Following DM Haloes with the Time-RG

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In collaboration with

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Outline

* Failures of Perturbation Theory
* Resummations and the TRG
* Introducing DM haloes
* Results

The future of precision cosmology: non-linear scales

matter density

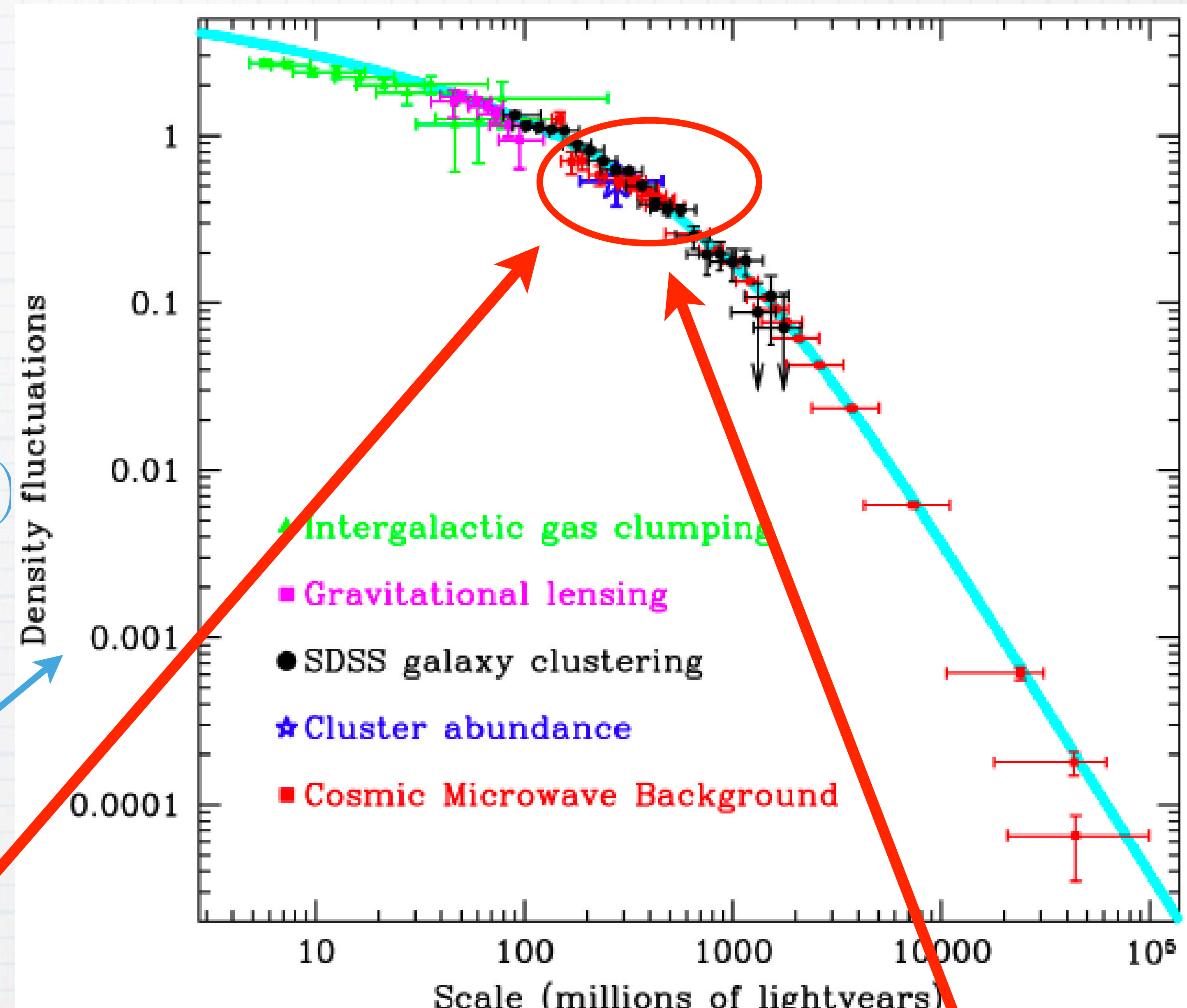
$$\rho(\mathbf{x},\,\tau) \equiv \bar{\rho}(\tau)[1+\delta(\mathbf{x},\,\tau)]$$

power spectrum

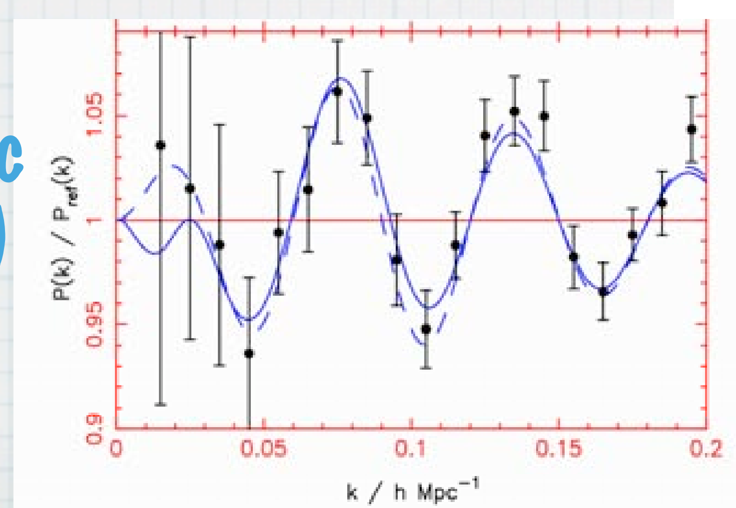
$$\langle \delta(\mathbf{k},\,\tau)\,\delta(\mathbf{k}',\,\tau)\rangle = P(k,\,\tau)\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

`size´ of the fluctuations at different scales/epochs:

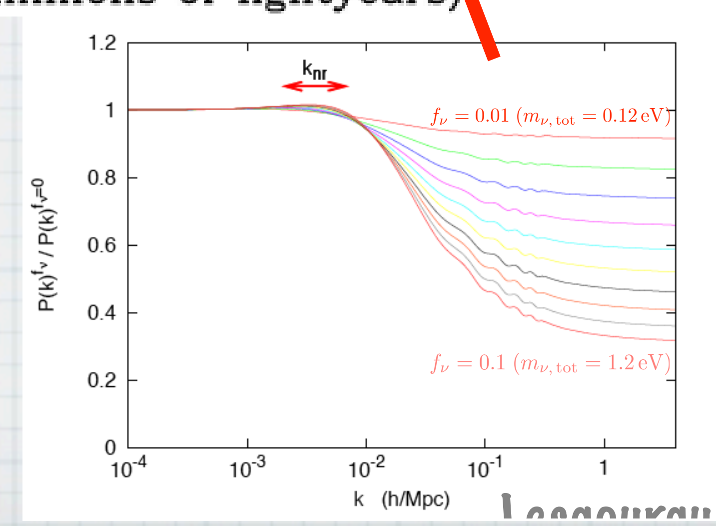
$$\Delta^2(k,\tau) = 4\pi k^3 P(k,\tau)$$



Baryonic Acoustic Oscillations (BAO)



Neutrino mass bounds



Theoretical tools

Linear perturbation theory

badly fails for $z<2\text{-}3$ and $k> 0.05h$/Mpc

N-body simulations

In principle ok, but need large volumes/resolutions:

- practically impossible to scan over cosmological models;
- non-standard but interesting scenarios are problematic: (massive neutrinos, non-gaussianity, DE-DM coupling...)

