COSMO/COSPA 2010

A spherical collapse model with massive neutrinos

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K. Ichiki and M. Takada, to be submitted

Cosmic Neutrinos

- Thermal Relic From the Big Bang (not directly observed yet)
  - Fermi distribution: $f = \dfrac{1}{1 + e^{E/kT_\nu}}$
  - Present density (per flavor) $n_\nu = 113\mathrm{cm}^{-3}$

Small mass (sub-eV) makes significant contribution to the total mass density of the universe

- Neutrino oscillation experiments
  - At least two flavors of neutrino are non-relativistic at present universe

$$\sum m_\nu \gtrsim 0.056(0.095)\mathrm{eV} \quad \longrightarrow \quad \Omega_\nu \gtrsim 1.1 \times 10^{-3} \left(\frac{h}{0.72}\right)^{-2}$$

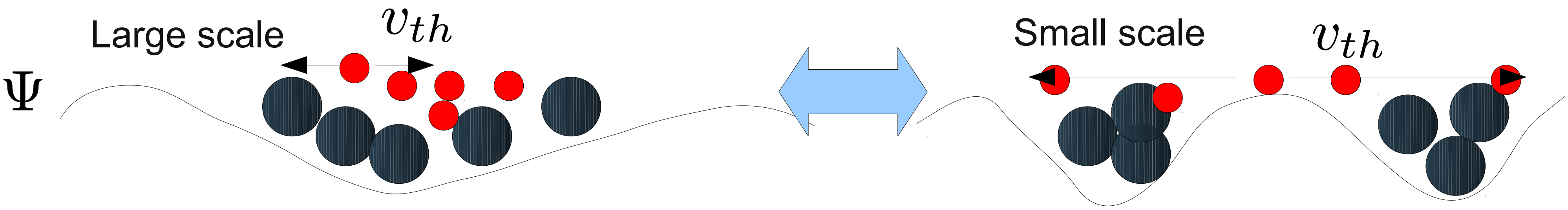
Neutrinos in Cosmology
-- free streaming effect --

- Small mass → Large thermal (random) velocity

$$v_{\rm th} = \frac{<p>}{m} = \frac{3T_\nu}{m} \sim 150 \frac{a_0}{a} \left(\frac{1~{\rm eV}}{m} \right) ~{\rm km/s}$$

- Neutrinos do not clump below free streaming scale
  - Maximum free streaming scale

$$\lambda_{\rm fs} \lesssim \lambda_{\rm nr} = 350 \sqrt{\Omega_m} \left(\frac{m}{1{\rm eV}} \right)^{1/2} ~ h^{-1}~{\rm Mpc}$$

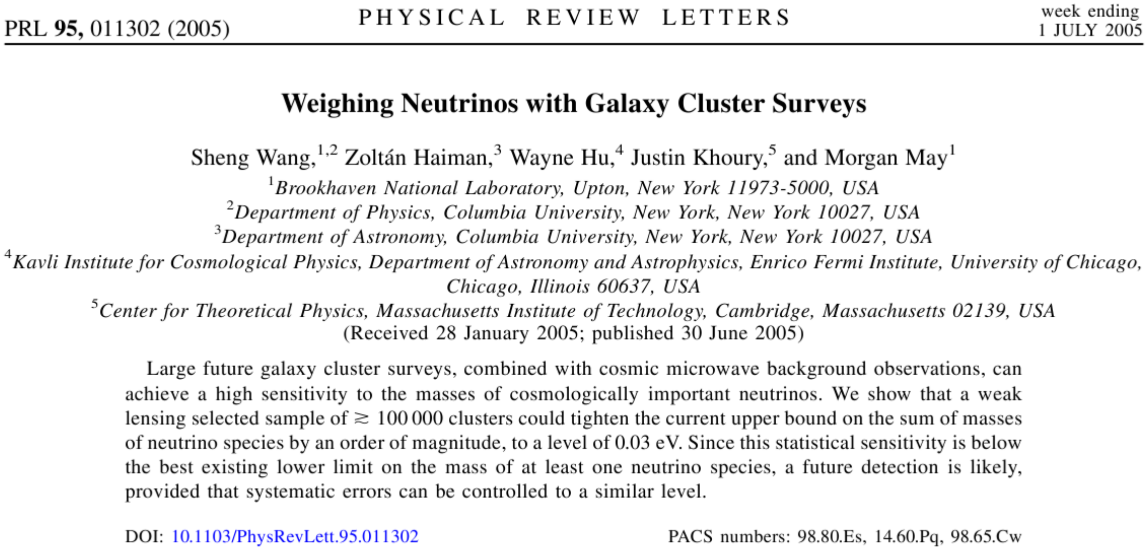


Both CDM and neutrinos can cluster

Only CDM can cluster
Slower gravitaional collapse

Purpose of this work

- Constraining neutrino masses from cosmology, in particular, from galaxy clusters (cluster number counts, SZ effect, ...)
- Need to include effects of neutrino masses on the non-linear structure formation, which has been neglected in the literature.
- Consider the spherical collapse model, including massive neutrinos, and apply it to the halo mass function



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Weighing Neutrinos with Galaxy Cluster Surveys

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Large future galaxy cluster surveys, combined with cosmic microwave background observations, can achieve a high sensitivity to the masses of cosmologically important neutrinos. We show that a weak lensing selected sample of $\gtrsim$ 100 000 clusters could tighten the current upper bound on the sum of masses of neutrino species by an order of magnitude, to a level of 0.03 eV. Since this statistical sensitivity is below the best existing lower limit on the mass of at least one neutrino species, a future detection is likely, provided that systematic errors can be controlled to a similar level.

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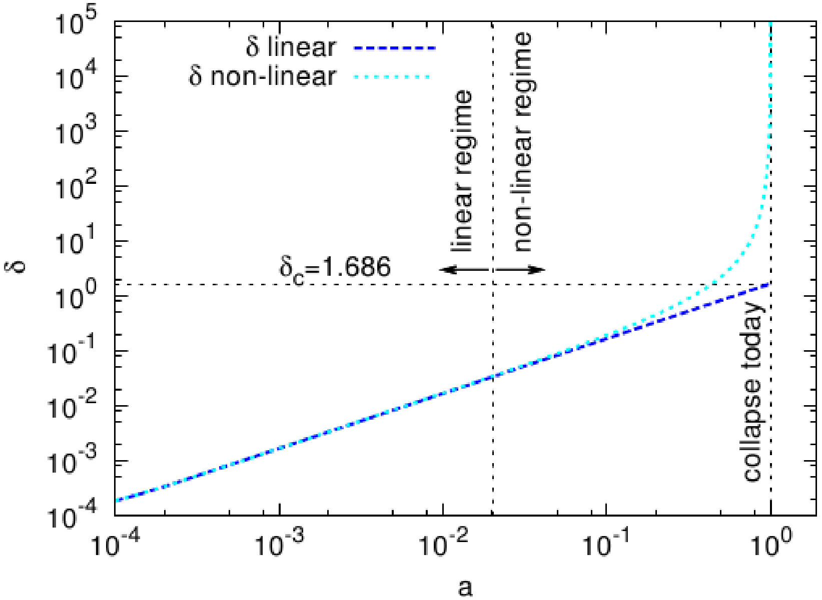
Spherical Collapse model(1)

- Consider a overdense region with a top-hat profile
  - It expands slower than the background, reaches a maximum radius, and collapses.
  - much easier than N-body simulation
  - important applications (e.g., halo mass function)
  - An important quantity:
    - critical overdensity

$$\delta_c \approx 1.686$$

- One-to-one correspondance between Initial δ to the collapse time
- Number of halo can be predicted using linear theory and δc



Taken from Bartelmann et al.

Spherical collapse model(2)

