Clustering and Velocities of Dark Matter Halos & Primordial non-Gaussianity

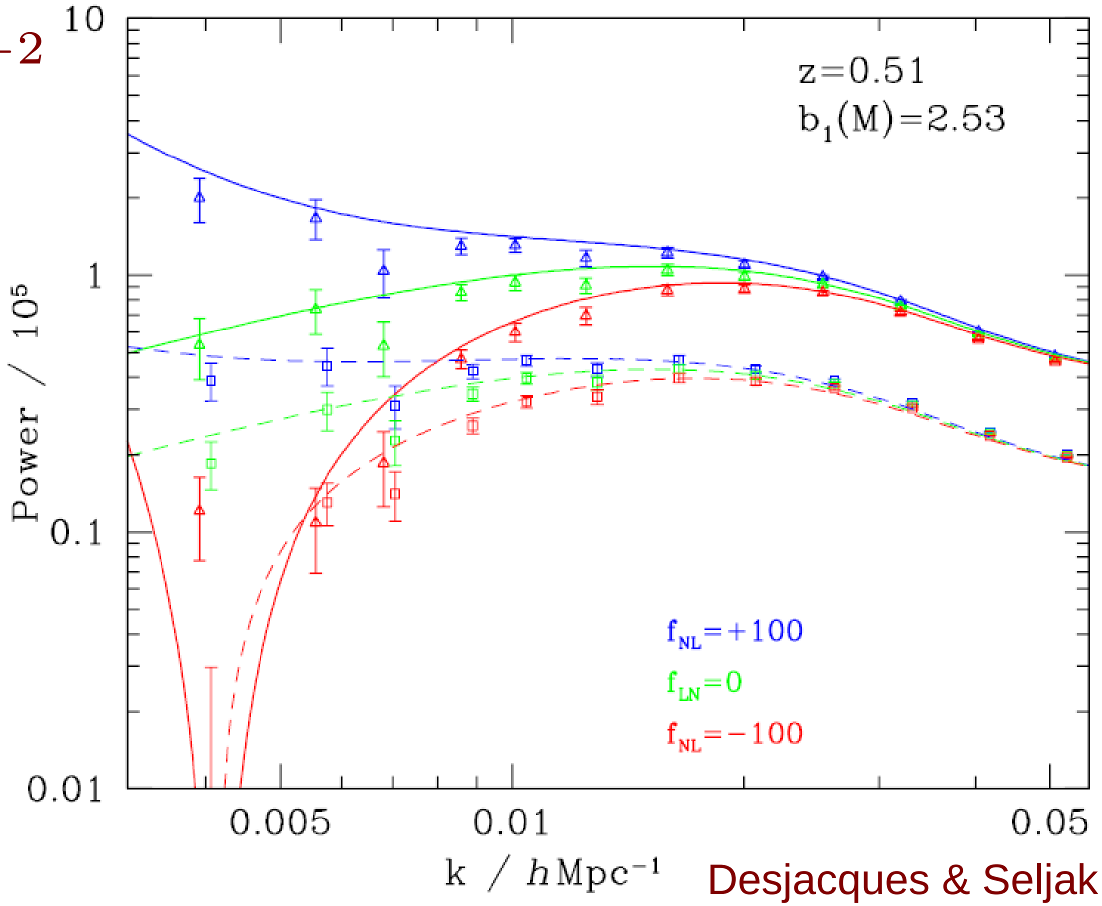
Fabian Schmidt

FS, Marc Kamionkowski, arXiv:**1008.0638**
FS, arXiv:**1005.4063**

COSMO/CosPA, 2010

Non-Gaussianity and LSS

- Large Scale Structure (LSS):
  - *Local* non-Gaussianity has unique effect on LSS tracers: $\frac{\Delta P_g(k)}{P_g(k)} \propto k^{-2}$



Desjacques & Seljak

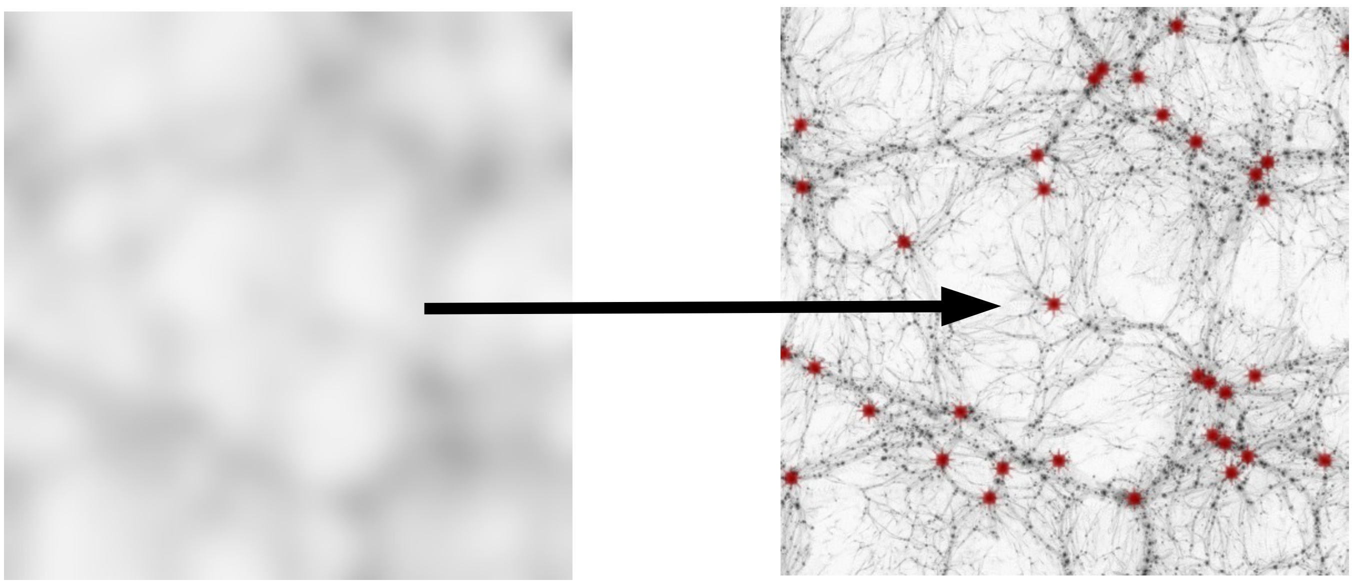
Non-Gaussianity and LSS

- Large Scale Structure (LSS):
  - *Local* non-Gaussianity has unique effect on LSS tracers: $$\frac{\Delta P_g(k)}{P_g(k)} \propto k^{-2}$$ Dalal et al 08
  - Current observational constraints using this effect already competitive with CMB Slosar et al 08