Fluid equations for Cold Dark Matter

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

In Fourier space, (defining $\theta(\mathbf{x},\tau) \equiv \nabla\cdot\mathbf{v}(\mathbf{x},\tau)$),

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\alpha(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\delta(\mathbf{k_2},\tau) = 0$$

$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_M\mathcal{H}^2\delta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\beta(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\theta(\mathbf{k_2},\tau) = 0$$

mode-mode coupling controlled by:

$$\alpha(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{(\mathbf{k_1}+\mathbf{k_2})\cdot\mathbf{k_1}}{k_1^2}$$

$$\beta(\mathbf{k_1},\mathbf{k_2}) \equiv \frac{|\mathbf{k_1}+\mathbf{k_2}|^2(\mathbf{k_1}\cdot\mathbf{k_2})}{2k_1^2k_2^2}$$

linear approximation: $\alpha(\mathbf{k_1},\, \mathbf{k_2}) = \beta(\mathbf{k_1},\, \mathbf{k_2}) = 0$

no mode-mode coupling

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) = 0$$

$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_M \mathcal{H}^2 \delta(\mathbf{k},\tau) = 0$$

$$\Omega_M = 1 \rightarrow \mathcal{H} \sim a^{-1/2}$$

↓

$$\delta(\mathbf{k},\, \tau) \quad = \quad \delta(\mathbf{k},\, \tau_i)\left(\frac{a(\tau)}{a(\tau_i)}\right)^m$$

$$-\frac{\theta(\mathbf{k},\, \tau)}{\mathcal{H}} \quad = \quad m\,\delta(\mathbf{k},\, \tau)$$

$$m = \begin{cases} 1 & \text{growing mode} \\ -\frac{3}{2} & \text{decaying mode} \end{cases}$$

Compact Perturbation Theory

Crocce, Scoccimarro '05

Consider again the continuity and Euler equations

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

define $\begin{pmatrix} \varphi_1(\eta,\mathbf{k}) \\ \varphi_2(\eta,\mathbf{k}) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta(\eta\,,\mathbf{k}) \\ -\theta(\eta\,,\mathbf{k})/\mathcal{H} \end{pmatrix}$ with $\eta = \log\frac{a(\tau)}{a(\tau_i)}$ $\quad \mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

then we can write (we assume an EdS model):

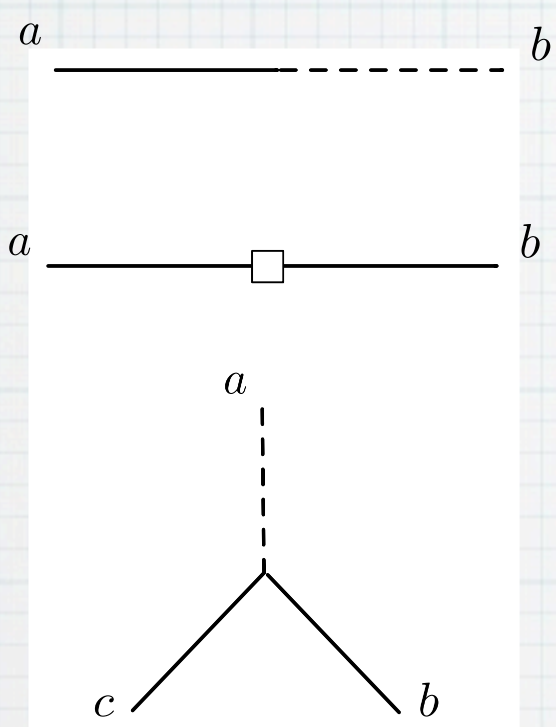
$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^\eta\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

and the only non-zero components of the mode-mode coupling are

$$\gamma_{121}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \gamma_{112}(\mathbf{k_1},\,\mathbf{k_3},\,\mathbf{k_2}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{\alpha(\mathbf{k_2},\,\mathbf{k_3})}{2}$$

$$\gamma_{222}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\beta(\mathbf{k_2},\,\mathbf{k_3})$$

Perturbation Theory: Feynman Rules

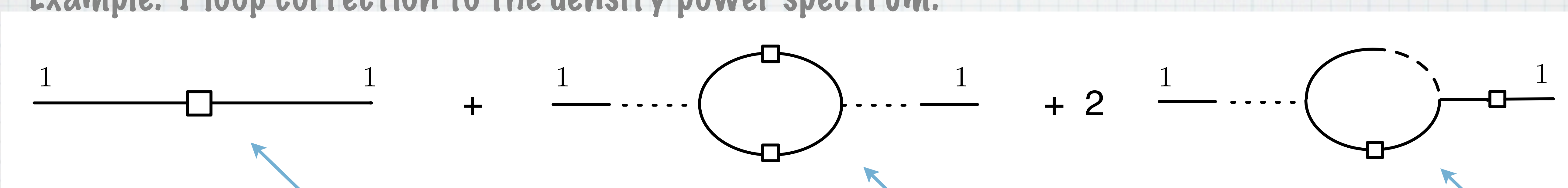


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Example: 1-loop correction to the density power spectrum:



Linear Power spectrum

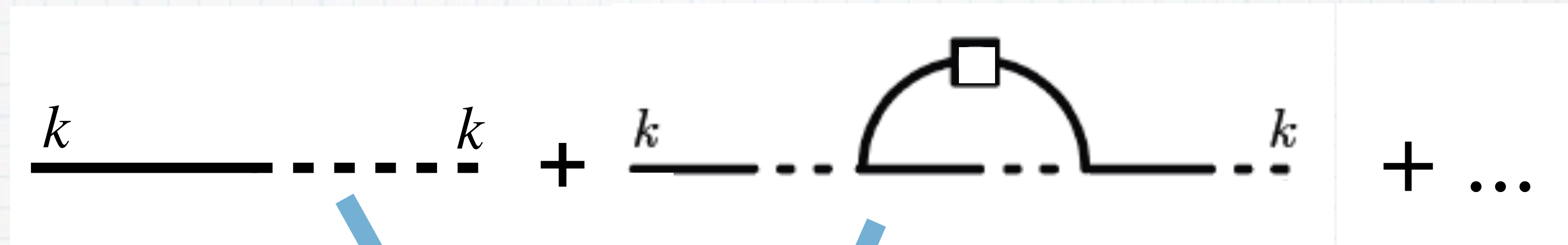
"P22"

"P13"

All known results in cosmological perturbation theory are expressible in terms of diagrams in which <u>only a trilinear fundamental interaction</u> appears

Problems with PT

1-loop propagator @ large k:



$$G_{ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\left[1 - k^2\sigma^2\,\frac{(e^{\eta_a} - e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)$$

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

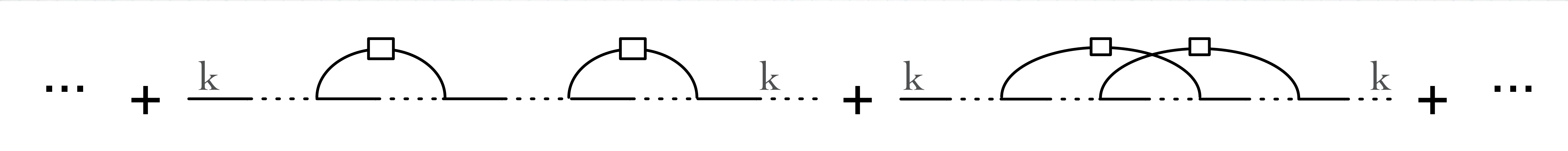
$(\sigma\, e^{\eta_a})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$ in the BAO range!

2-loop

the PT series blows up in the BAO range

But it can be resummed!!

(Crocce-Scoccimarro '06)



$$G(k;\eta,\eta_{in}) = \frac{\langle \delta(k,\eta)\delta(k,\eta_{in})\rangle}{\langle \delta(k,\eta_{in})\delta(k,\eta_{in})\rangle} \sim e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$$

physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes: memory loss

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

damping in the BAO range!

