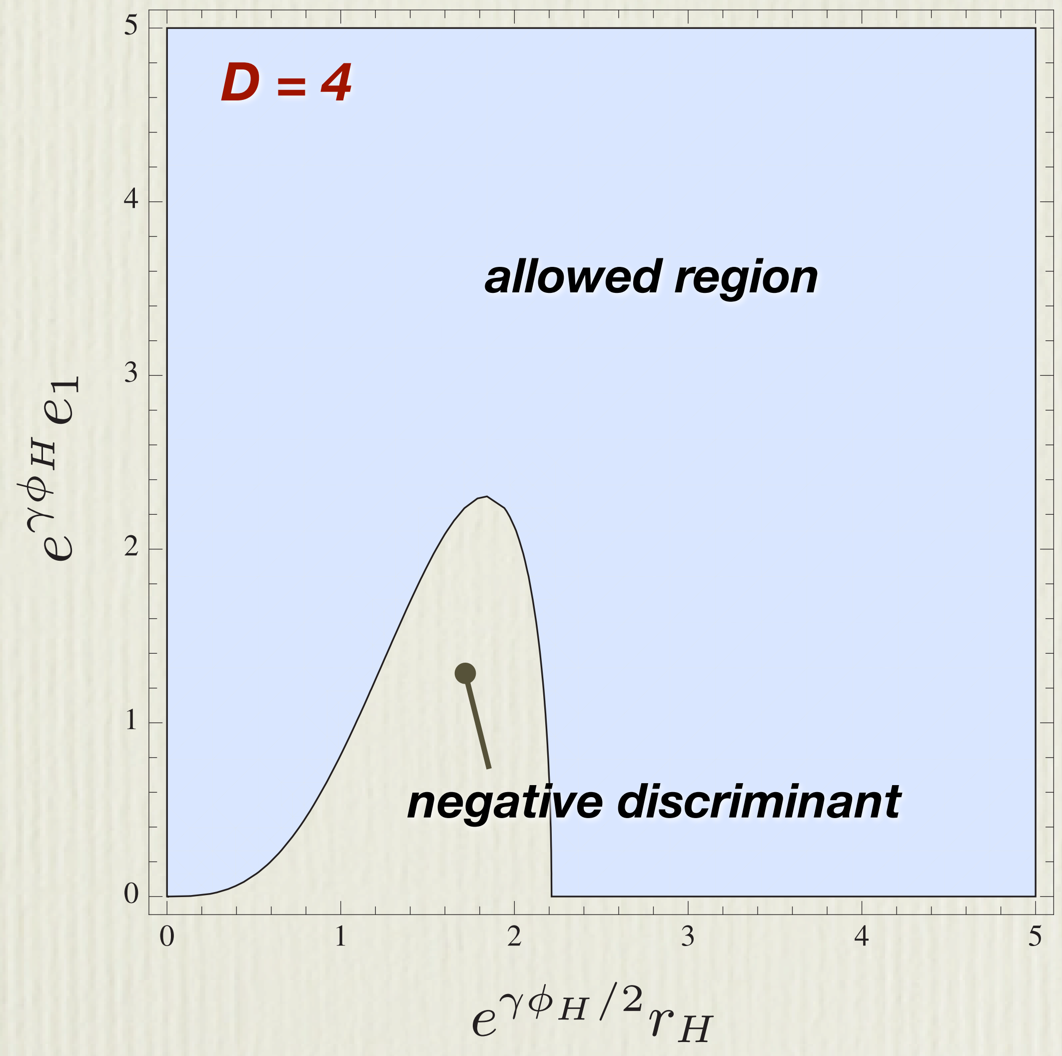
$$a\phi_H'^2 + b\phi_H' + c = 0 \quad \Rightarrow \quad \phi' = F(\phi)$$