- *Motivation to study & model this effect carefully !* (also for other, non-local models of NG)

Large Scale Structure

- Statistics (clustering) of LSS tracers



(dramatization)

- Key theoretical problem:
  - how to map *initial linear fluctuations* to observed *non-linear density field* of tracer (on large scales)

Large Scale Structure

- Problem usually phrased as mapping
  - *linear matter overdensity* $\delta = \frac{\delta\rho}{\bar{\rho}}$ to *galaxy density* δ_g

$$\delta(k,z) = \mathcal{M}(k,z)T(k)\phi(k) \propto k^{-2}\,\phi(k)$$

Large Scale Structure

- Problem usually phrased as mapping
  - *linear matter overdensity* $\delta = \frac{\delta\rho}{\bar{\rho}}$ to *galaxy density* δ_g

$$\delta(k,z) = \mathcal{M}(k,z)T(k)\phi(k) \propto k^{-2}\,\phi(k)$$

- In the following, focus on *halos:*
  - collapsed, virialized dark matter structures
  - Work in *Lagrangian picture* throughout: positions in *initial* density field

Describing Halo Clustering

Two approaches to halo number density $n_h(\vec{x})$:

I. $n_h(\vec{x}) = n_h[\delta]$

Physical, non-Gaussian,
initial linear matter overdensity

-> *local biasing*

Local Biasing

- Assume halo density is *local function* of initial matter density:

Fry & Gaztanaga 93

$$n_h(\vec{x}) = F(\rho_L(\vec{x})) = \bar{n}_h \cdot \left(1 + b_1 \delta(\vec{x}) + \frac{b_2}{2}\delta^2(\vec{x}) + ...\right),$$

Bias parameters

Local Biasing

- Assume halo density is *local function* of initial matter density:

Fry & Gaztanaga 93

$$n_h(\vec{x}) = F(\rho_L(\vec{x})) = \bar{n}_h \cdot \left(1 + b_1 \delta(\vec{x}) + \frac{b_2}{2} \delta^2(\vec{x}) + ...\right),$$

Bias parameters

- Clustering of halos on large scales *(Gaussian case)*:

$$\xi_h(r) = \langle \delta_h(\vec{x}) \delta_h(\vec{x} + \vec{r}) \rangle = b_1^2 \xi_m(r) + \mathcal{O}(\delta^4)$$

$$P_h(k) = b_1^2 P_m(k) + \mathcal{O}(\delta^4)$$

Effect of Non-Gaussianity

- In principle, n_h *sensitive to all higher moments* of matter density

Matarrese et al, 1986
Verde & Matarrese 08

Effect of Non-Gaussianity

- In principle, n_h *sensitive to all higher moments* of matter density
- Halo correlation function *if δ is NG:*

Matarrese et al, 1986
Verde & Matarrese 08

$$\xi_h(r) = b_1^2 \xi_m(r) + b_1^2\, b_2\, \zeta(\vec{x}, \vec{x}, \vec{x} + \vec{r})$$

$$P_h(k) = b_1^2 P(k) + b_1^2\, b_2\, \mathcal{P}^{\delta\delta\delta}(k)$$

Bispectrum of initial density field

$$\mathcal{P}^{\delta\delta\delta}(k) = \int \frac{d^3 k_1}{(2\pi)^3} B_m(k_1, |\vec{k}_1 - \vec{k}|, k)$$

$$B_m(k_1, k_2, k_3) = \left(\prod_{i=1}^{3} \mathcal{M}(k_i) T(k_i) \right) B_\phi(k_1, k_2, k_3) \propto k_1^2 k_2^2 k_3^2 B_\phi(k_1, k_2, k_3)$$

Describing Halo Clustering

Two approaches to halo number density $n_h(\vec{x})$:

I. $n_h(\vec{x}) = n_h[\delta]$

Physical, non-Gaussian,
initial linear matter overdensity

-> *local biasing*

II. $n_h(\vec{x}) = n_h[\delta_G, \phi_G]$ via $\delta = f[\delta_G, \phi_G]$

Gaussian fields related by Poisson equation

-> *Peak-background split (PBS)*

Gaussian -> Non-Gaussian

- For general, quadratic NG:

$$\phi(\vec{x}) = \phi_G(\vec{x}) + f_{\rm NL} \int d^3\vec{y}\, d^3\vec{z}\, W(\vec{y}-\vec{x}, \vec{z}-\vec{x})\phi_G(\vec{y})\phi_G(\vec{z})$$

$$\tilde{\phi}(\vec{k}) = \tilde{\phi}_G(\vec{k}) + f_{\rm NL} \int \frac{d^3\vec{k}_1}{(2\pi)^3}\, \widetilde{W}(\vec{k}_1, \vec{k}-\vec{k}_1)\tilde{\phi}_G(\vec{k}_1)\tilde{\phi}_G(\vec{k}-\vec{k}_1)$$

Gaussian -> Non-Gaussian

- For general, quadratic NG:

$$\phi(\vec{x}) = \phi_G(\vec{x}) + f_{\rm NL} \int d^3\vec{y}\, d^3\vec{z}\, W(\vec{y}-\vec{x}, \vec{z}-\vec{x})\phi_G(\vec{y})\phi_G(\vec{z})$$

$$\tilde{\phi}(\vec{k}) = \tilde{\phi}_G(\vec{k}) + f_{\rm NL} \int \frac{d^3\vec{k}_1}{(2\pi)^3} \widetilde{W}(\vec{k}_1, \vec{k}-\vec{k}_1)\tilde{\phi}_G(\vec{k}_1)\tilde{\phi}_G(\vec{k}-\vec{k}_1)$$

  - Homogeneity and isotropy:

$$\widetilde{W}(\vec{k}_1, \vec{k}_2) = \widetilde{W}(k_1, k_2, \hat{k}_1 \cdot \hat{k}_2)$$

  - In most models, *W scale-free:* $\widetilde{W} = \widetilde{W}(k_1/k_2, \hat{k}_1 \cdot \hat{k}_2)$

Relation to Bispectrum

- $B_\phi(k_1,k_2,k_3) = 2f_{\rm NL}\widetilde{W}(\vec{k}_1,\vec{k}_2)P_\phi(k_1)P_\phi(k_2) + \text{perm.}$
  - To leading order in $f_{\rm NL}$...
  - Note: *bispectrum does not specify W uniquely*
  - One possible choice:

$$\widetilde{W}(k_1,k_2,k_3) = \frac{1}{2f_{\rm NL}}\frac{B_\phi(k_1,k_2,k_3)}{P_{\phi 1}P_{\phi 2}+P_{\phi 1}P_{\phi 3}+P_{\phi 2}P_{\phi 3}}$$