RPT: use G, and not g, as the linear propagator

Partial (!) list of contributors to the field

* "traditional" P.T.: see Bernardeau et al, Phys. Rep. 367, 1, (2002), and refs. therein; Jeong-Komatsu; Saito et al; Sefusatti;...
* resummation methods: Valageas; Crocce-Scoccimarro; McDonald; Matarrese-M.P.; Matsubara; Taruya-Hiratamatsu; M.P.; Bernardeau-Valageas; Bernardeau-Crocce-Scoccimarro;...

Time-RG (M.P. '08)

(also for cosmologies with $D^{\pm} = D^{\pm}(k,\ z)$)

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta, \mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k},\ -\mathbf{k_1},\ -\mathbf{k_2})\,\varphi_b(\eta, \mathbf{k_1})\,\varphi_c(\eta, \mathbf{k_2})$$

$$\partial_\eta \varphi = -\Omega\,\varphi + e^\eta \gamma\,\varphi\,\varphi$$

$$\partial_\eta \langle \varphi\,\varphi \rangle = -\sum \Omega\,\langle \varphi\,\varphi \rangle + \sum e^\eta \gamma\,\langle \varphi\,\varphi\,\varphi \rangle$$

$$\partial_\eta \langle \varphi\,\varphi\,\varphi \rangle = -\sum \Omega\,\langle \varphi\,\varphi\,\varphi \rangle + \sum e^\eta \gamma\,\langle \varphi\,\varphi\,\varphi\,\varphi \rangle$$

$$\ldots$$

infinite tower of equations

can be obtained from the RG-like physical requirement

$$\frac{d}{\eta_{in}} Z[J, \Lambda; \eta_{in}] = 0$$

Advantages

Works also for cosmologies with $\Omega_{ab} = \Omega_{ab}(k, \eta)$

not only for $\Omega_{ab} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

e.g. massive neutrinos,
Scalar-tensor theories

Power spectrum ($\langle \varphi \varphi \rangle$) and bispectrum ($\langle \varphi \varphi \varphi \rangle$) from a single run!

Systematic approximation scheme straightforward

Equations to solve:

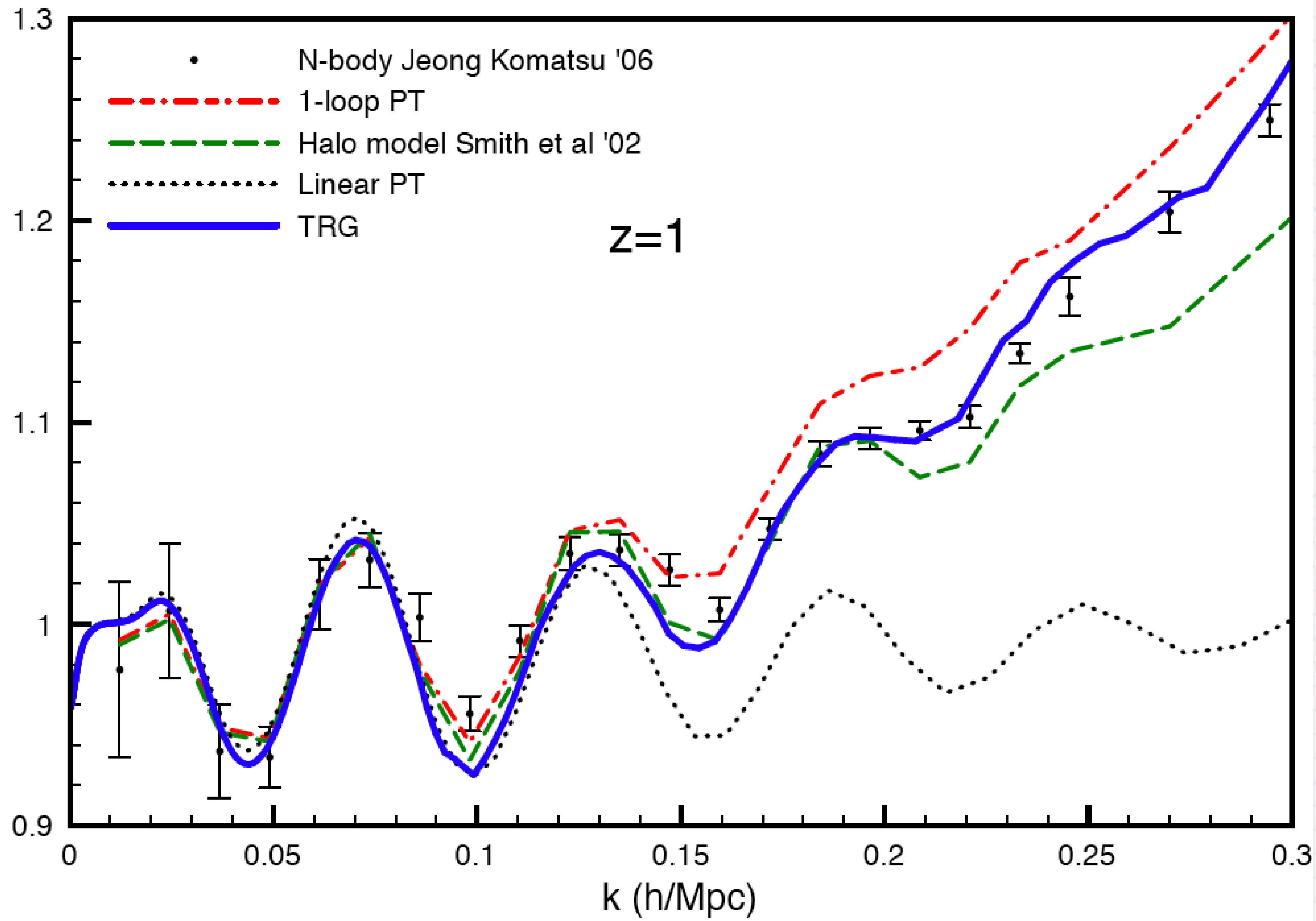
$$\partial_\eta P_{ab}(\mathbf{k}, \eta) = -\Omega_{ac}(\mathbf{k}, \eta) P_{cb}(\mathbf{k}, \eta) - \Omega_{bc}(\mathbf{k}, \eta) P_{ac}(\mathbf{k}, \eta)$$
$$+ e^\eta \int d^3q \, [\gamma_{acd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}) B_{bcd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$+ B_{acd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta) \gamma_{bcd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k})] \,,$$

$$\partial_\eta B_{abc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta) = -\Omega_{ad}(\mathbf{k}, \eta) B_{dbc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$- \Omega_{bd}(-\mathbf{q}, \eta) B_{adc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$- \Omega_{cd}(\mathbf{q}-\mathbf{k}, \eta) B_{abd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$+ 2e^\eta [\gamma_{ade}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}) P_{db}(\mathbf{q}, \eta) P_{ec}(\mathbf{k}-\mathbf{q}, \eta)$$
$$+ \gamma_{bde}(-\mathbf{q}, \mathbf{q}-\mathbf{k}, \mathbf{k}) P_{dc}(\mathbf{k}-\mathbf{q}, \eta) P_{ea}(\mathbf{k}, \eta)$$
$$+ \gamma_{cde}(\mathbf{q}-\mathbf{k}, \mathbf{k}, -\mathbf{q}) P_{da}(\mathbf{k}, \eta) P_{eb}(\mathbf{q}, \eta)] \,.$$