$$\begin{aligned}
a &= C\gamma r_H^{2D+2}\Big[-2(D-3)\Big\{C\Big((D-2)\gamma^2[2C(D-3)-1]+D-4\Big)+1\Big\}e_1^2 r_H^4 \\
&\quad + C(D-2)r_H^{2D}\Big\{C(D-4)\Big(C(D-2)\gamma^2[2C(D-5)(D-3)+3D-11]+C(D-4)_5+3D-11\Big)+2(D-3)\Big\}\Big] \\
b &= \Big[-C(D-2)r_H^{4D+1}\Big\{C^2(D-4)(D-1)_2\gamma^2[(D-4)_5C^2-2C-2]+[C(D-4)_5+2(D-3)][C(D-4)+1]^2\Big\} \\
&\quad + 2(D-3)e_1^2 r_H^{2D+5}\Big\{2C(D-2)\gamma^2[2C^2(D-4)-(2D-9)C-1]+[C(D-4)+1]^2\Big\}+4C(D-3)^2e_1^4r_H^9\gamma^2\Big] \\
c &= -\frac{1}{2}C^2(D-2)^2(D-1)\gamma\Big\{C(D-4)[C(D-4)(D+1)+4]-2(D-2)\Big\}r_H^{4D}+2(D-3)^2e_1^4r_H^8\gamma \\
&\quad - 2(D-2)_3e_1^2\gamma[3(D-4)C^2+6C-1]r_H^{2D+4} = 0.
\end{aligned}$$