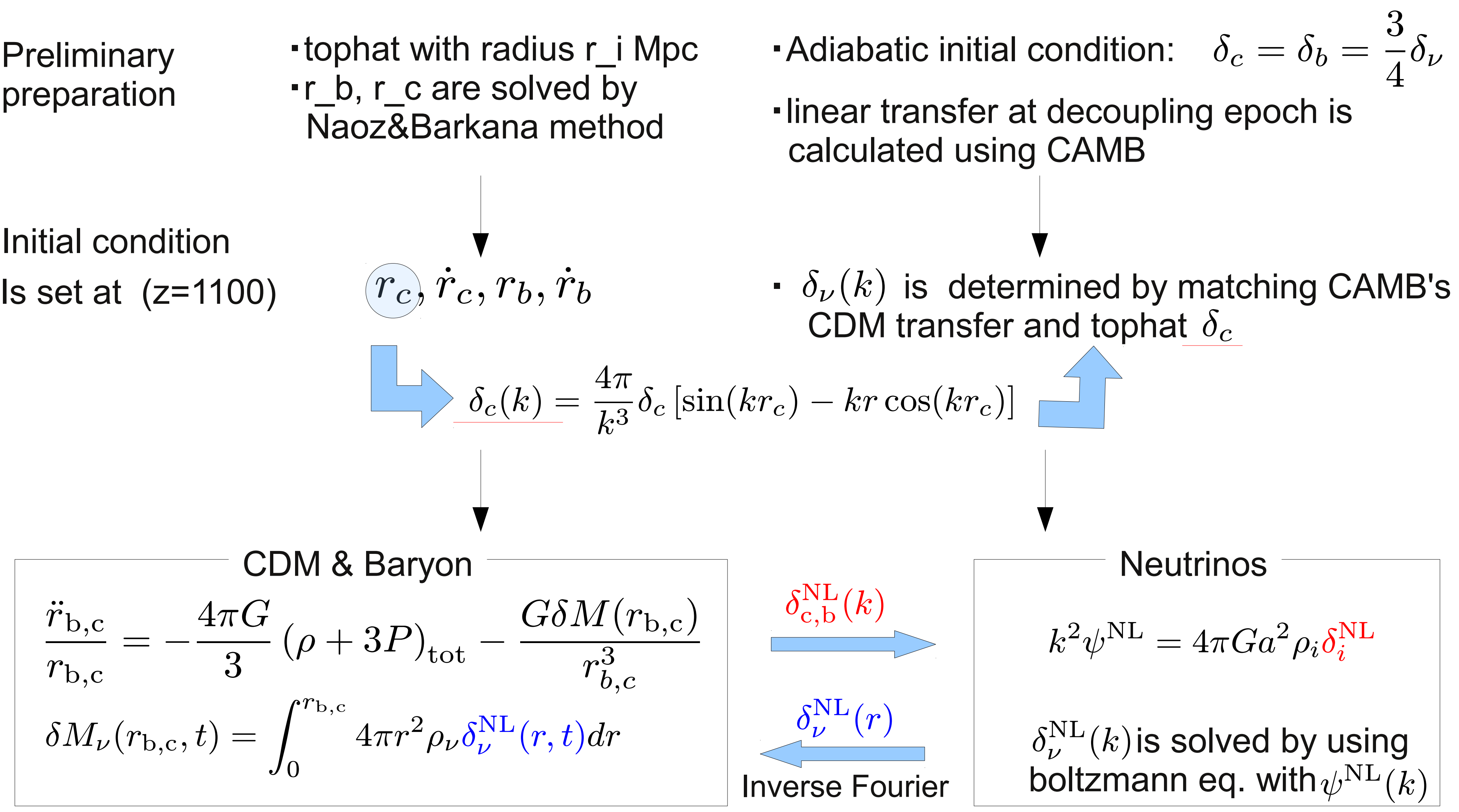
- History of the Spherical Collapse model
  - CDM only (e.g., Tomita, PTP, '69, Gunn&Gott, ApJ, '72)
  - CDM + Λ (e.g., Lahav et al., MNRAS, '91)
  - CDM + Quintessence (e.g., Wang&Steinhardt, ApJ, '98)
  - CDM + Λ + Baryon (Naoz & Barkana, MNRAS, '06)
  - CDM + coupled quintessence
    (e.g., Nunes & Mota, MNRAS, '06)
  - Decaying DM+Λ (Oguri,Takahashi,Kotake,'03)
  - CDM + early dark energy
    (e.g., Bartelmann et al., A&A, '06, Francis et al., MNRASL, '08)
  - CDM + clustering dark energy (Bjaelde&Wong, 1009.0010)
  - CDM + Λ + massive ν

This work!

Spherical Collapse with ν (1)

- Setup and assumptions
  - Top-hat profile of CDM and baryon remains unchanged (which significantly simplifies the calc.)
  - δ_ν is kept small during the collapse of CDM and baryon (check later)
    - Work with multipole-expanded boltzmann equation in Fourier space (e.g., Ma&Bartschinger, ApJ '95)
    - Insert non-linear (Newtonian) gravitational potential calculated from δ_c, δ_b and δ_ν (Singh&Ma, PRD, '03, Ringwald&Wang, JCAP, '04)

Spherical collapse with ν (2)



Preliminary preparation

- tophat with radius r_i Mpc
- r_b, r_c are solved by Naoz&Barkana method

- Adiabatic initial condition: $\delta_c = \delta_b = \frac{3}{4}\delta_\nu$
- linear transfer at decoupling epoch is calculated using CAMB

Initial condition

Is set at (z=1100)

$r_c, \dot{r}_c, r_b, \dot{r}_b$

- $\delta_\nu(k)$ is determined by matching CAMB's CDM transfer and tophat δ_c

$$\delta_c(k) = \frac{4\pi}{k^3}\delta_c\left[\sin(kr_c) - kr\cos(kr_c)\right]$$

CDM & Baryon

$$\frac{\ddot{r}_{\rm b,c}}{r_{\rm b,c}} = -\frac{4\pi G}{3}(\rho + 3P)_{\rm tot} - \frac{G\delta M(r_{\rm b,c})}{r^3_{b,c}}$$

$$\delta M_\nu(r_{\rm b,c}, t) = \int_0^{r_{\rm b,c}} 4\pi r^2 \rho_\nu \delta_\nu^{\rm NL}(r,t)dr$$

$\delta^{\rm NL}_{\rm c,b}(k)$

$\delta^{\rm NL}_\nu(r)$

Inverse Fourier

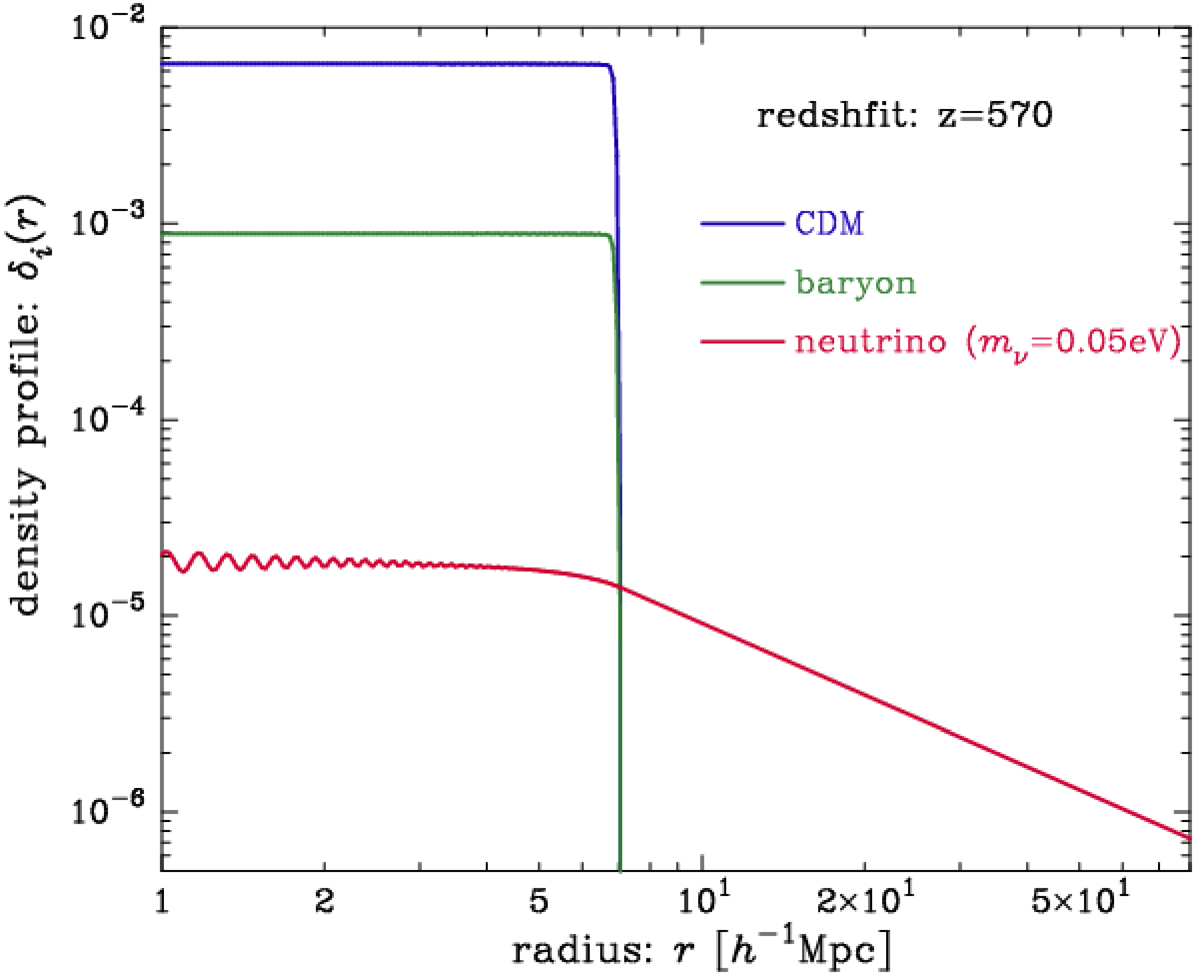
Neutrinos

$$k^2\psi^{\rm NL} = 4\pi G a^2 \rho_i \delta_i^{\rm NL}$$

$\delta_\nu^{\rm NL}(k)$ is solved by using boltzmann eq. with $\psi^{\rm NL}(k)$

$\mathcal{O}(10^4)$ coupled diff. equations

Watch a movie!