initial conditions given at $\eta = 0$, corresponding to $z = z_{in}$

Only approximation: $T_{abcd} = 0$

Full equation: numerical results



M. Pietroni 0806.0971 (JCAP)

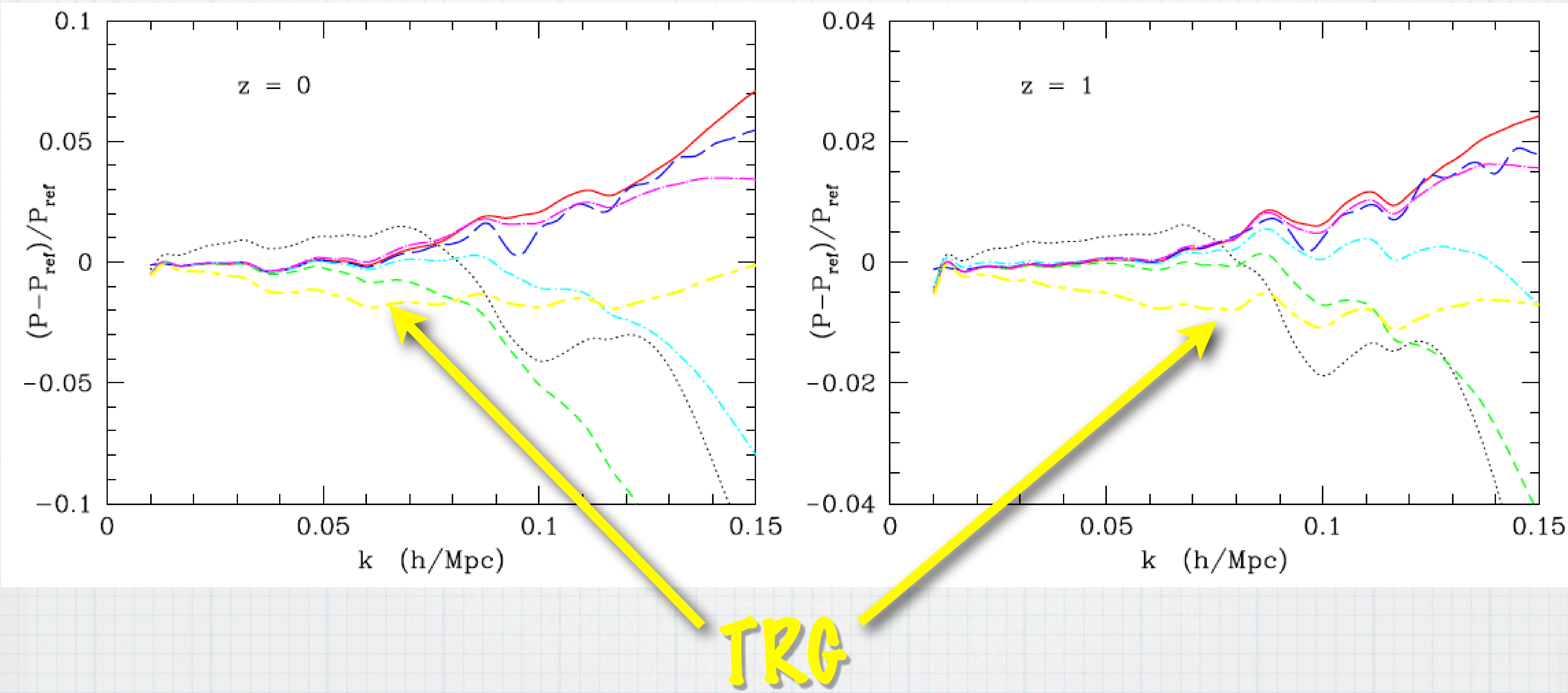
initial conditions:

$$P_{ab}(\mathbf{k}, 0) = P_{\mathrm{Lin}}(\mathbf{k}, z_{in}) u_a u_b$$

$$B_{abc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}, 0) = 0$$

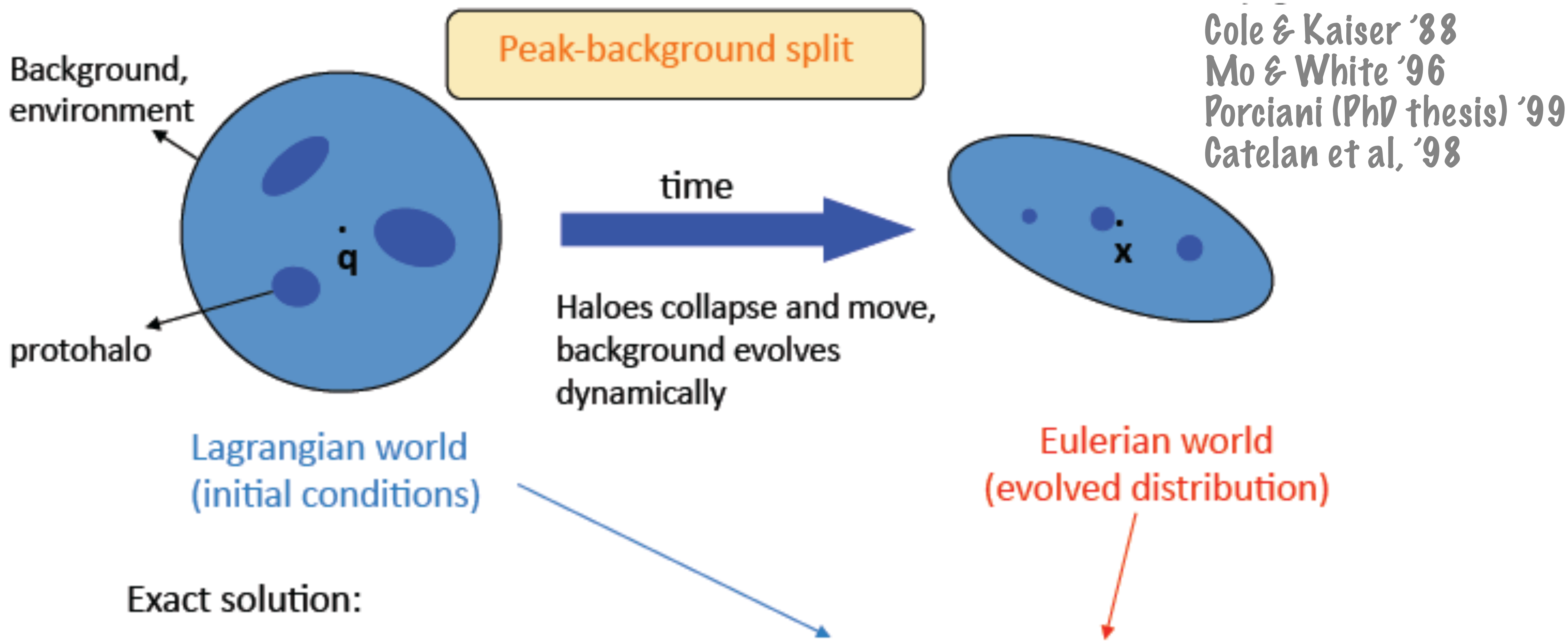
Comparison with other methods

Carlson, White, Padmanabhan, '09



Fractional difference w.r.t. high resolution N-body below 2% in the BAO range down to z=0!

From DM fluid to DM haloes



$$1 + \delta_h(\mathbf{x},z) = [1 + \delta_h(\mathbf{q})]\,[1 + \delta(\mathbf{x},z)]$$

the bias of DM haloes is stocastic, non-linear, and non-local

The dynamics of proto-haloes

$$\frac{\partial \delta_h}{\partial \tau} + \nabla \cdot [(1+\delta_h)\mathbf{v_h}] = 0\,,$$

proto-haloes are conserved!

$$\frac{\partial \mathbf{v_h}}{\partial \tau} + \mathcal{H}\mathbf{v_h} + (\mathbf{v_h}\cdot\nabla)\mathbf{v_h} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$

gravitational potential determined by the DM fluid

$$|\mathbf{v_h} - \mathbf{v}| \sim 1/a$$

3-fluid system: $\delta,\ \mathbf{v},\ \delta_h$

alternative approach: follow the peak distribution (Desjacques et al '10)