$$C = \frac{2(D-3)e^{-\gamma\phi_H}}{\tilde{r}_H^2}.$$

X_H : evaluated at the horizon

EOM & Boundary conditions

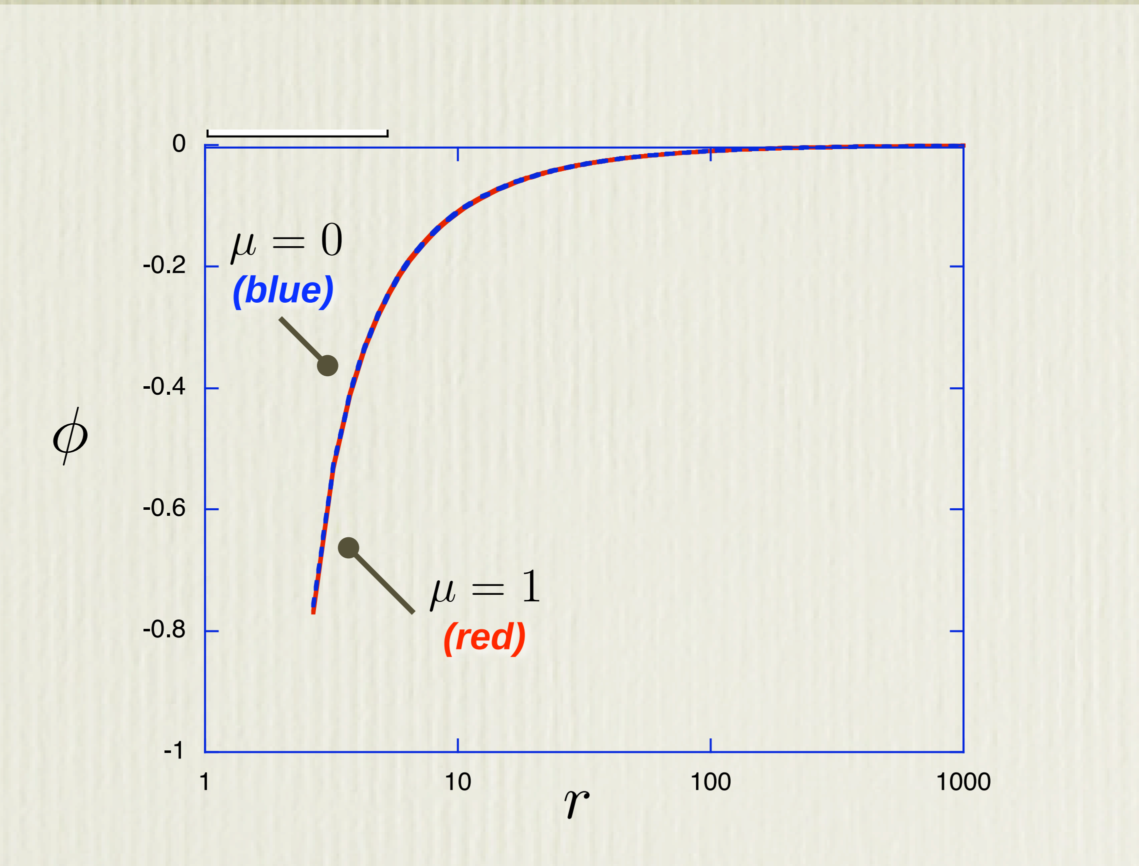


- ***D = 4*** ***: There is a region where the discriminant becomes negative and no black hole solution.***
- ***D > 4*** ***: no restriction for the parameters.***

cf. Maeda-Ohta-Sasagawa 2009

4-dimensions (neutral)

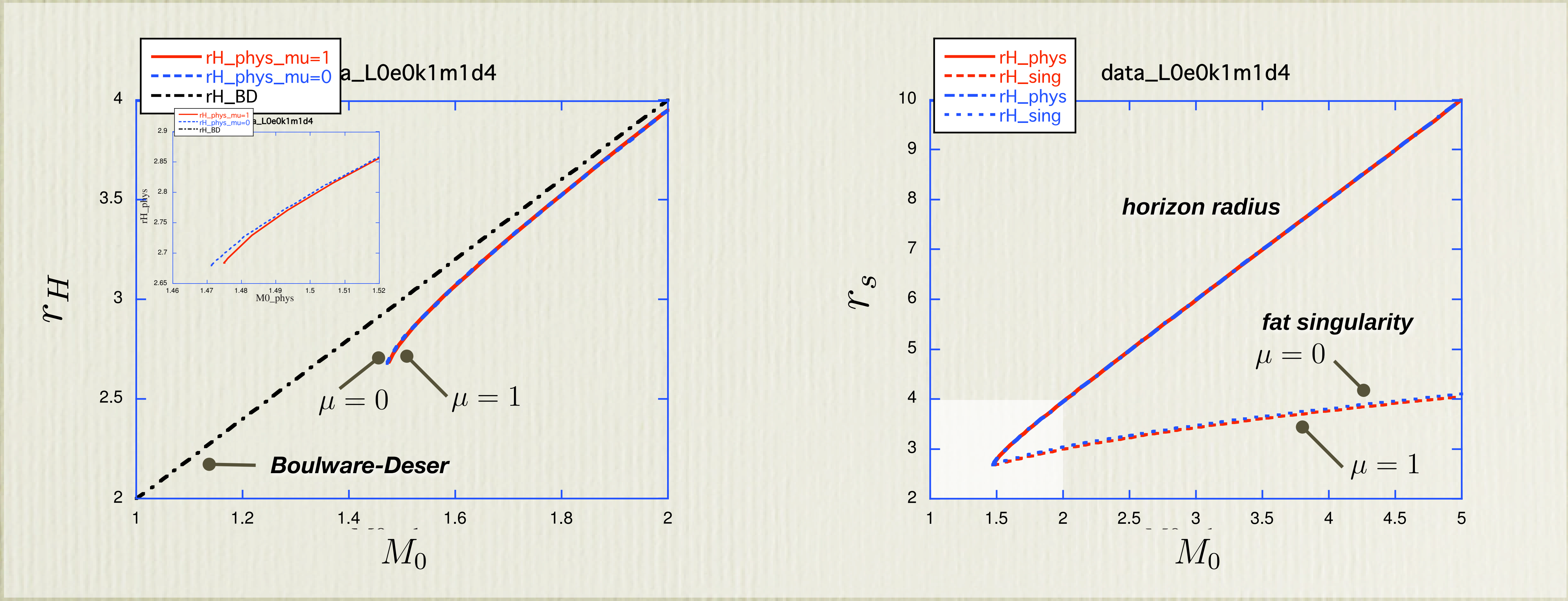
➢ ***Configuration of the dilaton field:*** ϕ