FS & M. Kamionkowski

Local Non-Gaussianity

- Simplest form of non-Gaussianity:

$$\phi(\vec{x}) = \phi_G(\vec{x}) + f_{\rm NL}\phi_G^2(\vec{x})$$

$$\Rightarrow W(\vec{y},\vec{z}) = \delta_D(\vec{y})\delta_D(\vec{z}) \quad \Leftrightarrow \quad \widetilde{W}(\vec{k}_1,\vec{k}_2) = 1$$

Peak-Background Split (PBS)

- Perturbations of *Gaussian* fields written as:

$$\delta = \delta_l + \delta_s,\ \ \phi = \phi_l + \phi_s,\ \ \ldots$$

- Definition of bias:

$$b_1 = \frac{\partial \ln n_h}{\partial \delta_l} - 1$$

Lagrangian bias

Peak-Background Split (PBS)

- Perturbations of *Gaussian* fields written as:

$$\delta = \delta_l + \delta_s,\ \ \phi = \phi_l + \phi_s,\ \ \ldots$$

- Definition of bias:

$$b_1 = \frac{\partial \ln n_h}{\partial \delta_l} - 1$$

Lagrangian bias

- $n_h = n_h(\rho, P(k_s))$ encapsulates physics of halo formation
  - Usually, $n_h = n_h(\rho, \sigma_R) = n_h(\rho, \nu)$

e.g., Slosar et al 08

$$\nu = \delta_c / \sigma_R$$

Gaussian Halo Bias in PBS

- Large-scale δ changes collapse threshold:

$$\delta_c \to \delta_c - \delta_l \quad \Rightarrow \quad b_1 = -\frac{1}{\sigma_R}\frac{\partial \ln n_h}{\partial \nu}$$

Mo & White 96

- In the Gaussian case, δ_l does not affect $P(k_s)$

Non-Gaussianity in PBS

FS & M. Kamionkowski

- Non-Gaussianity couples "s" and "l" pieces:

$$\hat{\delta}_s(\vec{x}) = \delta_s(\vec{x}) + 2 f_{\rm NL} \int d^3x \int d^3y \, W_0(\vec{y}, \vec{z}) \left\{ \phi_l \delta_s + \phi_s \delta_l + \phi_s \delta_s \right\}$$

Fictitious Gaussian fields

  - From Poisson equation *(T(k) = 1)*
  - High-δ peaks assumed: $(\vec{\nabla}\phi)^2$ terms neglected

Non-Gaussianity in PBS

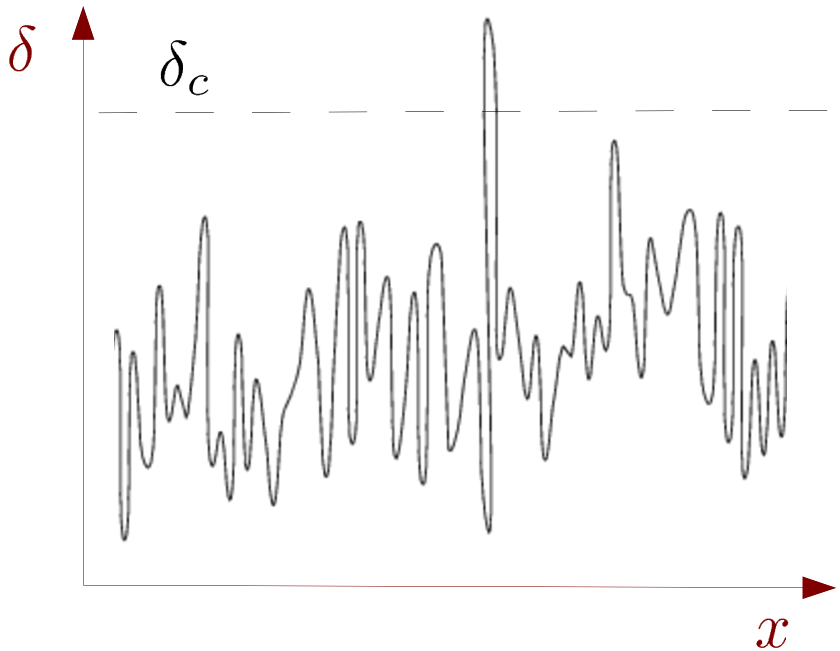
FS & M. Kamionkowski

- With NG, δ_l has influence on $P(k_s)$:

$$\frac{\Delta P(k_s)}{P(k_s)} = 4 f_{\rm NL} \int \frac{d^3 k_1}{(2\pi)^3} \widetilde{W}_0(\vec{k}_1, \vec{k}_s - \vec{k}_1) \phi_l(\vec{k}_1)$$

$k_s \gg k_1$

Squeezed limit

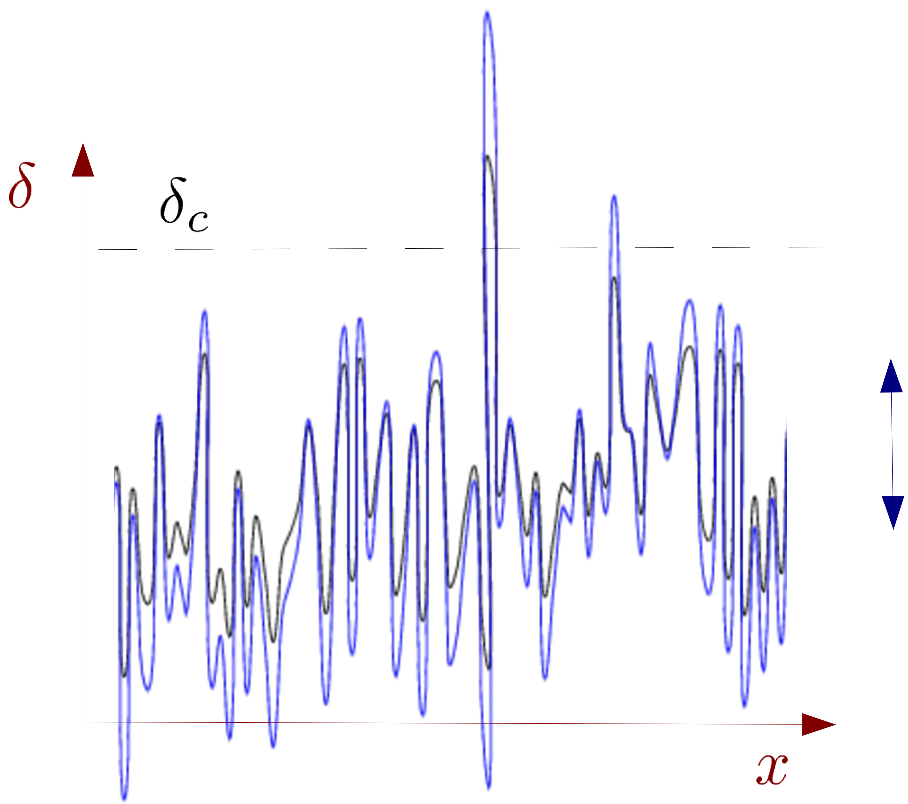


Non-Gaussianity in PBS

FS & M. Kamionkowski

- With NG, δ_l has influence on $P(k_s)$:

$$\frac{\Delta P(k_s)}{P(k_s)} = 4 f_{\rm NL} \int \frac{d^3 k_1}{(2\pi)^3} \widetilde{W}_0(\vec{k}_1, \vec{k}_s - \vec{k}_1) \phi_l(\vec{k}_1)$$