Compact notation

$$\eta \equiv \ln(D_+/D_{+in}), \qquad \begin{pmatrix} \varphi_1(\mathbf{k},\eta) \\ \varphi_2(\mathbf{k},\eta) \\ \varphi_3(\mathbf{k},\eta) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta_m(\mathbf{k},\eta) \\ -\theta(\mathbf{k},\eta)/(\mathcal{H} f_+) \\ \delta_h(\mathbf{k},\eta) \end{pmatrix}$$

$$\partial_\eta \varphi_a(\mathbf{k},\eta) = -\Omega_{ab}(\eta)\varphi_b(\mathbf{k},\eta) + e^\eta \gamma_{abc}(\mathbf{k}, -\mathbf{p}, -\mathbf{q})\varphi_b(\mathbf{p},\eta)\,\varphi_c(\mathbf{q},\eta),$$

$$\Omega(\eta) = \begin{pmatrix} 1 & -1 & 0 \\ -\frac{3}{2}\frac{\Omega_m}{f_+^2} & \frac{3}{2}\frac{\Omega_m}{f_+^2} & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

Matrix containing cosmology information

Vertex with non-vanishing components:

$$\gamma_{121}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \frac{1}{2}\,\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\alpha(\mathbf{p},\mathbf{q})\,,$$

$$\gamma_{222}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\beta(\mathbf{p},\mathbf{q})\,,$$

$$\gamma_{112}(\mathbf{k},\, \mathbf{q},\, \mathbf{p}) = \gamma_{121}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}),$$

$$\gamma_{323}(\mathbf{k},\, \mathbf{p},\, \mathbf{q}) = \gamma_{332}(\mathbf{k},\, \mathbf{q},\, \mathbf{p}) = \gamma_{121}(\mathbf{k},\, \mathbf{p},\, \mathbf{q})$$

Initial conditions

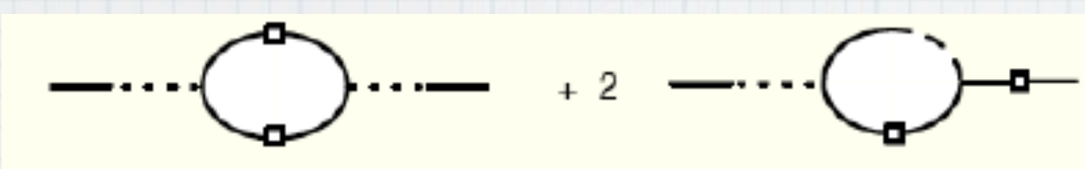
DM haloes identified at z=0 and traced back to z_{in}=50

Two possibilities: 1) assume/fit the functional relation: $\delta_h(\mathbf{k}, z_{in}) = \mathcal{F}[\delta_m]$

2) assume/fit all the n-point (cross)correlators:

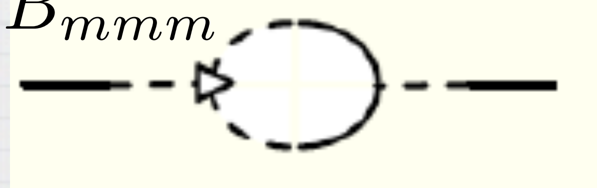
$$P_m(k, z_{in}),\ P_{mh}(k, z_{in}),\ P_h(k, z_{in})$$

$$B_{mmm}(k_1, k_2, k_3; z_{in}),\ B_{mmh}(k_1, k_2, k_3; z_{in}), \cdots$$



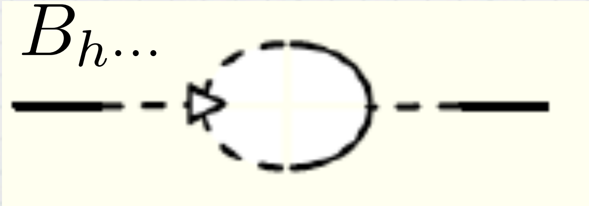
$\sim D^4$

B_{mmm}



$\sim D^3$

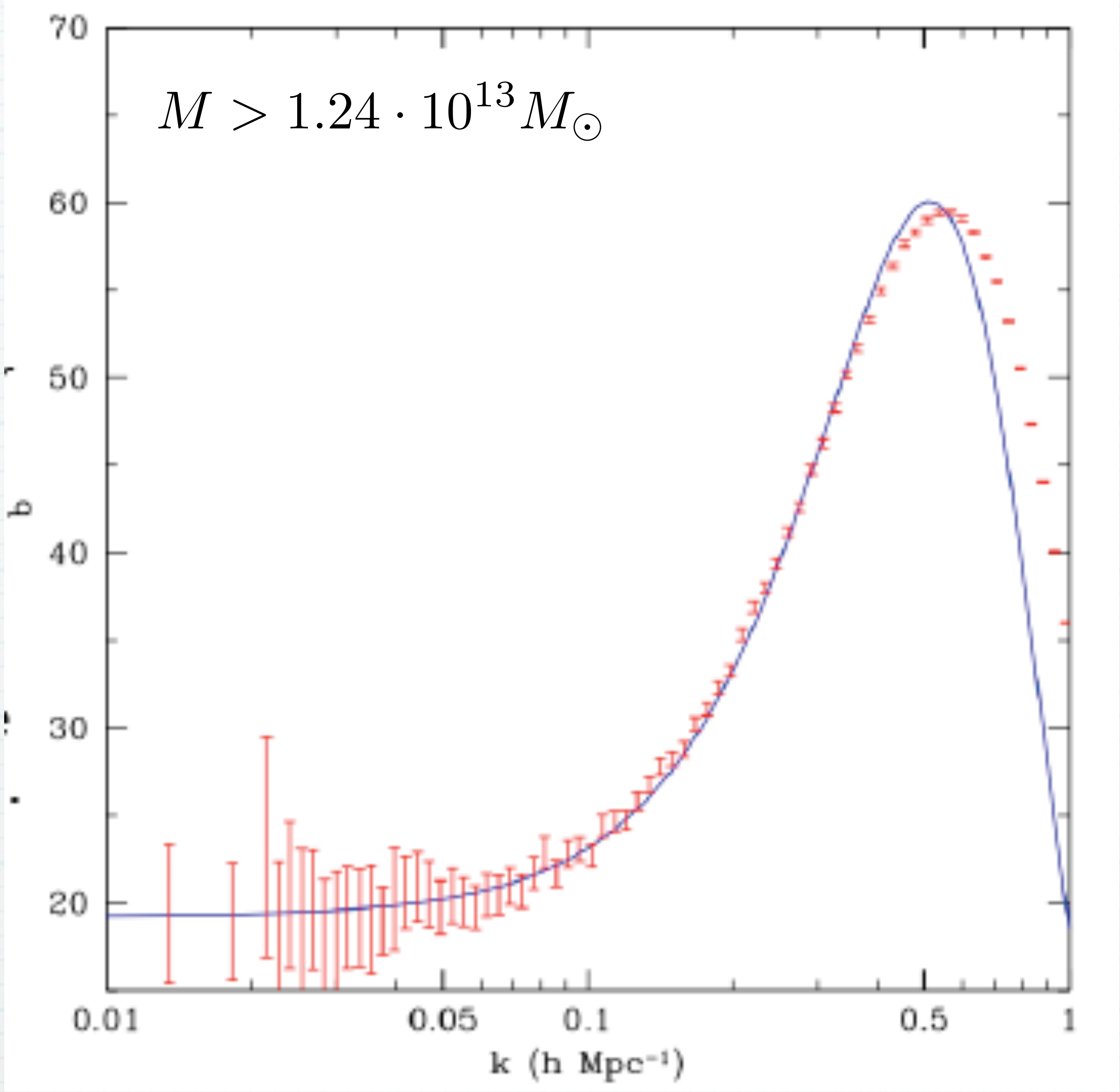
$B_{h\ldots}$



$\sim D^2$ initial values for $B_{h\ldots}$ are irrelevant!