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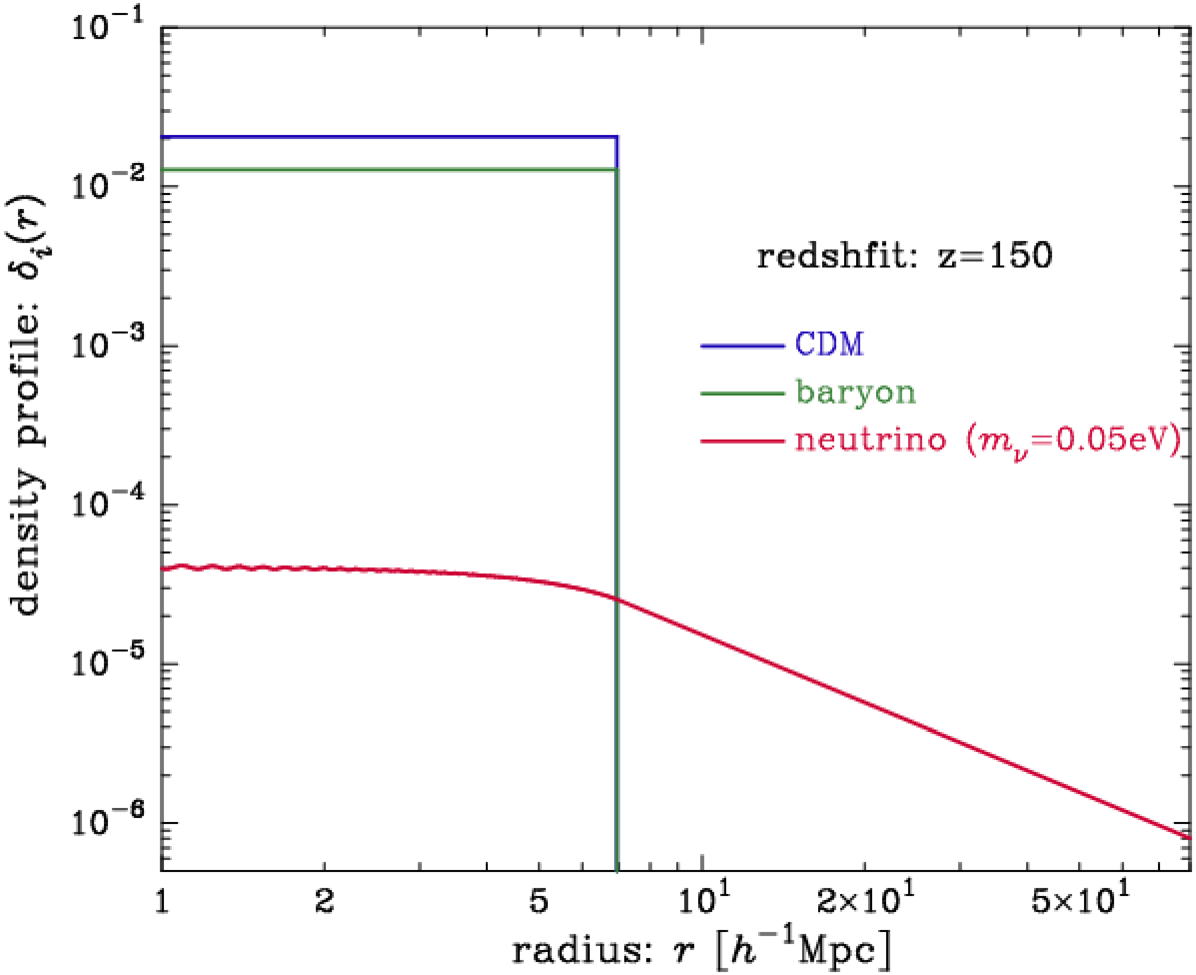

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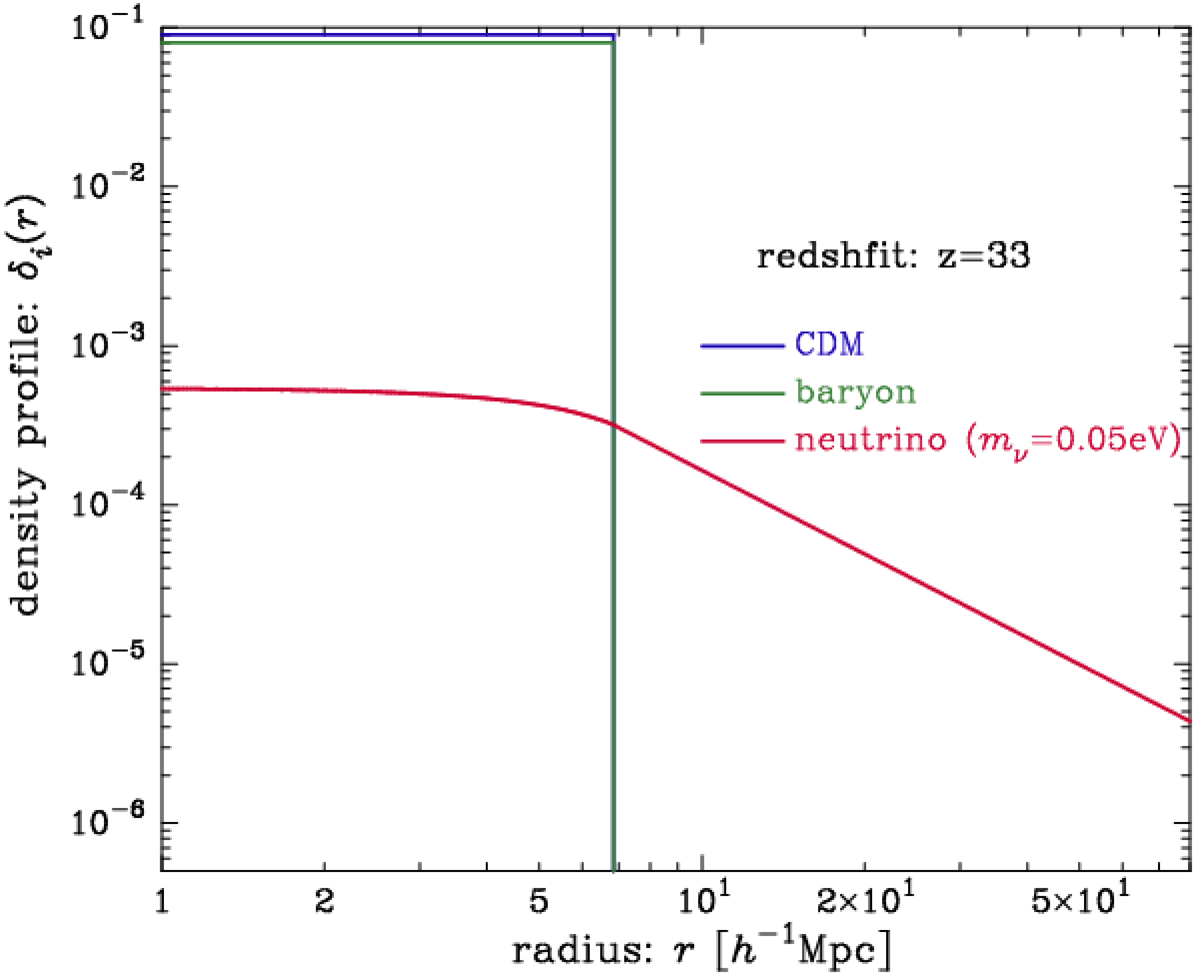

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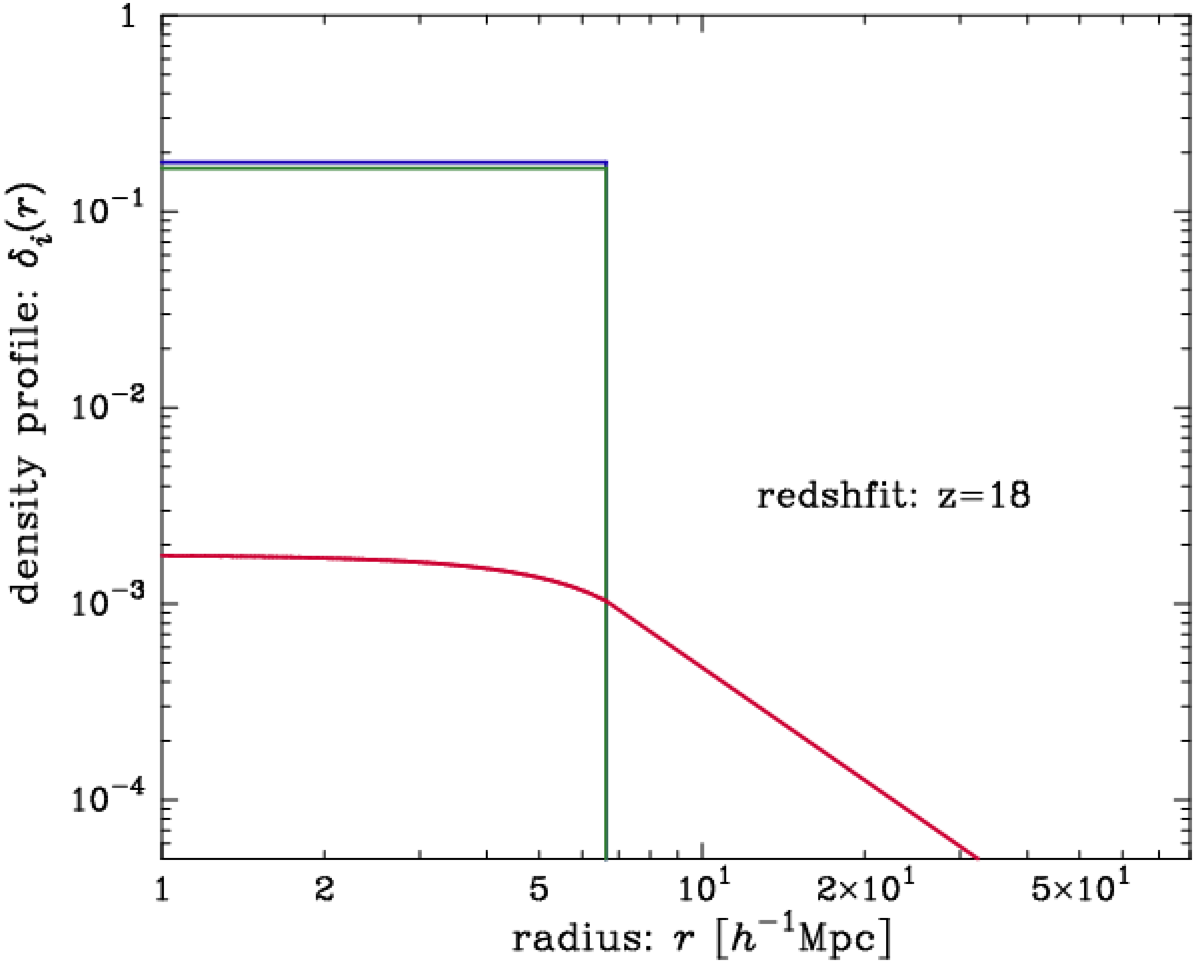