$k_s \gg k_1$

Squeezed limit

Non-Gaussianity in PBS

FS & M. Kamionkowski

- With NG, δ_l has influence on $P(k_s)$:

$$\frac{\Delta P(k_s)}{P(k_s)} = 4f_{\rm NL} \int \frac{d^3k_1}{(2\pi)^3} \widetilde{W}_0(\vec{k}_1, \vec{k}_s - \vec{k}_1)\phi_l(\vec{k}_1)$$

- Non-Gaussian halo bias:

$$b_1 = \frac{\partial \ln n_h}{\partial \delta_l} - 1$$

Gaussian δ !

$$\Delta b_1(k) = 2f_{\rm NL}\mathcal{M}^{-1}(k)\frac{\partial \ln n}{\partial \ln \sigma_R}\frac{\sigma_W^2(k)}{\sigma_R^2},$$

$$\frac{\partial \phi_l}{\partial \delta_l} \propto k^{-2}$$

$$\sigma_W^2(k) \equiv \int \frac{d^3k_s}{(2\pi)^3} P(k_s) F_R^2(k_s) \widetilde{W}_0(\vec{k}, \vec{k}_s - \vec{k})$$

Smoothing kernel (e.g. F.T. of tophat)

Non-Gaussianity in PBS

FS & M. Kamionkowski

- With NG, δ_l has influence on $P(k_s)$:

$$\frac{\Delta P(k_s)}{P(k_s)} = 4 f_{\rm NL} \int \frac{d^3 k_1}{(2\pi)^3} \widetilde{W}_0(\vec{k}_1, \vec{k}_s - \vec{k}_1) \phi_l(\vec{k}_1)$$

- Non-Gaussian halo bias:

$$b_1 = \frac{\partial \ln n_h}{\partial \delta_l} - 1$$

Gaussian δ !

$$\Delta b_1(k) = 2 f_{\rm NL} \mathcal{M}^{-1}(k) \frac{\partial \ln n}{\partial \ln \sigma_R} \frac{\sigma_W^2(k)}{\sigma_R^2},$$

$\frac{\partial \ln n}{\partial \ln \sigma_R} = b_1\, \delta_c$

$$\sigma_W^2(k) \equiv \int \frac{d^3 k_s}{(2\pi)^3} P(k_s) F_R^2(k_s) \widetilde{W}_0(\vec{k}, \vec{k}_s - \vec{k})$$

Smoothing kernel (e.g. F.T. of tophat)

Scale-dependent Bias

FS & M. Kamionkowski

- Non-Gaussian halo bias:

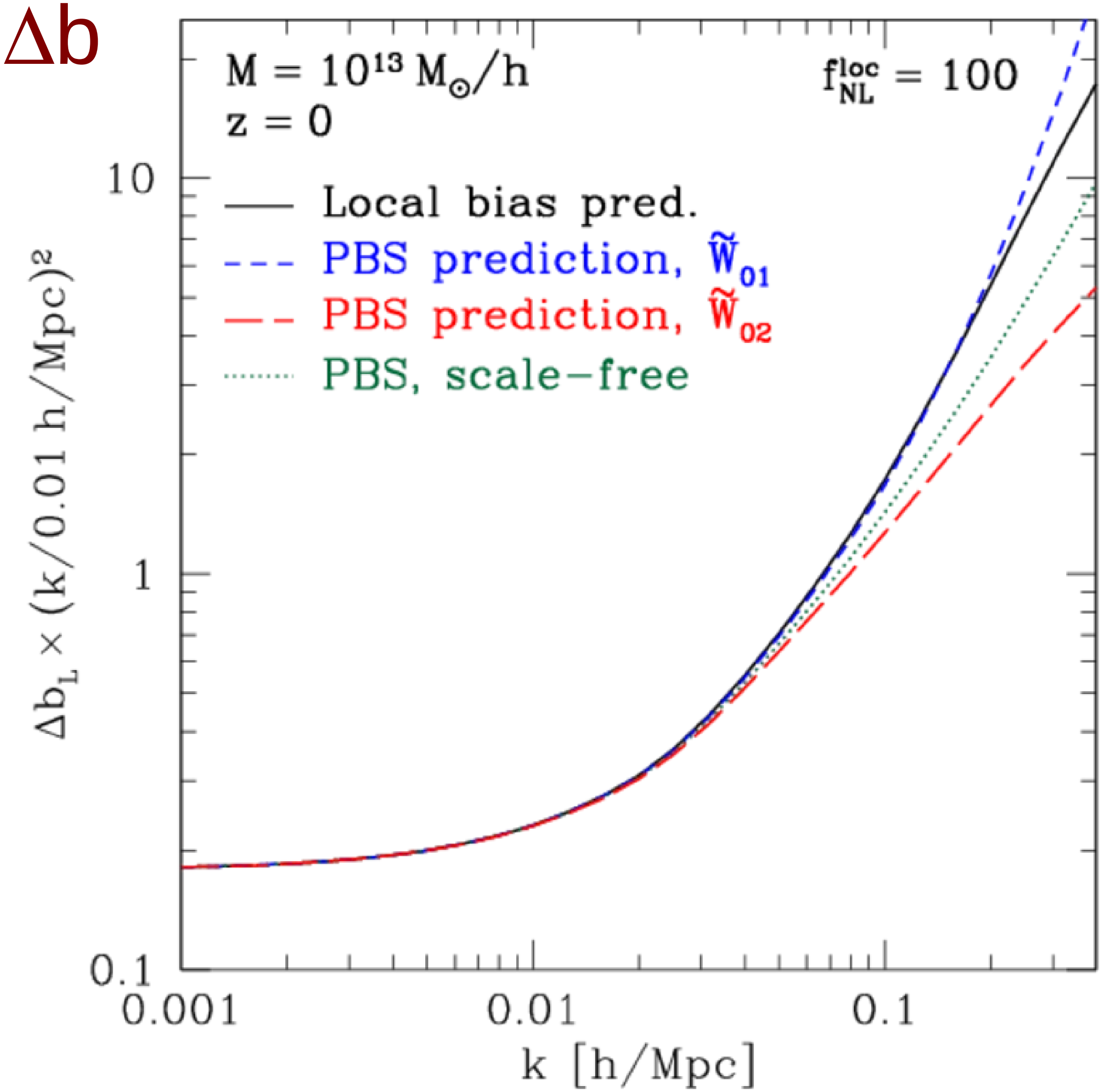
$$\Delta b_1(k) = 2 f_{\rm NL} \mathcal{M}^{-1}(k) \frac{\partial \ln n}{\partial \ln \sigma_R} \frac{\sigma_W^2(k)}{\sigma_R^2}$$