- *dilaton*場の値は小さく，傾きも緩やかになっている．

4-dimensions (neutral)

➢ Horizon radius & singularity



- *There is no black hole solution blow the critical mass (horizon radius).*

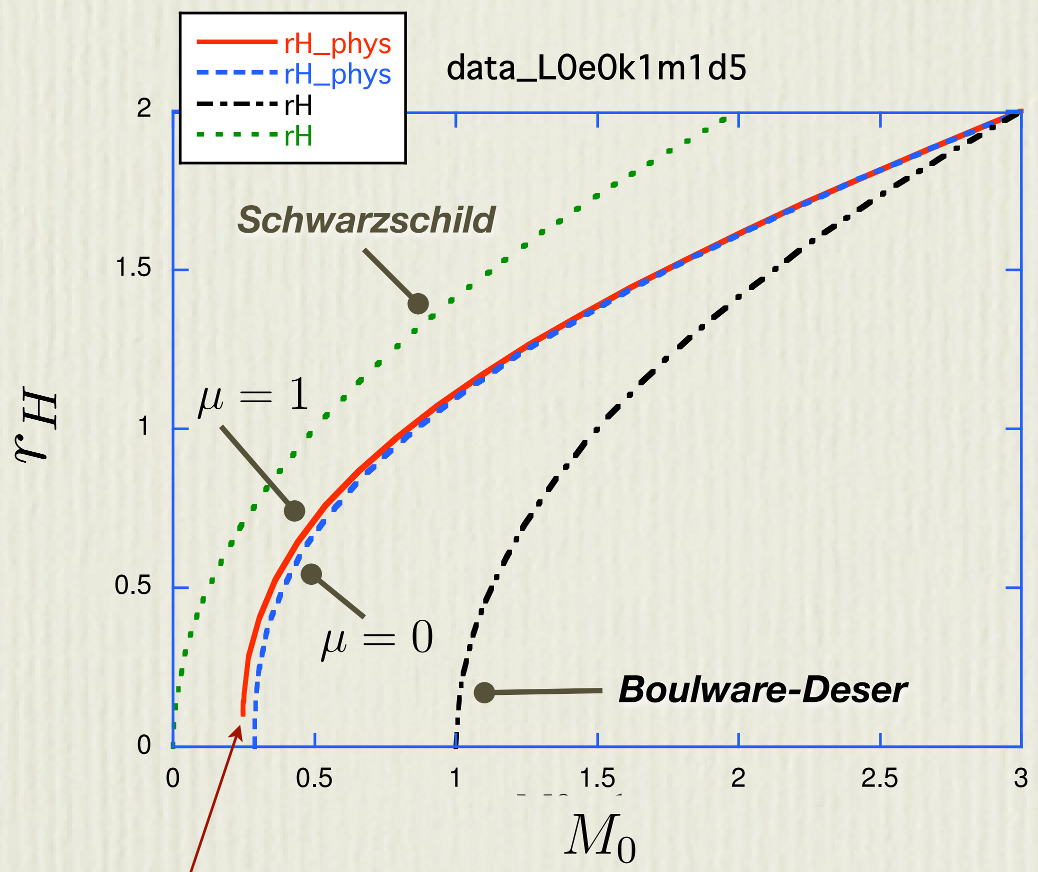
$$M_0 = 1.47132 \ (\mu = 0), \quad M_0 = 1.47502 \ (\mu = 1)$$