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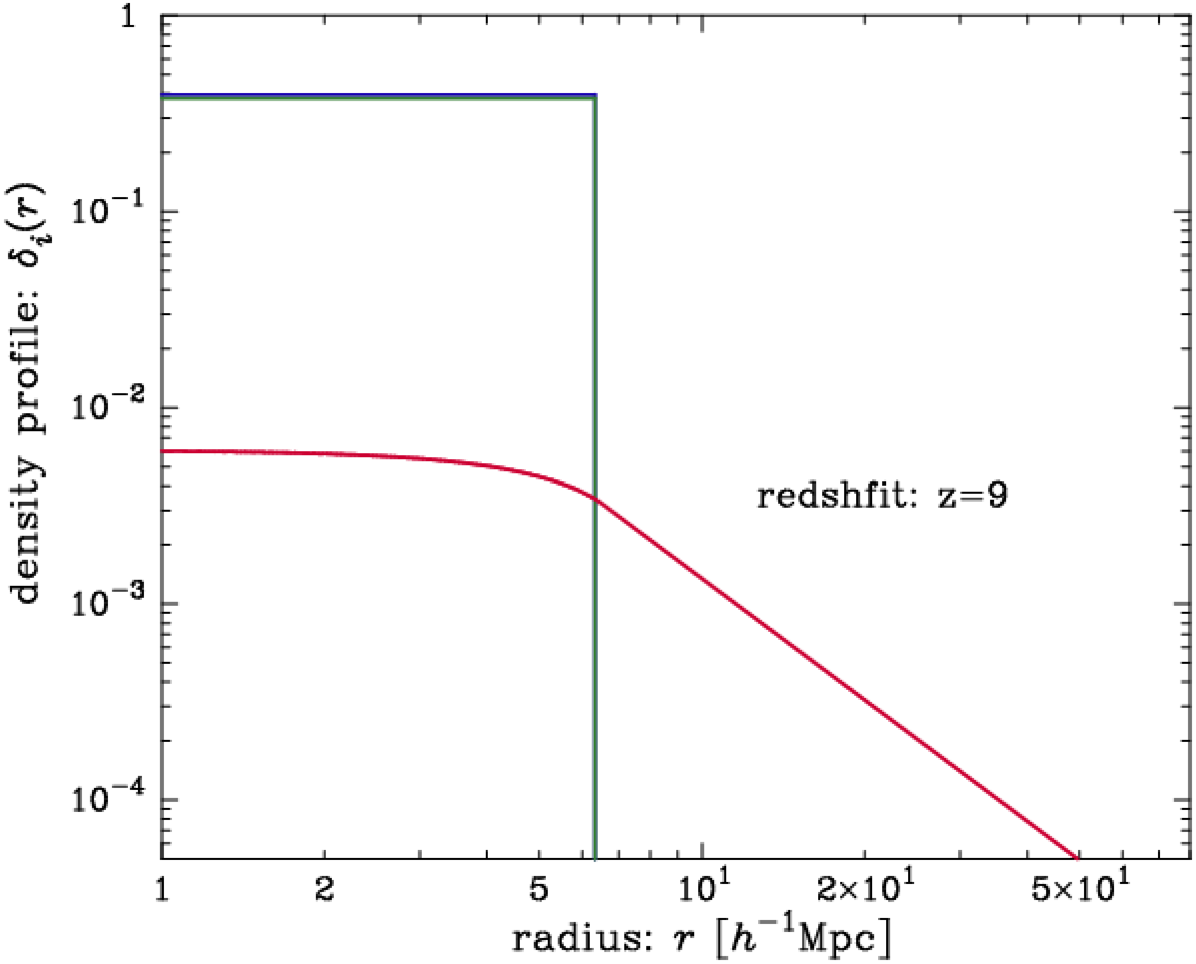

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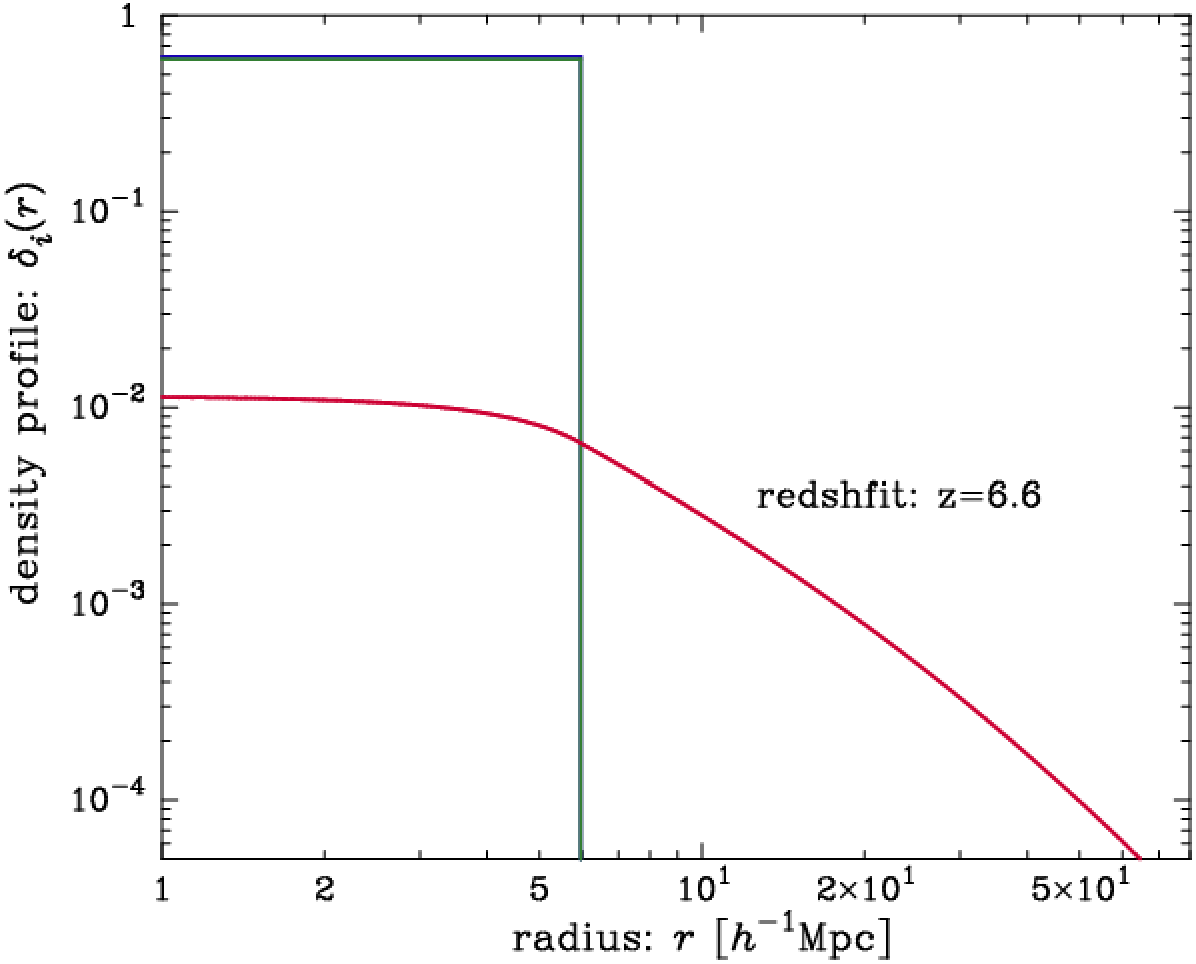