- Recall that $P_h(k) = b_1^2 P(k)$
  - Change in halo power spectrum: $\dfrac{\Delta P_h(k)}{P_h(k)} = 2 \dfrac{\Delta b(k)}{b_1}$

Quantitative Results

FS & M. Kamionkowski

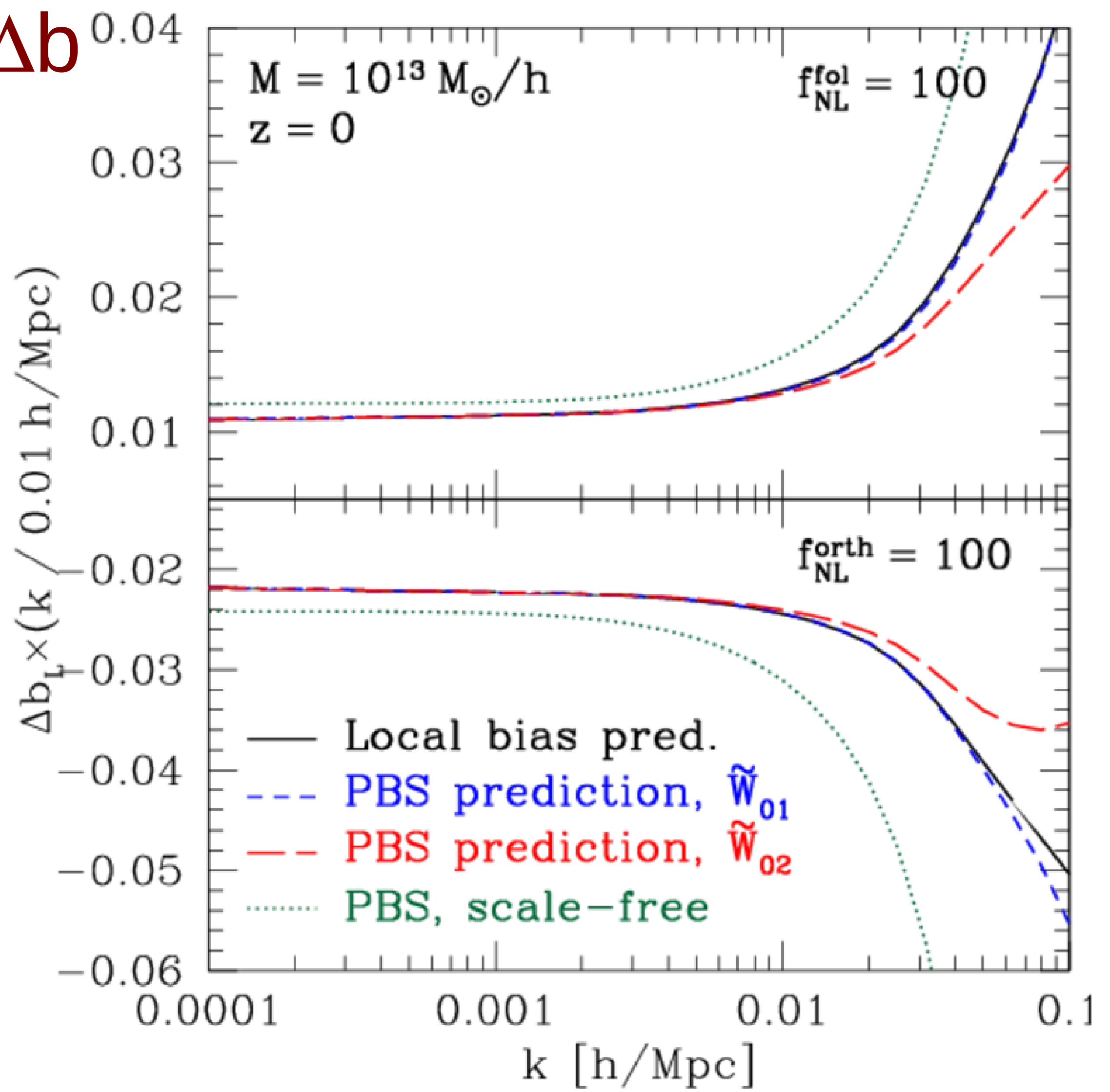
- NG bias correction Δb as function of scale
  - *Local* model
  - $\widetilde{W}_0 \to 1 \Rightarrow \Delta b_1 \propto k^{-2}$
  - Scaled by k^2