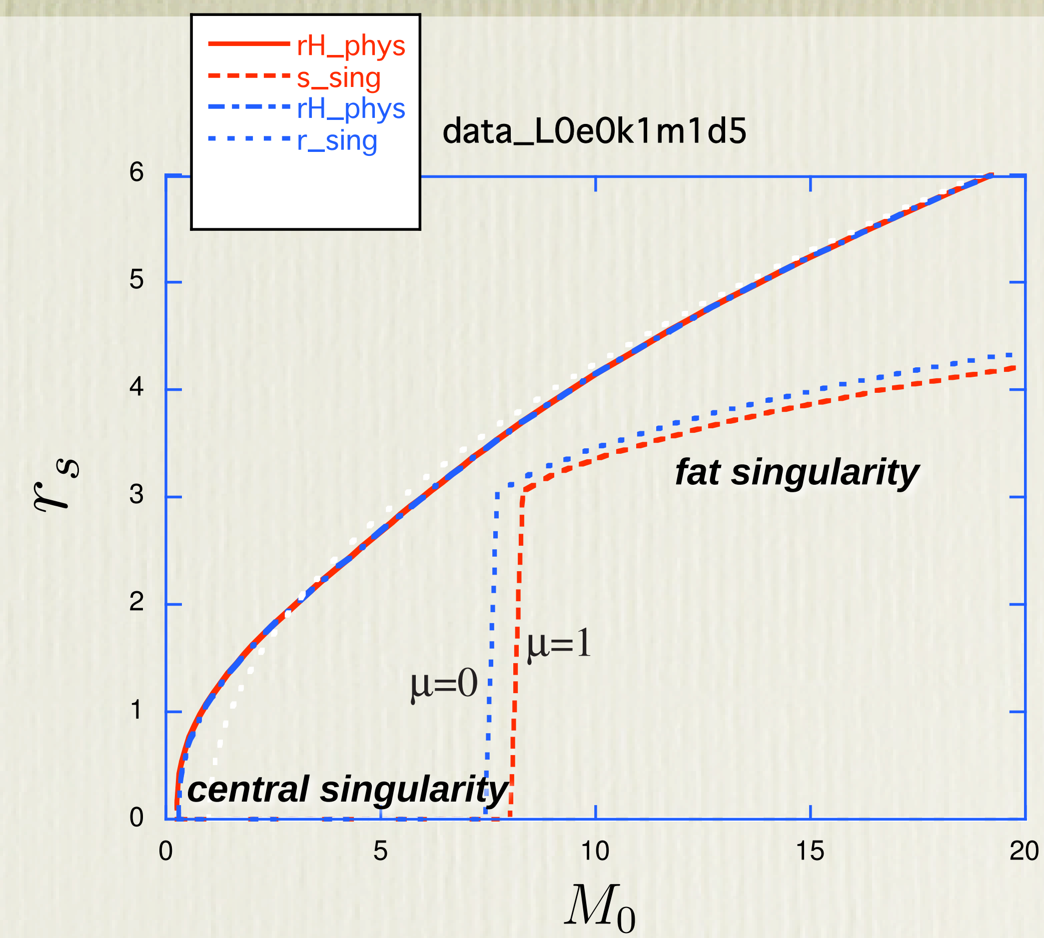
➡ *The regularity condition at the horizon is broken*

- *The fat singularity appears at non-zero radius.*

5-dimensions (neutral)