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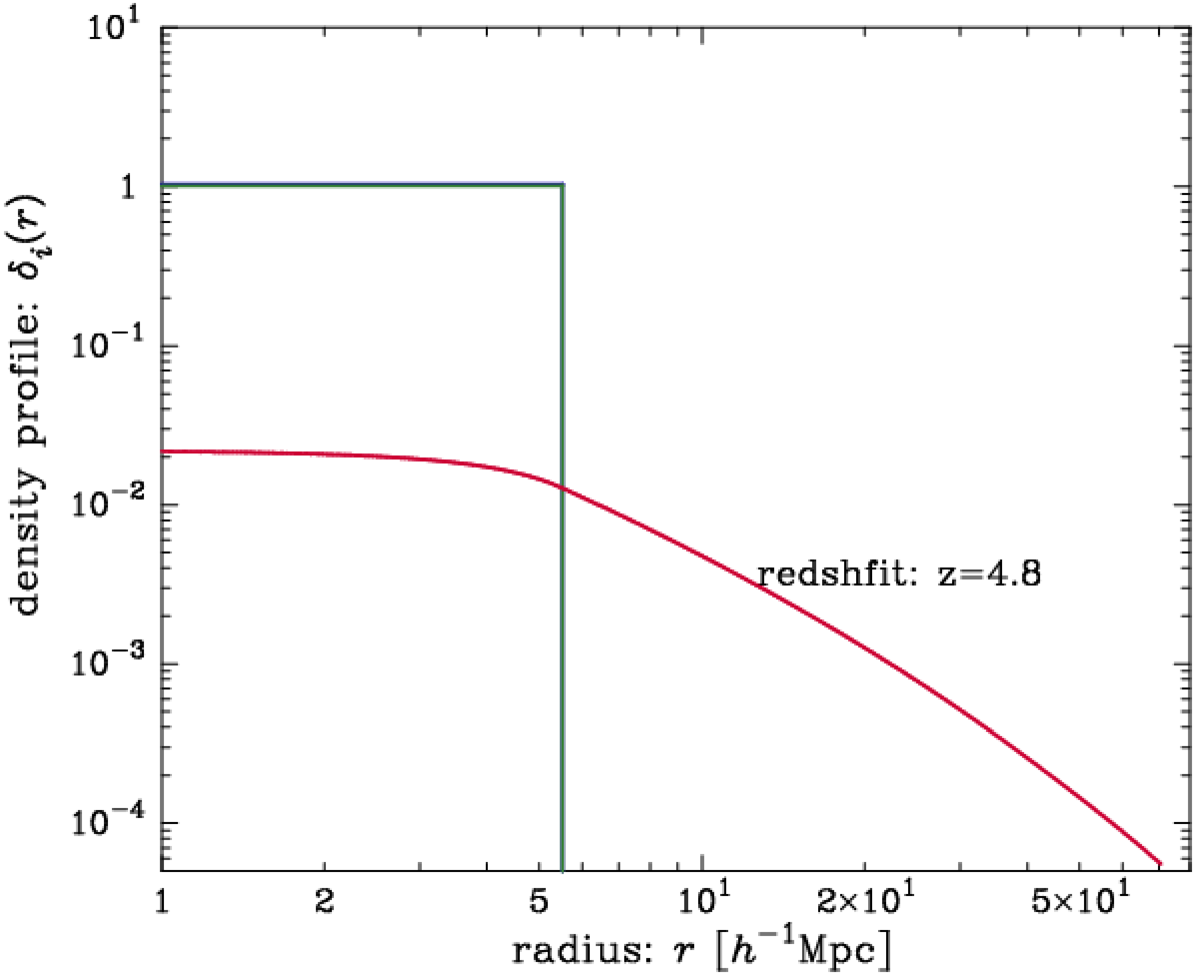

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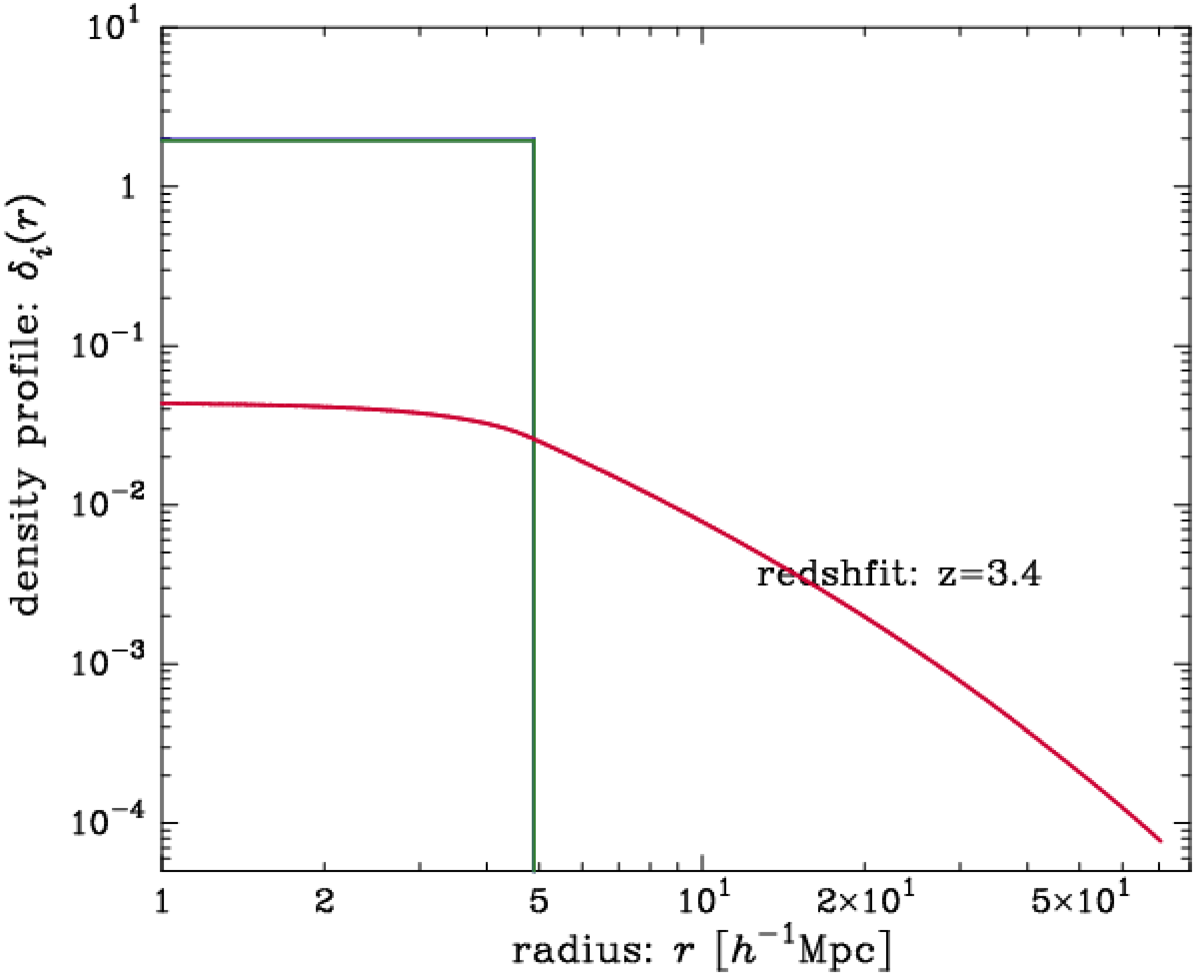

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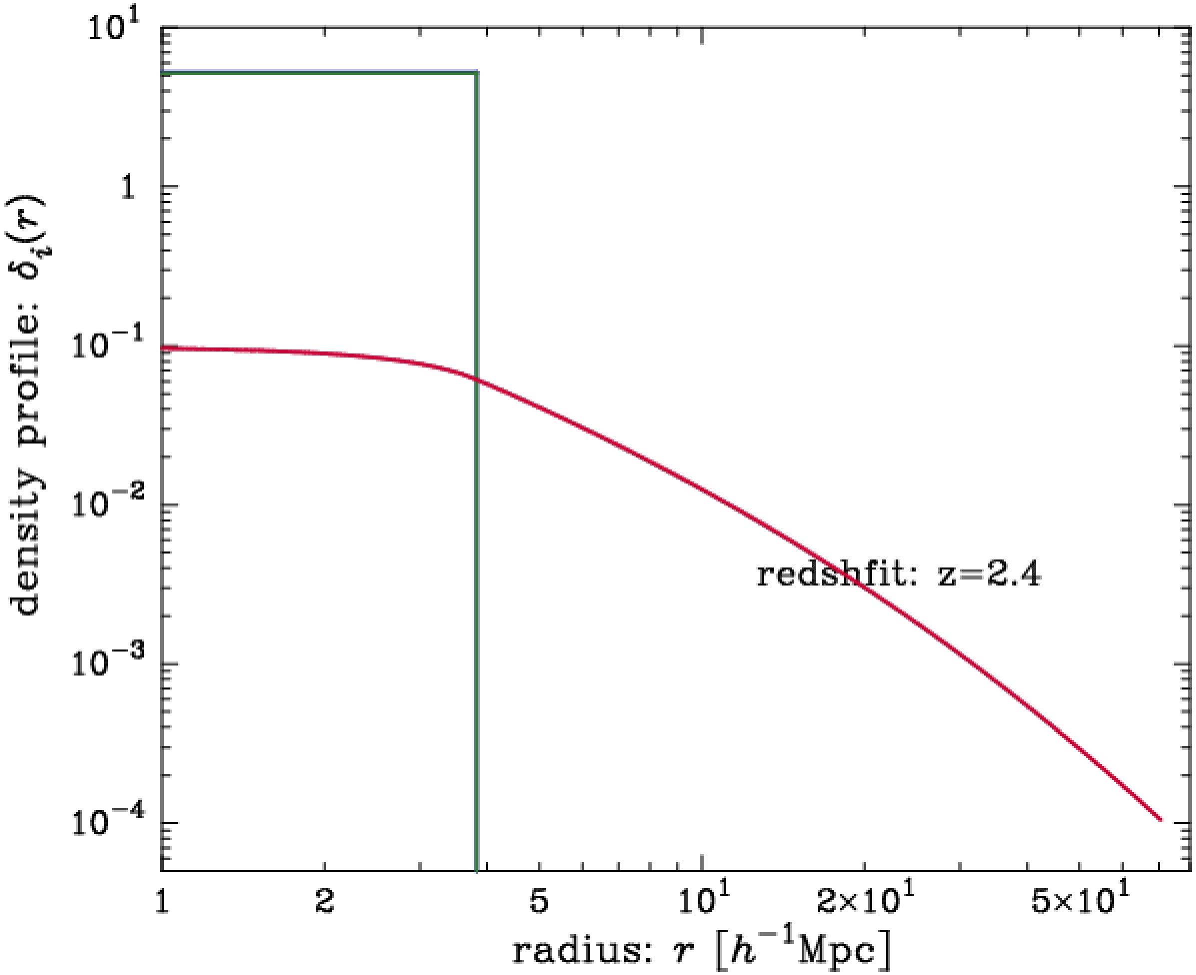