Quantitative Results

FS & M. Kamionkowski

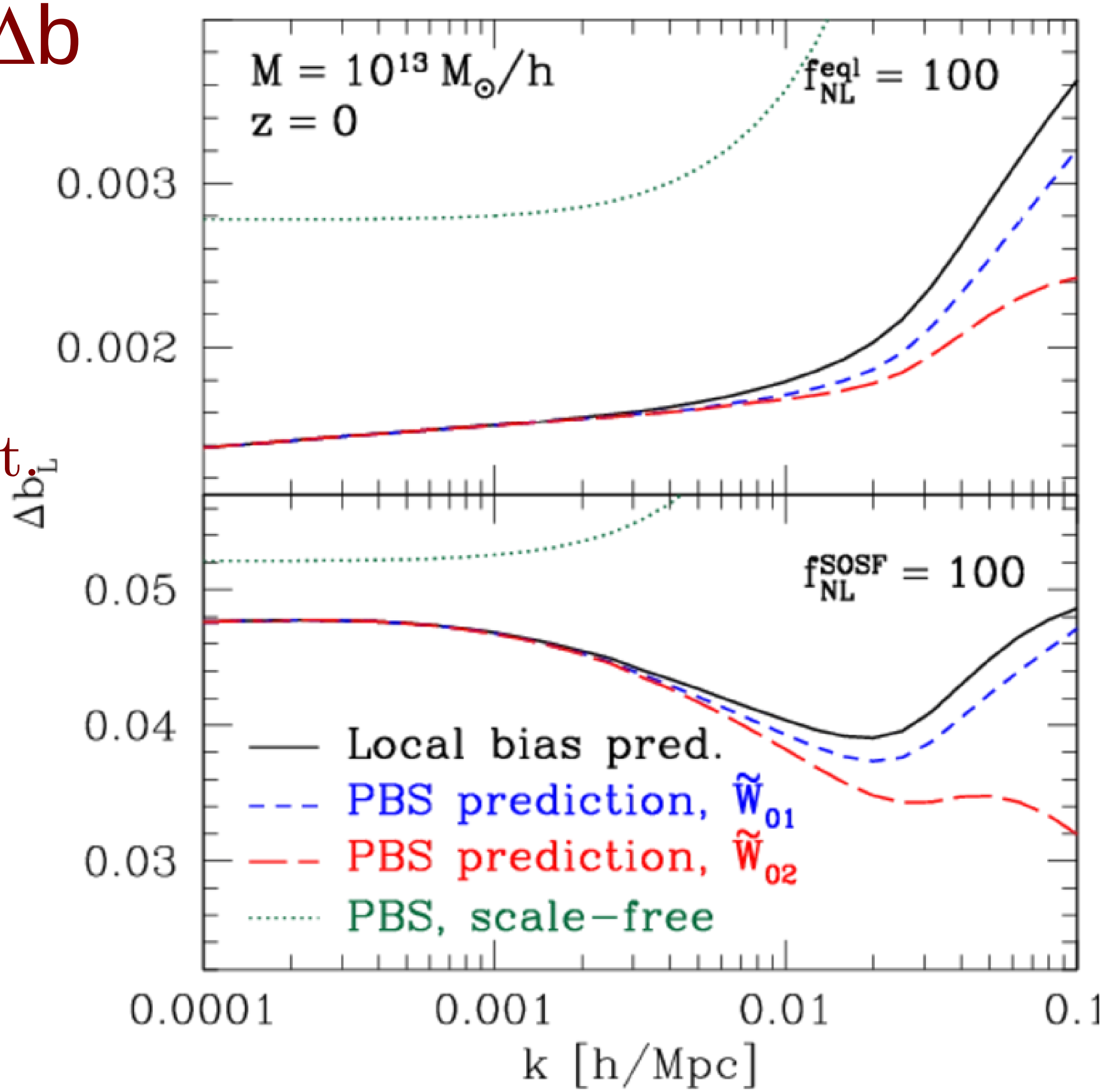
- NG bias correction Δb as function of scale
  - *Folded / orthogonal* models
  - $\widetilde{W}_0 \propto k \Rightarrow \Delta b_1 \propto k^{-1}$
  - Scaled by k



Quantitative Results

FS & M. Kamionkowski

- NG bias correction Δb as function of scale
  - *Equilateral / SOSF* models
  - $\widetilde{W}_0 \propto k^2 \Rightarrow \Delta b_1 \approx \text{const.}$



Choice of Kernel

- Bispectrum does not uniquely specify kernel
  - For any function $g(k_1, k_2, k_3)$, can construct kernel producing any given bispectrum

- However, *squeezed limit* is unique:

$$\widetilde{W}(\vec{k}_1, \vec{k}_2) \overset{k_2 \gg k_1}{=} \frac{B_\phi(k_1, k_2, k_3)}{2 f_{\rm NL} P_\phi(k_1) P_\phi(k_2)} \left[1 + \mathcal{O}(k_1/k_2)\right]$$

(assuming kernel is non-singular for all $\vec{k}_1, \vec{k}_2$)

PBS vs Local Biasing

- $\Delta b(k)$ from LB equivalent to PBS prediction - *in large-scale limit*
  - Geometric correction factors going as $1 + \mathcal{O}(k/k_1)$
  - Consequence of separation of “l” and “s” scales

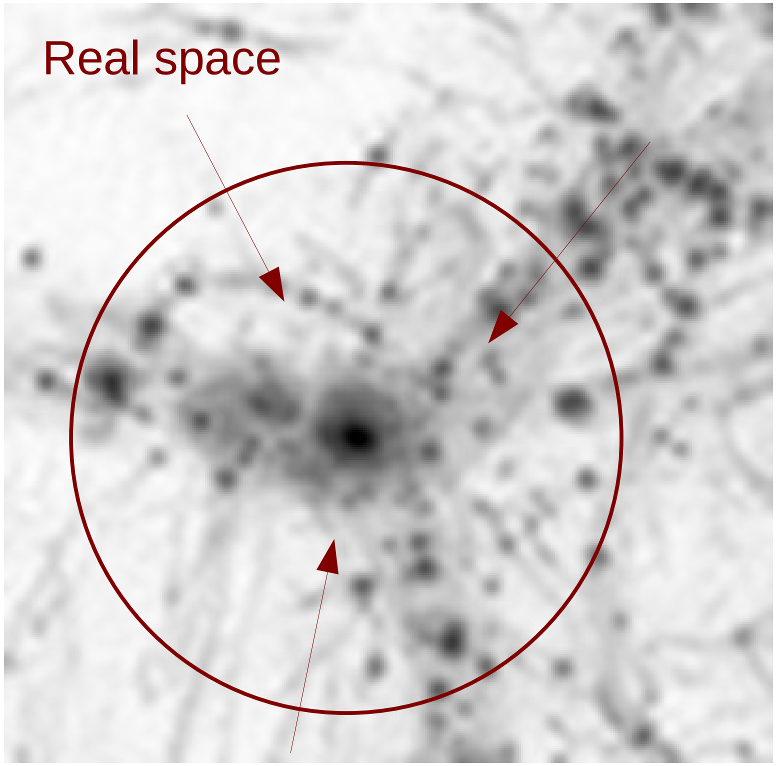
PBS vs Local Biasing

- $\Delta b(k)$ from LB equivalent to PBS prediction - *in large-scale limit*
  - Geometric correction factors going as $1+\mathcal{O}(k/k_1)$
  - Consequence of separation of “l” and “s” scales

- Agreement in amplitude restricted to *high-peak limit* of Press-Schechter theory
  - *PBS:* effect scales with $\frac{\partial \ln n_h}{\partial \ln \sigma_R} = b_1 \delta_c$
  - *LB:* determined by b_2

II. Halo Velocities and NG

- Future redshift surveys will be extremely sensitive probes of NG
- Positions of objects measured in *redshift space*