➢ ***Horizon radius & singularity***



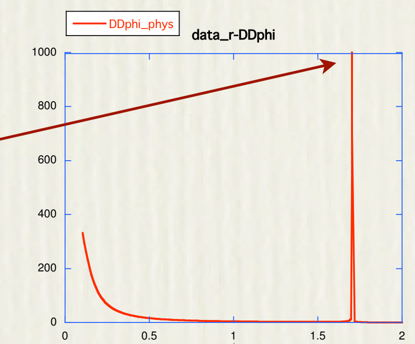


- ***There is lower limit for the horizon radius. (cf. the $\mu = 0$ case)***

$M_0 = 0.246494$, $r_H = 0.103555$

- ***For the critical solution ϕ" diverges at some radius in d.o.c.***

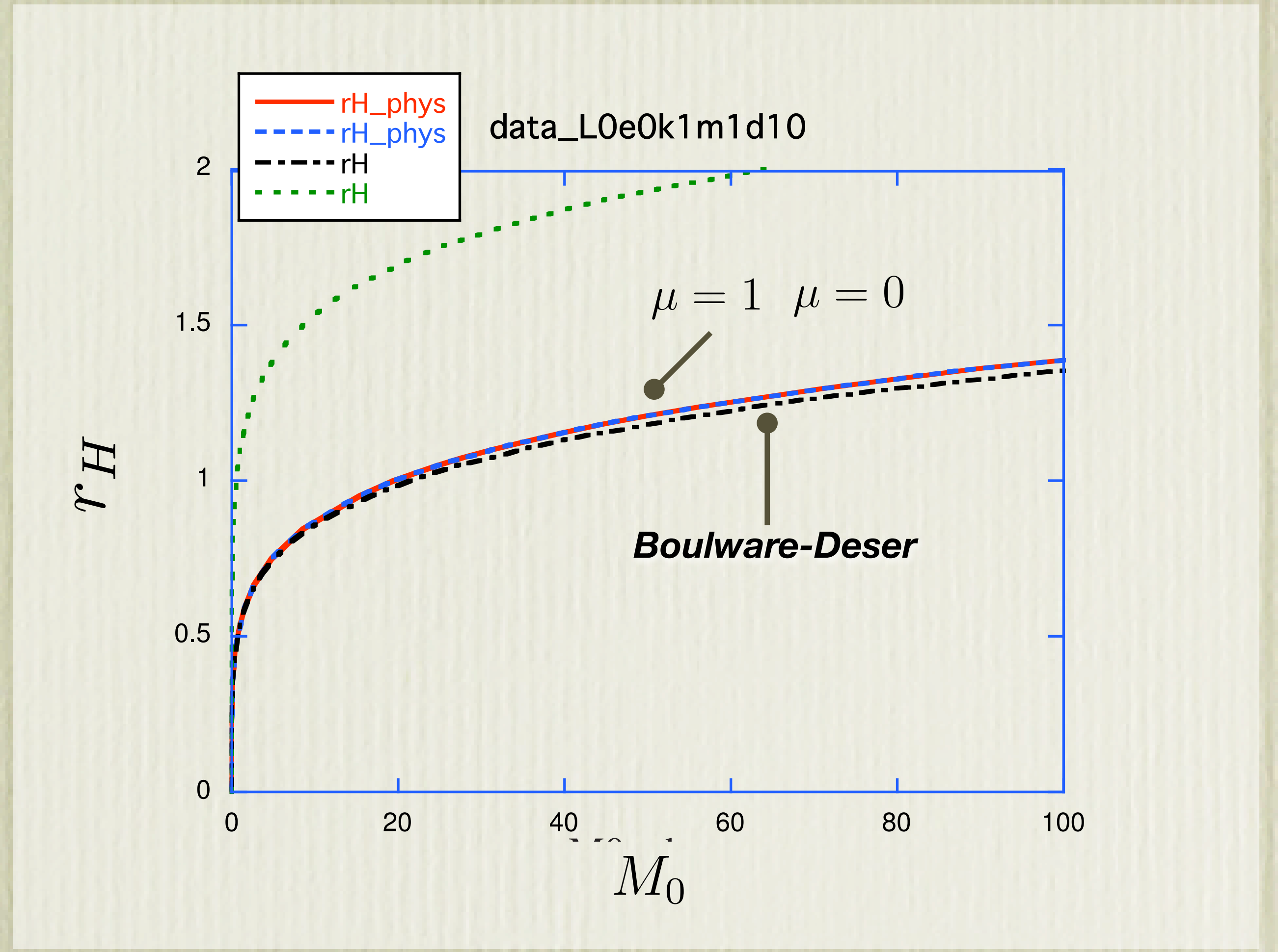
- ***The fat singularity appears for the "large" BH solution***