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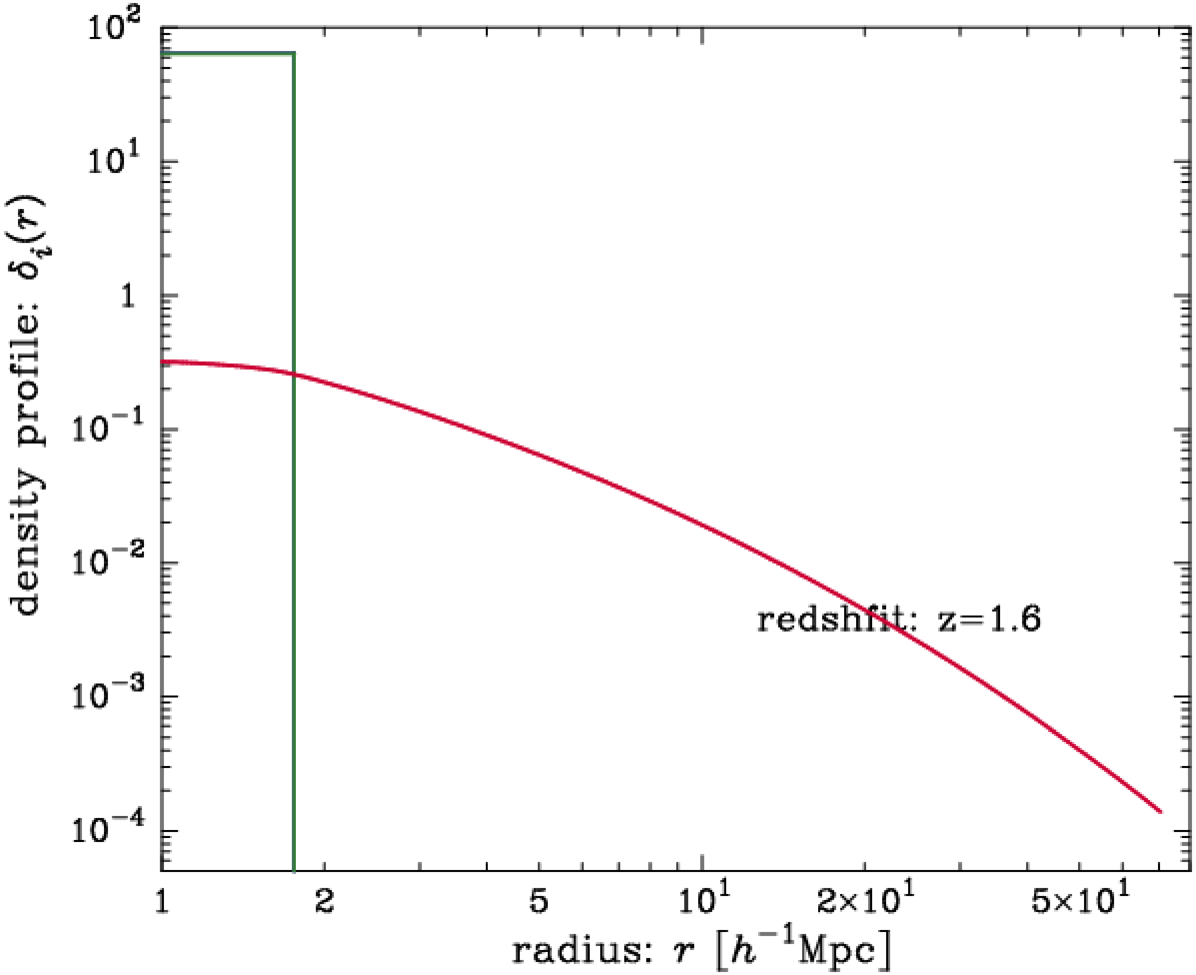

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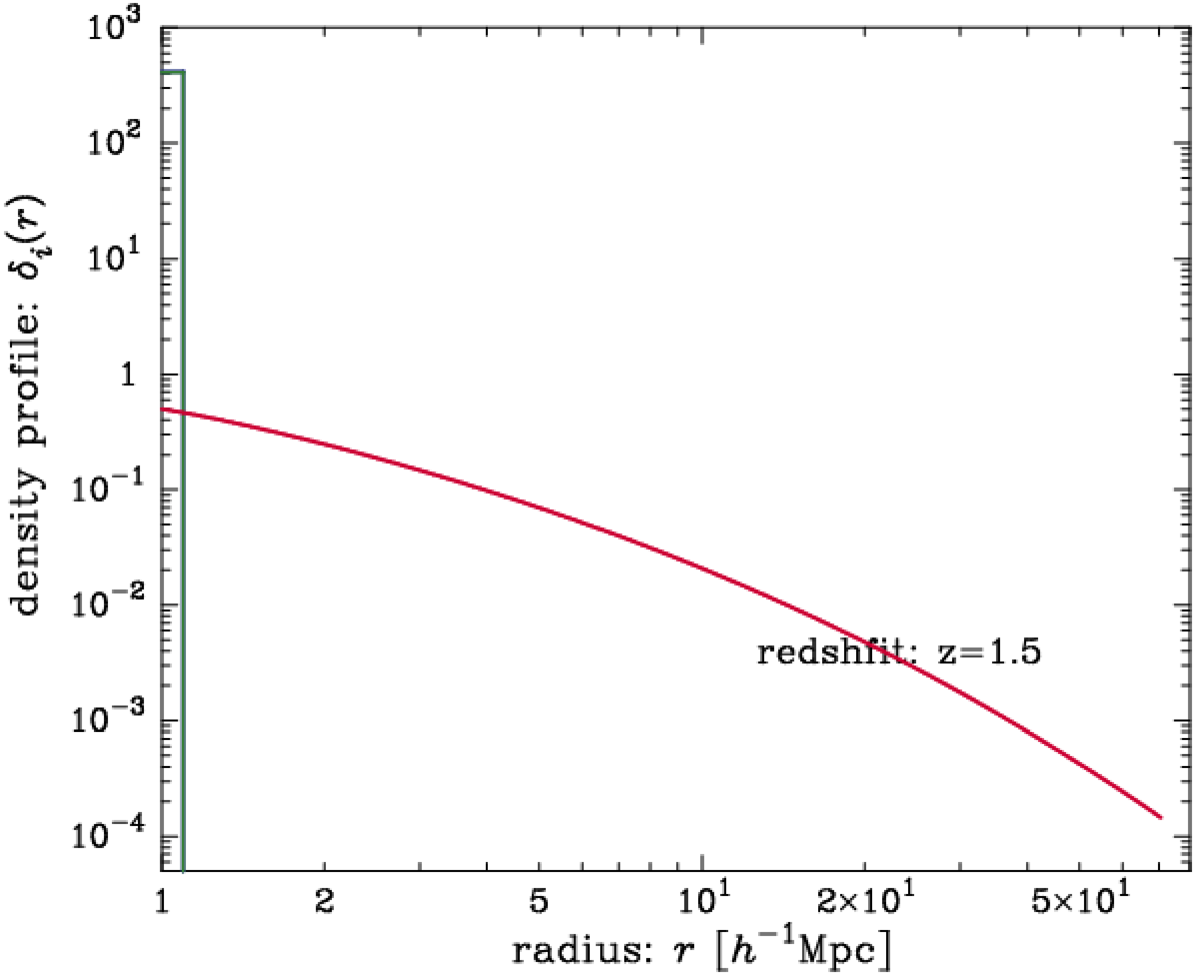
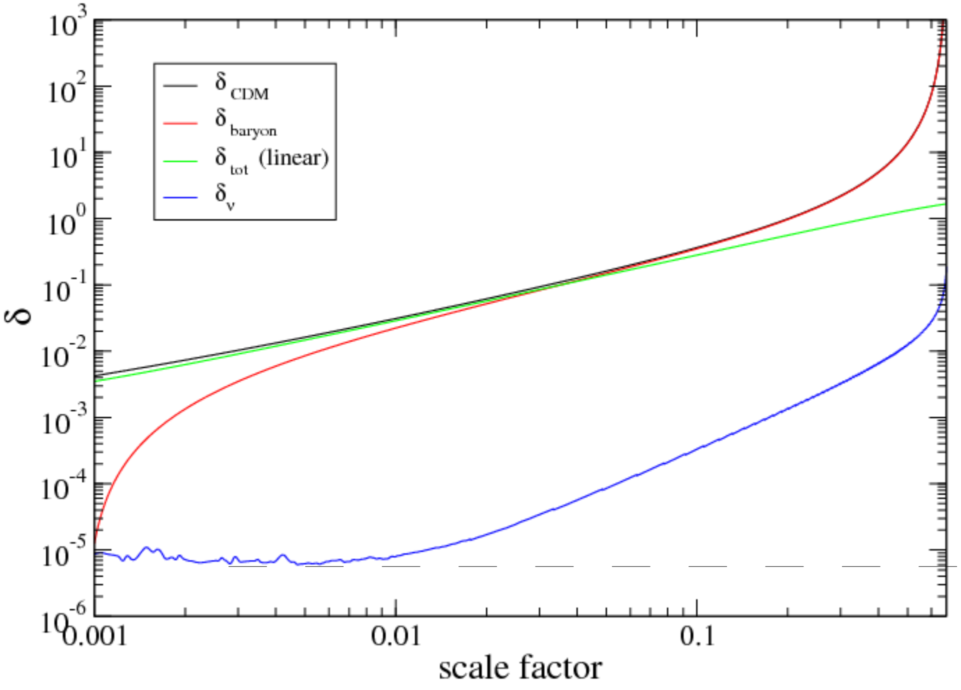