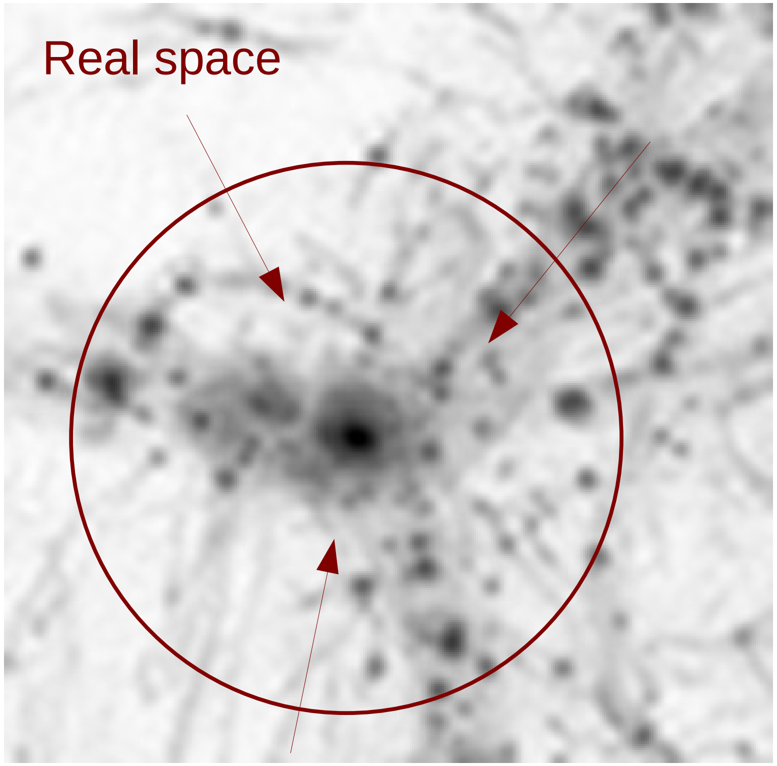


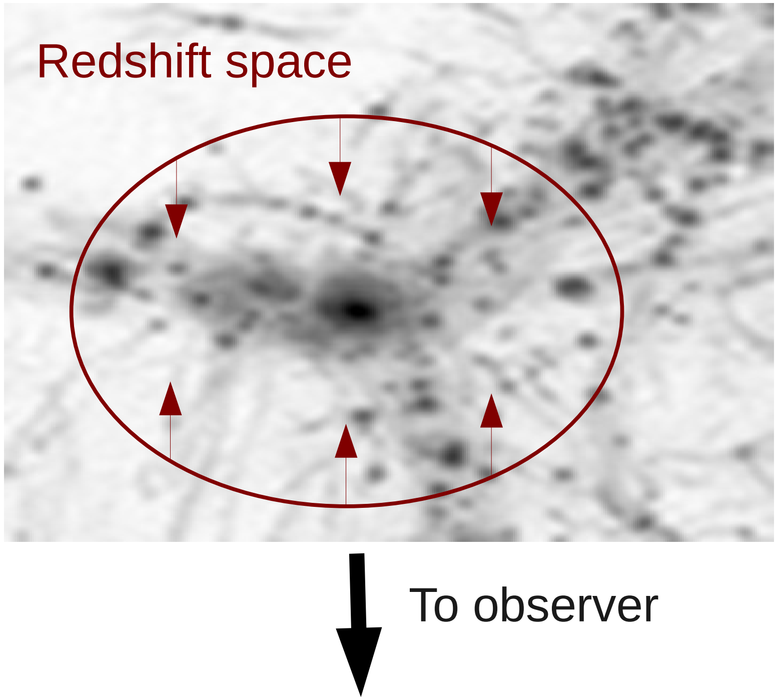
$$\vec{s} = \vec{x} + \frac{v_z}{aH}\hat{z}$$

II. Halo Velocities and NG

- Future redshift surveys will be extremely sensitive probes of NG
- Positions of objects measured in *redshift space*

$$\vec{s} = \vec{x} + \frac{v_z}{aH}\hat{z}$$





Halo Velocities and NG

- Well-known formula for large scales:

$$P_{g,s}(k,\mu) = \left(1 + \frac{f}{b_1}\mu^2\right)^2 P_g(k), \quad f = d\ln D/d\ln a$$

Kaiser, 1987

k_z/k

  - Derived assuming local biasing and Gaussian IC

- What happens to halo velocities in the presence of NG ?

Velocity Difference PDF

FS, 2010

- Evaluate PDF of δu, assuming halos at $\vec{r}_1,\ \vec{r}_2$
  - using moments theorem from statistical field theory
  - Equivalent to Edgeworth expansion