Initial conditions: lagrangian halo bias

DM haloes identified at z=0
and traced back to z_{in}=50



initial (lagrangian) bias well
reproduced by the non-local relation: $P_{mh}(k) = (b_1 + b_2\, k^2) P_m(k) e^{-k^2 R^2/2}$

(Matsubara '99, Desjacques '08)

Linear approximation: debiasing

$$\varphi_a(\mathbf{k};\eta) = g_{ab}(\eta)\varphi_b(\mathbf{k};0)\,, \qquad g_{ab}(\eta) = \Bigg[\begin{pmatrix} 3/5 & 2/5 & 0 \\ 3/5 & 2/5 & 0 \\ 3/5 & 2/5 & 0 \end{pmatrix}$$

Growing mode

Std. decaying mode

$$+e^{-5/2\eta}\begin{pmatrix} 2/5 & -2/5 & 0 \\ -3/5 & 3/5 & 0 \\ 2/5 & -2/5 & 0 \end{pmatrix}$$

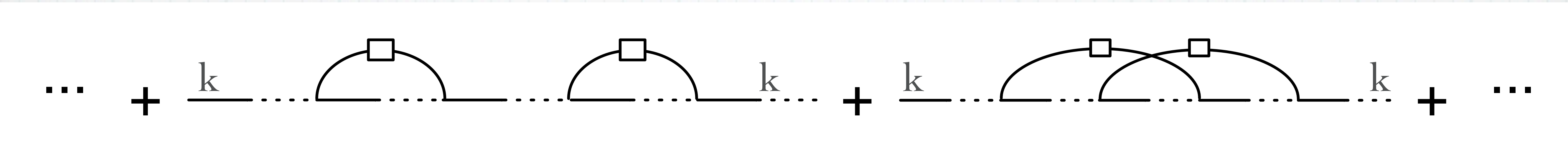
New decaying mode

$$+e^{-\eta}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}\Bigg]\theta(\eta)\,,$$

$$\varphi_a(\mathbf{k};0) = \begin{pmatrix} \varphi(\mathbf{k}) \\ \varphi(\mathbf{k}) \\ \varphi_h(\mathbf{k}) \end{pmatrix} \qquad \varphi_a(\mathbf{k};\eta) = \begin{pmatrix} \varphi(\mathbf{k}) \\ \varphi(\mathbf{k}) \\ \varphi(\mathbf{k}) + e^{-\eta}(\varphi_h(\mathbf{k}) - \varphi(\mathbf{k})) \end{pmatrix}$$

Initial conditions → (Time) → Evolved fields: “debiasing”, the initial bias is progressively erased (see also Fry 1996)

Beyond linear order: Large-k resummation for the propagator



$$G_{ab}(k;\eta) = \left(g_{ab}(\eta) + \delta_{a3} f_b \Delta^{(2)} g_{31}(k;\eta)\right) \exp\left[-k^2\sigma^2 \frac{(e^{\eta}-1)^2}{2}\right]$$

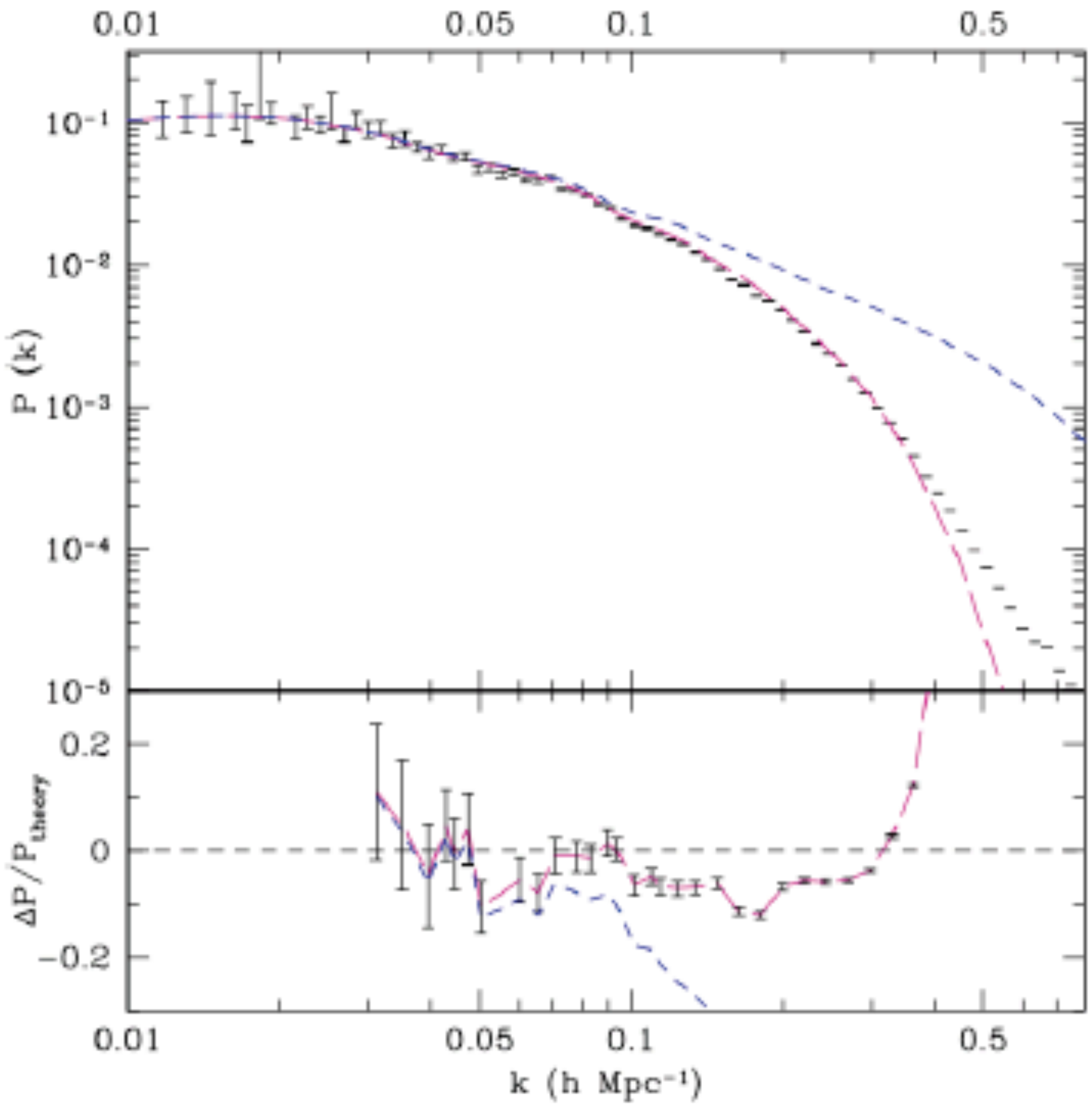
↑ 1-loop

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q \, \frac{P^0(q)}{q^2}\right)$$

The exact result of Crocce and Scoccimarro generalizes to the protohalo propagator