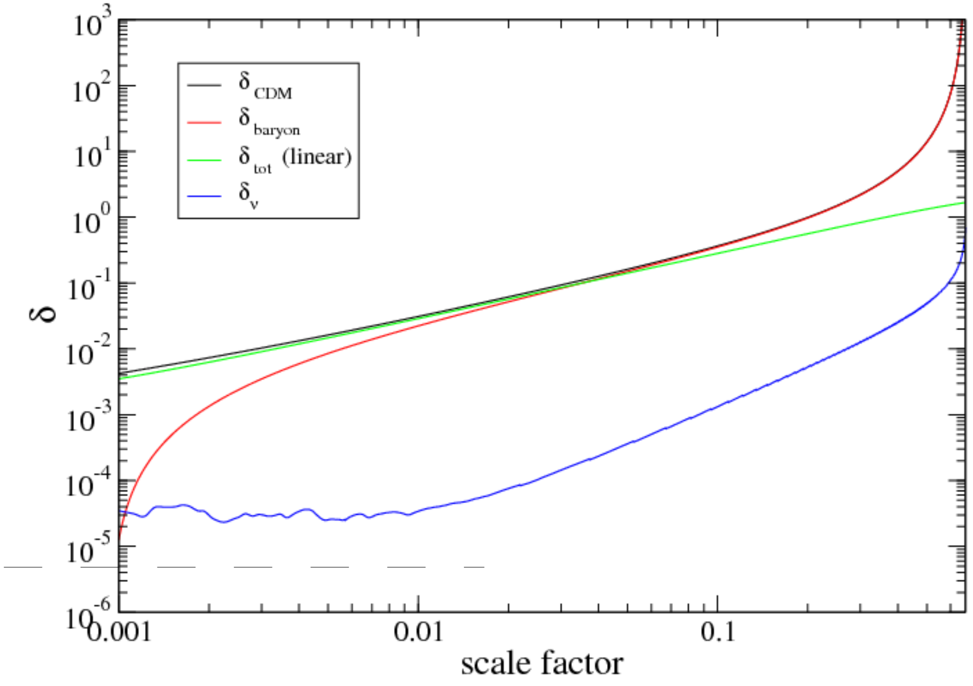
Result (1) scale dependence

- In both figure, we start calculations from the same initial amplitude and the same redshift for $m_\nu = 0.05\mathrm{eV}$

R=6.89 Mpc $10^{14}M_\odot$



R=14.84 Mpc $10^{15}M_\odot$

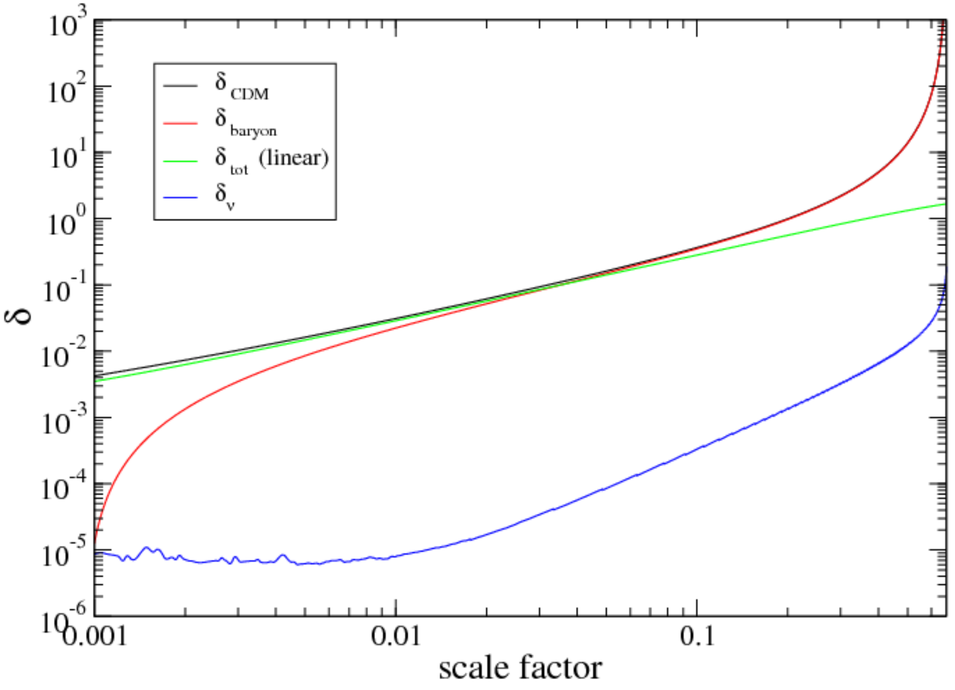


- The larger neutrino clustering is expected for the larger scales

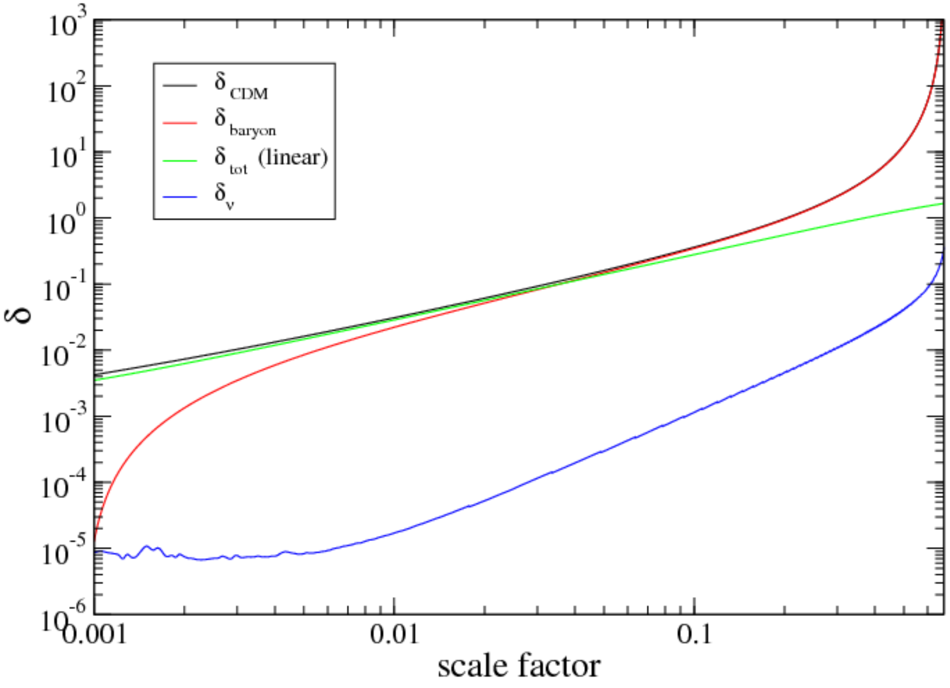
Result (2) mass dependence

- In both figure, we start calculations from the same initial amplitude and the same redshift for R=6.89 Mpc