ϕ''-r diagram

10-dimensions (neutral)

➢ ***Horizon radius & singularity***



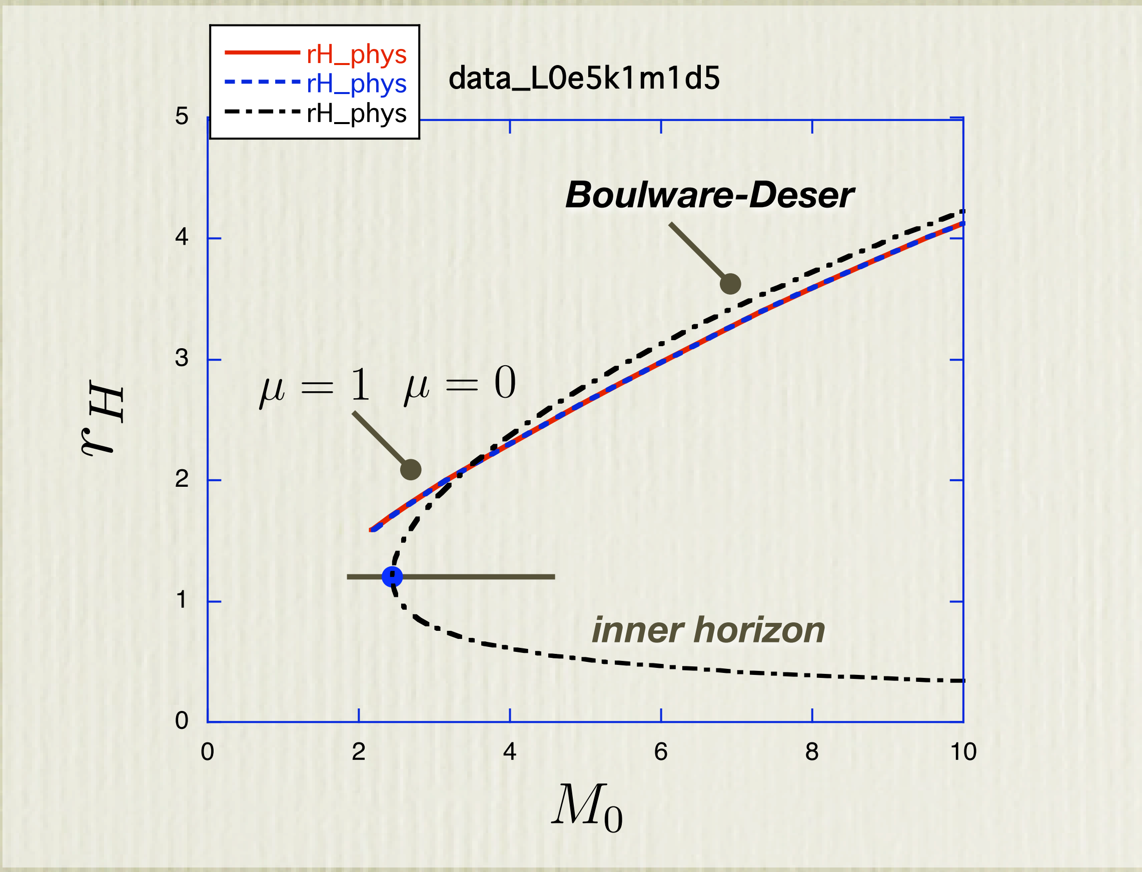
- There is a black hole solution for any mass.

- The singularity locates at the center as the non-dilatonic solutions (central singularity).

Charged Solutions (5-D)

➢ **Horizon radius**

$$D = 5, \quad e_1 = 5$$



- There is no black hole solution blow the critical mass (horizon radius).

- The critical solution is not extreme limit (cf. Boulware-Deser solution).