Cross-correlation at different times

Cross spectrum between the evolved δ_h (at z=0) and the initial δ_m



N-body

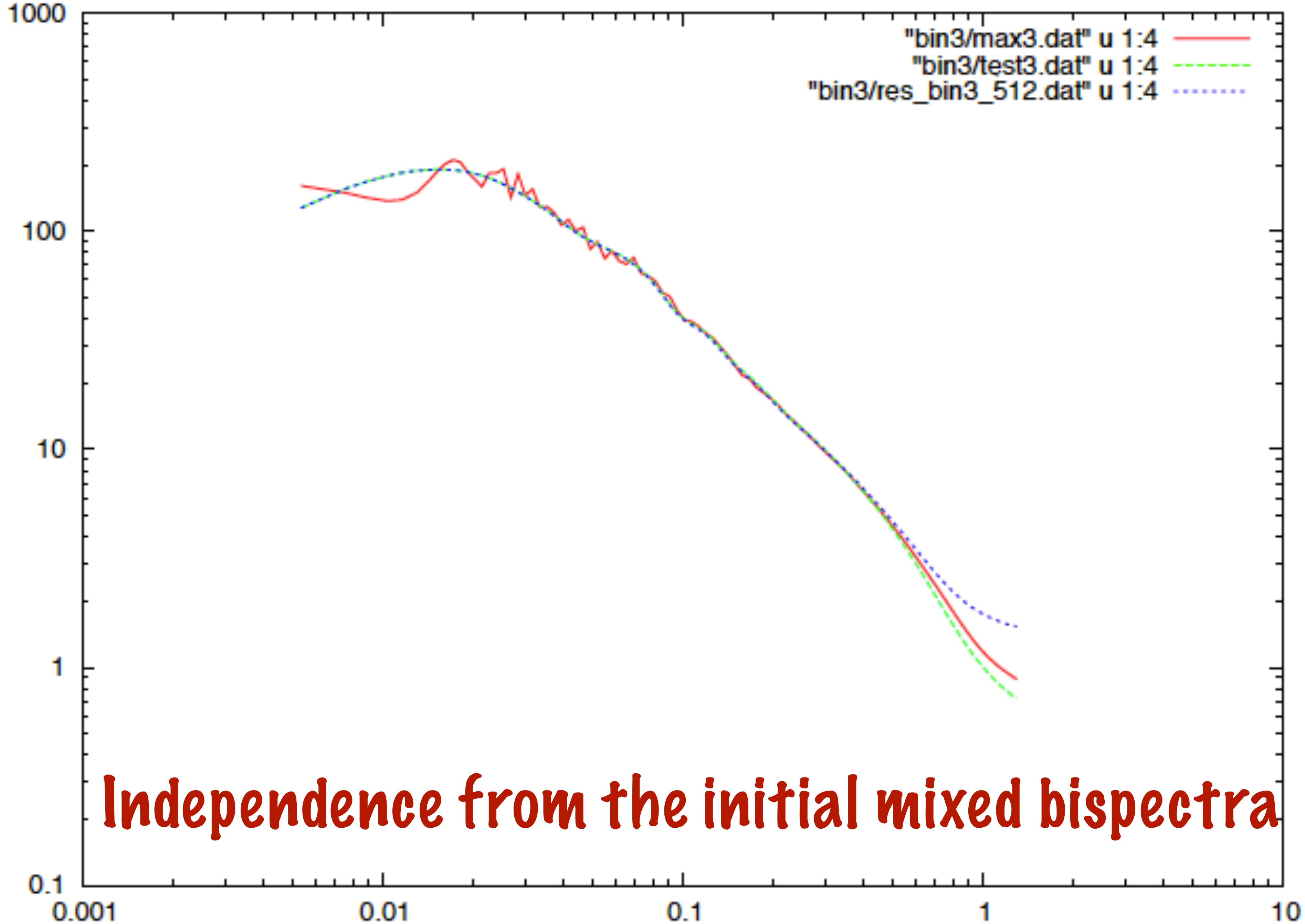
Linear

Resummed

$$P_{31}(k,\eta,0)=G_{31}(k,\eta)\,P_{11}^{(0)}(k)+G_{32}(k,\eta)\,P_{21}^{(0)}(k)+G_{33}(k,\eta)\,P_{31}^{(0)}(k)$$