$$m_\nu = 0.05\mathrm{eV}$$

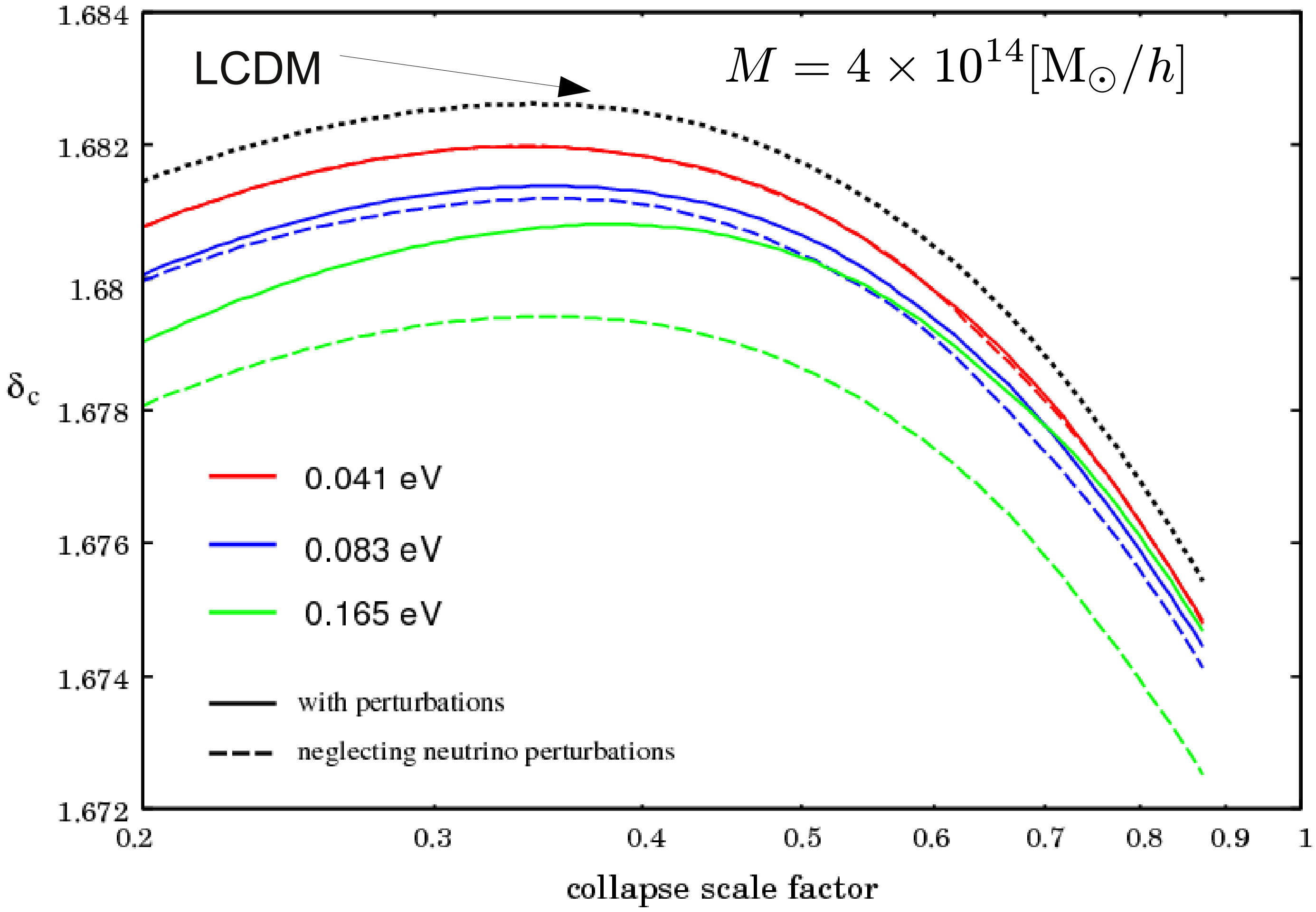


$$m_\nu = 0.1\mathrm{eV}$$



- The larger mass we consider, collapse redshift becomes smaller (e.g., 1+z = 1.47 → 1.50) and critical overdensity becomes smaller.

Result(3) critical over density



Adding neutrino mass

δ_c becomes smaller

(～0.2%)

- δc becomes larger if neutrino clustering is considered（0.1%）.
- neutrino clustering effect on δ_c is negligible for m < 0.05eV

Neutrino overdensity helps CDM gravitaional collapse at linear stage,
While it is less important at non-linear stage.

Discussion

- Let's apply our result to the halo mass function
  (Press&Schechter, ApJ, '74, Sheth&Tormen, MNRAS, '99)
  - exponentially sensitive on : δ_c

$$\frac{dn}{dM} \propto \nu \exp[-\nu^2/2]$$

(Press&Schechter like mass function)

$$\nu \equiv \frac{\delta_c(z)}{\sigma(M,z)} \sim \frac{\delta_c(z)}{\sigma_8 g(z)} \qquad g(z) = \frac{\delta_L(z)}{\delta_L(z=0)}$$

$4 \times 10^{14} [\mathrm{M}_\odot/h]$

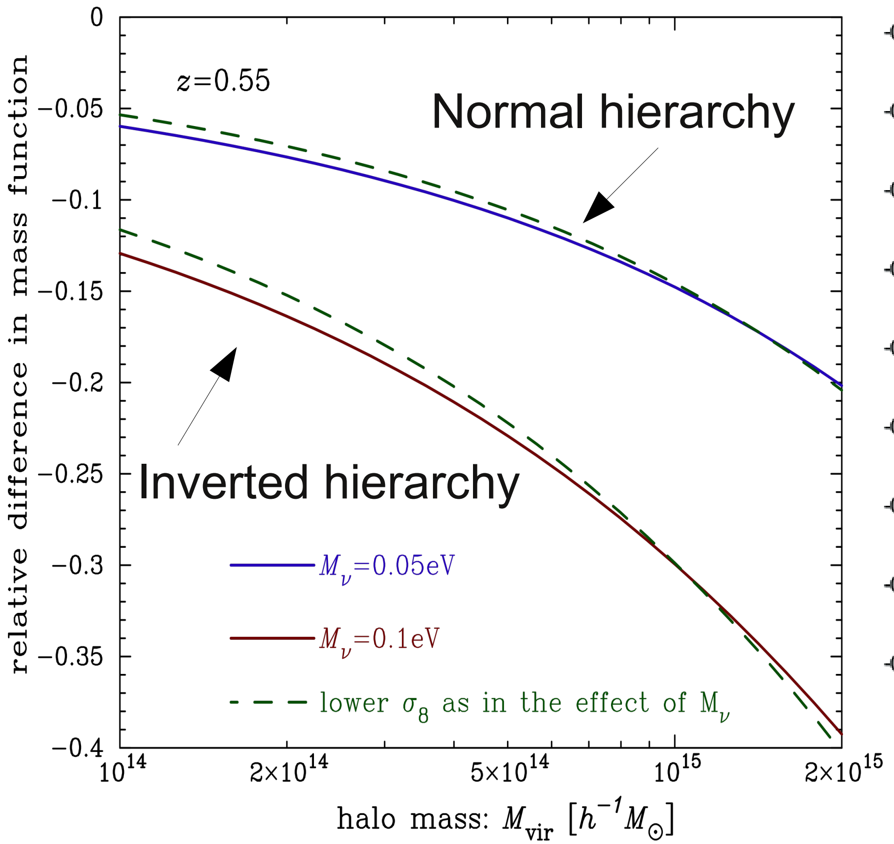
$m_\nu = 0.165\mathrm{eV}$

- Neutrino brings -0.2% change in δ_c
  (∗ 1.683 → 1.680; For comparison, Λ brings -0.7% change)
  → larger number of objects in the past than expected (σ_8 normalization)

5σ object $\frac{\Delta\left(\frac{dn}{dM}\right)}{\frac{dn}{dM}} = (1-\nu^2)\frac{\Delta\nu}{\nu} \sim 24 \times 0.2\% = 4.8\%$

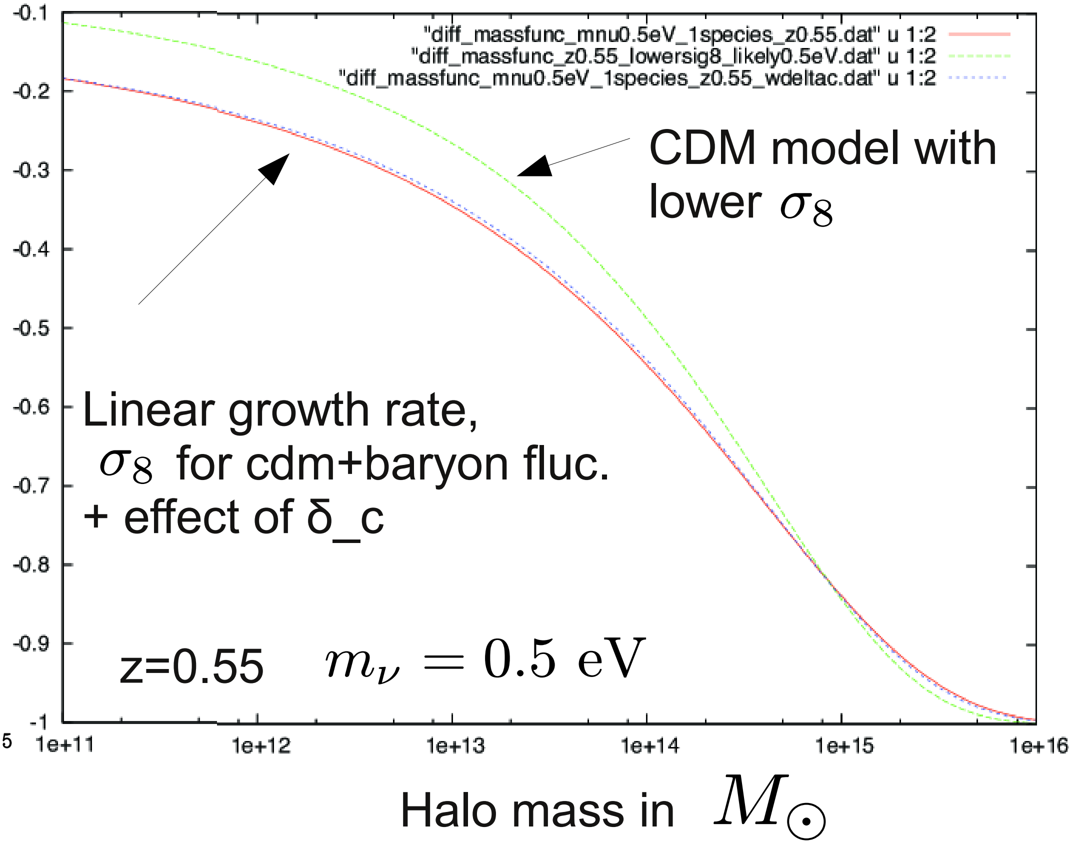
Discussion (2)

- Instead if one considers WMAP (CMB) normalization, mass of neutrinos (0.1eV) causes massive object less abundant: -30% for $\sim 10^{15}\mathrm{M}_\odot$
- CDM mass function with lower σ_8 does not capture the full feature



Normal hierarchy

Inverted hierarchy



CDM model with lower σ_8

Linear growth rate, σ_8 for cdm+baryon fluc. + effect of δ_c

z=0.55 $m_\nu = 0.5\ \mathrm{eV}$

Halo mass in $M_\odot$

Summary

- Spherical collapse model is revised by including massive neutrinos
  - Coupled linearlized Boltzmann eq. is solved using non-linear newtonian gravitational potential
    - CDM+baryon can collapse together at low redshifts
    - Neutrinos can not catch up with the collapse
    - Neutrinos delays the collapse of CDM overdense region
- Result : 0.2% change in critical over density
  - The change may bring 4.8% (1.6%) change in halo mass function for 5σ (3σ) object. However, the effect coming from the linear growth rate is much larger (good news!)
  - This is an independent support for the result of Brandbyge et al., (arXiv: 1004.4105) based on the hybrid N-body simulation including massive neutrinos.