Grinstein & Wise, 1984
Matarrese et al, 1986

- Result: halo velocities still *statistically unbiased* on large scales

Velocity Difference PDF

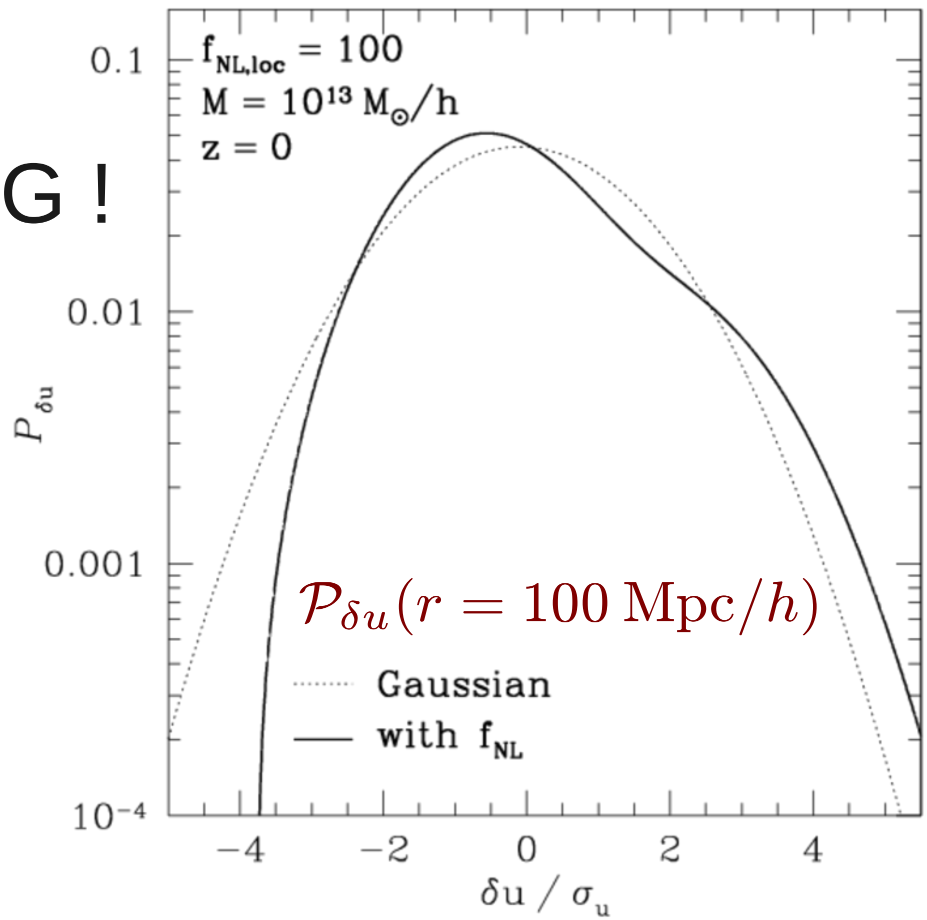
FS, 2010

- Evaluate PDF of δu, assuming halos at $\vec{r}_1,\ \vec{r}_2$

See also Lam et al, 2010

- PDF of δu sensitive to NG !

Catelan & Scherrer 95

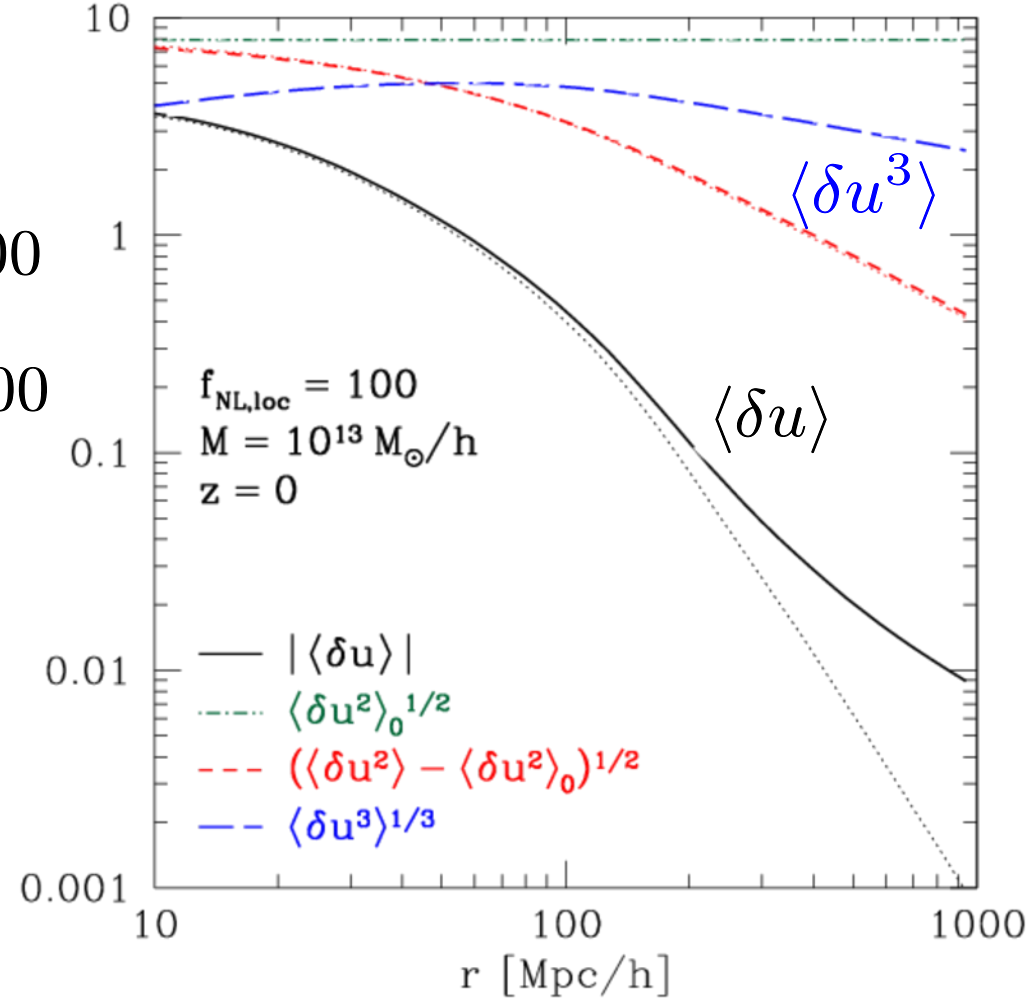


Velocity Difference PDF

- Velocity moments as function of scale
  - Significant departure from Gaussianity:

$$\frac{\langle \delta u^3 \rangle}{\sigma_u^3} \sim 0.2 \quad \text{for} \quad f_{\rm NL}^{\rm loc} = 100$$

$$\text{or} \quad f_{\rm NL}^{\rm eql} = 300$$



Redshift-space Halo Clustering

FS, 2010

- Result:

$$P_{s,h}(k,\mu) = \left(1 + \frac{f\mu^2}{b_1 + \Delta b_1}\right)^2 (b_1 + \Delta b_1)^2 P(k) + P_s^{\rm NG}(k,\mu)$$

Non-Gaussian bias correction

  - *Terms missed* by generalized Kaiser formula:

$$P_s^{\rm NG}(k,\mu) = -2k\mathcal{P}^{\delta u\delta}(k)\left[f\mu^2 b_1^2 + f^2\mu^4 b_1\right] + k^2\mathcal{P}^{uu\delta}(k)\left[f^2\mu^4 b_1 + f^3\mu^6\right]$$

PS in high-peak limit assumed

Redshift-space Halo Clustering

FS, 2010

- Result:

$$P_{s,h}(k,\mu) = \left(1 + \frac{f\mu^2}{b_1 + \Delta b_1}\right)^2 (b_1 + \Delta b_1)^2 P(k) + P_s^{\rm NG}(k,\mu)$$

Non-Gaussian bias correction

- *Terms missed* by generalized Kaiser formula:

$$P_s^{\rm NG}(k,\mu) = -2k\mathcal{P}^{\delta u\delta}(k)\left[f\mu^2 b_1^2 + f^2\mu^4 b_1\right] + k^2\mathcal{P}^{uu\delta}(k)\left[f^2\mu^4 b_1 + f^3\mu^6\right]$$

PS in high-peak limit assumed

From velocity 3-pt function
- there even for *unbiased tracers* !

Redshift-space Halo Clustering

FS, 2010

- Magnitude of *beyond-Kaiser* terms
  - Small due to weighting with powers of *k*
  - %-level at *k*~0.1
  - *Contaminant for z-surveys ?*
  - Expected to be more important for bispectrum