Power spectrum

Unlike the propagator, it cannot be resummed analytically.
Use the TRG

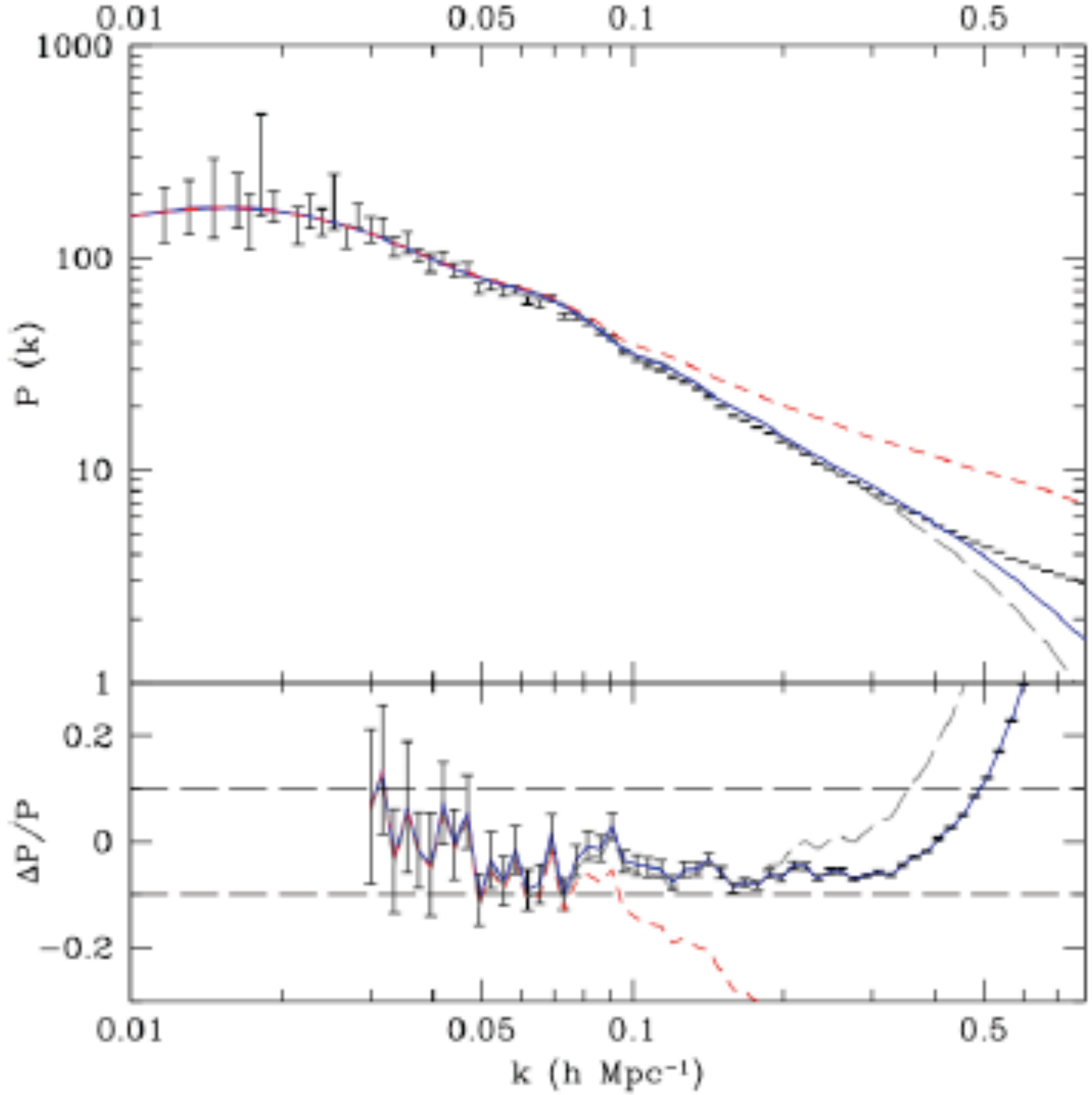


Independence from the initial mixed bispectra

Power spectrum

Comparison with simulations

Cross spectrum between the evolved δ_h and the evolved δ_m both at z=0

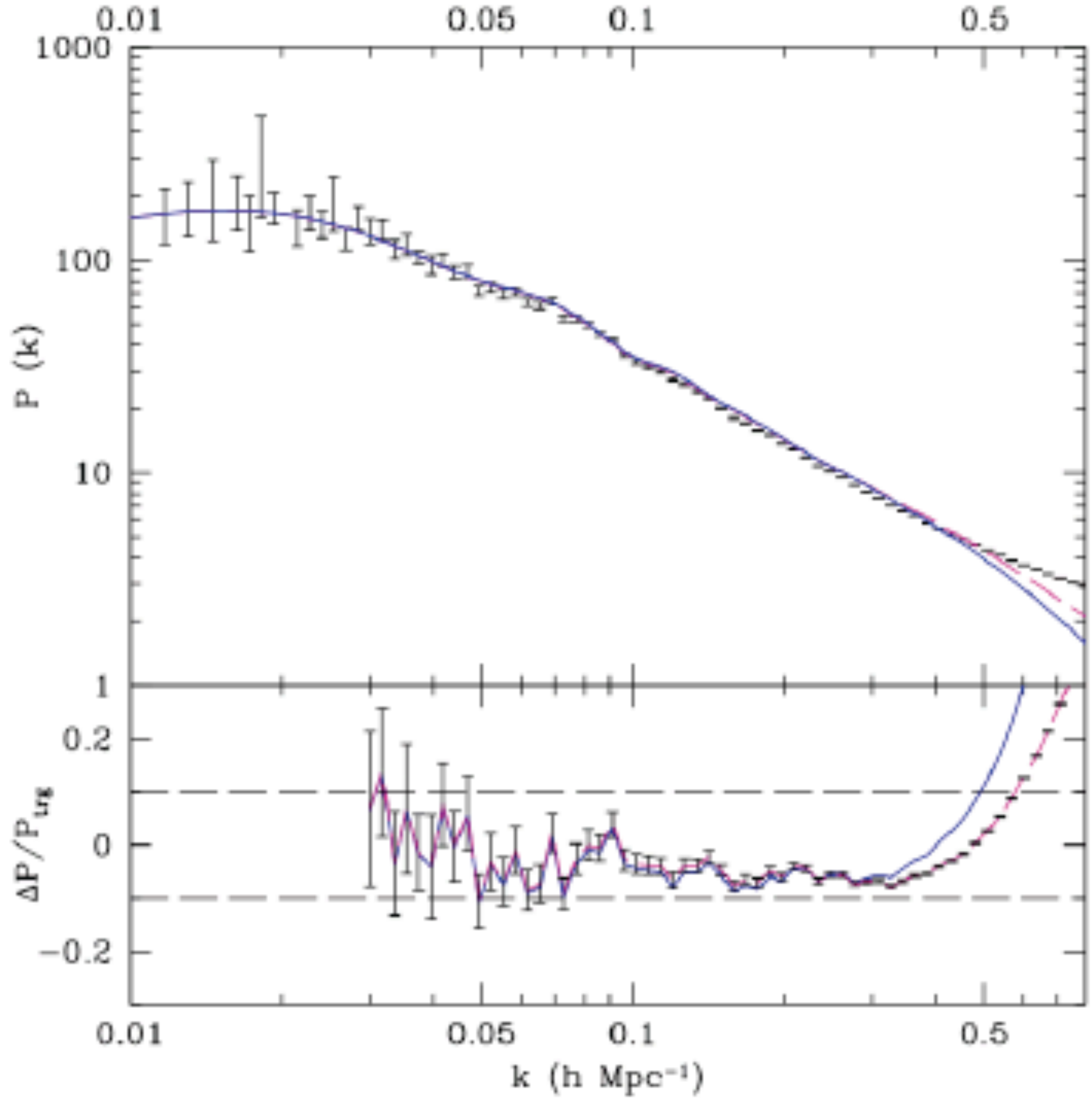


N-body (Pillepich et al '10)
1-loop
TRG (initial B=0)

Power spectrum

Comparison with simulations

Cross spectrum between the evolved δ_h and the evolved δ_m both at z=0



N-body
TRG (initial B=0)
TRG (initial B≠0)

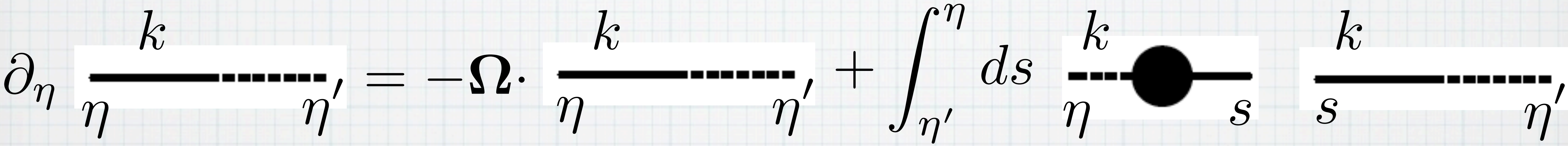
Conclusions

* Mildly non-linear scales are an unique opportunity to look for deviations from "vanilla" ΛCDM ($w \neq -1$, massive ν's, NonGaussianity, DE-DM interactions, exotic DM,...)
* Semi-analytic methods are needed to go beyond linear PT in a more transparent, flexible, and fast (!) way than NBody's
* First step towards bias. Open issues: velocity bias, from protohaloes to real haloes

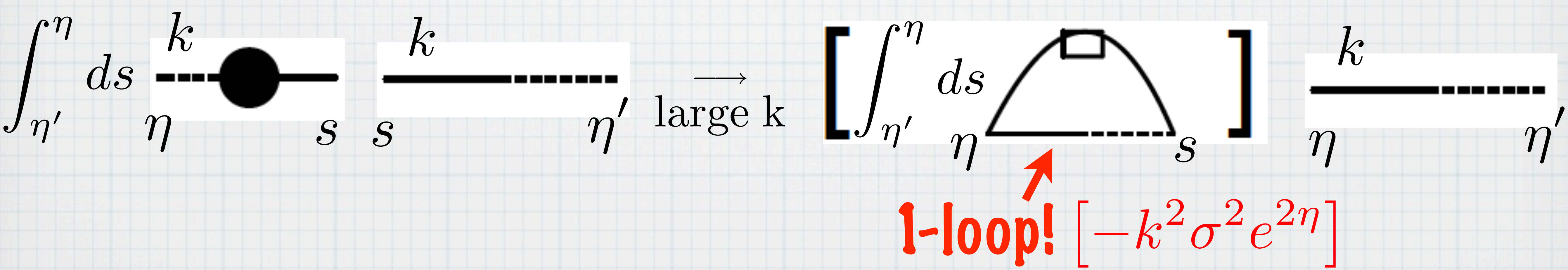
Time as the flow parameter

$$\partial_\eta G(k;\eta,\eta') = -\Omega \cdot G(k;\eta,\eta') + \int_{\eta'}^{\eta} ds\, \Sigma(k;\eta,s)\cdot G(k;s,\eta')$$

exact evolution equation for the propagator



large-momentum factorization



reproduce the Crocce-Scoccimarro resummation: $G = e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$

and allows to go beyond CS ... (Anselmi, MP, in preparation)

More General Cosmologies

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,,$$

deviation from geodesic
(e.g. DM-scalar field interaction)

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}(1+A(\vec{x},\,\tau))\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

$$\nabla^2\phi = 4\pi G\,(1+B(\vec{x},\,\tau))\,\rho\,a^2\,\delta$$

deviation from Poisson
(e.g. scale-dep. growth factor)

$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta,\,\mathbf{k}))\,\varphi_b(\eta,\mathbf{k}) = e^\eta\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

$$\boldsymbol{\Omega}_{ab} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(1+B(\eta,\,\mathbf{k})) & 2+\frac{\mathcal{H}'}{\mathcal{H}}+A(\eta,\,\mathbf{k}) \end{pmatrix} \qquad (\eta = \log a)$$

Ex: Scalar-Tensor: $\mathcal{A} = \alpha\, d\varphi/d\log a \qquad \mathcal{B} = 2\alpha^2 \qquad \alpha^2 = 1/(2\omega+3)$

see Saracco, MP, Tetradis, Pettorino, Robbers '09