Accelerating Universe in a $5D$ Cosmology

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Abstract

A model of accelerating universe is proposed in the framework of 5-dimensional cosmology. The $4D$ part as the Robertson-Walker metric is coupled to conventional perfect fluid, and its extra-dimensional part, endowed by a scalar field, is coupled to dark energy as a perfect fluid with constant density and pressure. By an appropriate rearrangement and interpretation of the scalar field terms in $5D$ Einstein equations an effective $4D$ Einstein equation arise which by an appropriate adjusting for the dark energy equation of state, may predict an accelerating universe with a ratio of matter to dark energy densities, in good agreement with observations.

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1 Introduction

According to the old suggestion of Kaluza and Klein the $5D$ vacuum Kaluza-Klein equations can be reduced under certain conditions to the $4D$ vacuum Einstein equations plus the $4D$ Maxwell equations. Recently, the idea that our four dimensional universe might have emerged from a higher dimensional space-time is receiving much attention [1]. One current interest is to find out in a more general way how the $5D$ field equations relate to the $4D$ ones. In this regard, a suggestion was made recently by Wesson in that the $5D$ equations without sources $R_{AB} = 0$ may be reduced to the $4D$ ones with sources $G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$, provided an appropriate definition is made for the energy momentum tensor of matter in terms of the extra part of the geometry [2]. Physically, the picture behind this interpretation is that curvature in $(4+1)$ space induces effective properties of matter in $(3+1)$ space-time. This idea is known as *space-time-matter* or *modern Kaluza-Klein* theory.

In a parallel way, the brane world scenario [3] assumes that our four-dimensional universe (the brane) is embedded in a higher dimensional space-time (the bulk). The important ingredient of the brane world scenario unlike the space-time-matter theory is that the matter exists apart from geometry and is confined to the brane, and the only communication between the brane and the bulk is through gravitational interaction. The brane world picture relies on a Z_2 symmetry and is inspired from string theory and its extensions [4]. This approach differs from the old Kaluza-Klein idea in that the size of the extra dimensions could be large, more or less similar to the idea in modern Kaluza-Klein theory.

On the other hand, the recent distance measurements of type Ia supernova suggest an accelerating universe [5]. This accelerating expansion is generally believed to be driven by an energy source which provides positive energy density and negative pressure, such as a positive cosmological constant [6], or a slowly evolving real scalar field called *quintessence* [7]. Since in a variety of inflationary models scalar fields have been used in describing the transition from the quasi-exponential expansion of the early universe to a power law expansion, it is natural to try to understand the present acceleration of the universe by constructing models where the matter responsible for such behavior is also represented by a scalar field [8].

In general, one would like to describe the recent behavior of the universe as a transition from a universe filled with dust-like matter ($p = 0$) to an accelerating one, and scalar fields are not the only possibility but there are (of course) alternatives. In particular, one can try to do it by using some perfect fluid but obeying exotic equations of state, the so-called Chaplygin gas [9]. This equation of state has recently raised a certain interest because of its many interesting and, in some sense, intriguingly unique features. For instance, the Chaplygin gas represents a possible unification of dark matter and dark energy, since its cosmological evolution is similar to an initial dust like matter and a cosmological constant for late times [10].

Beside the current acceleration of the universe there is another mysterious behavior in the current status of the universe, namely the *Coincidence scandal* or *Why now* problem that the observed vacuum energy and the current matter density have the same order of magnitude. Among possible solutions to this last problem the interesting idea of *holographic dark energy* has received much attention. This is based on the fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [11].

Taking into account the above questions and considerations, one may be interested in to try to combine them and ask the following question: *Is it possible to address the recent acceleration of the $4D$ universe and the coincidence scandal in a $5D$ model?* In this paper we will try to show that these behaviors may be effectively considered as a presence of the 5th dimension endowed by a scalar field.

2 The Model

We start with the $5D$ line element

$$dS^2 = g_{AB} dx^A dx^B, \tag{1}$$

in which A and B run over both the space-time coordinates α, β and one extra dimension indicated by 4. The space-time part of the metric $g_{\alpha\beta} = g_{\alpha\beta}(x^\alpha)$ is assumed to define the Robertson-Walker line element

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \tag{2}$$

where k takes the values $+1, 0, -1$ according to a close, flat or open universe, respectively. We also take the followings

$$g_{4\alpha} = 0, \quad g_{44} = \epsilon \Phi^2(x^\alpha),$$

where $\epsilon^2 = 1$ and the signature of the higher dimensional part of the metric is left general. This choice has been made because any fully covariant $5D$ theory has five coordinate degrees of freedom which can lead to considerable algebraic simplification, without loss of generality. We assume the fully covariant $5D$ Einstein equation

$$G_{AB} = 8\pi G T_{AB}, \tag{3}$$

where G_{AB} and T_{AB} are the $5D$ Einstein tensor and energy-momentum tensor, respectively. Note that unlike the Brane world gravity the $5D$ gravitational constant has been fixed to be the same value as the $4D$ one.

The $5D$ Ricci tensor in terms of the $5D$ Christoffel symbols is given by

$$R_{AB} = \partial_C \Gamma^C_{AB} - \partial_B \Gamma^C_{AC} + \Gamma^C_{AB}\Gamma^D_{CD} - \Gamma^C_{AD}\Gamma^D_{BC}. \tag{4}$$

Putting $A \to \alpha$, $B \to \beta$ in (4) gives us the $4D$ part of the $5D$ quantity. Expanding some summed terms on the r.h.s by letting $C \to \lambda, 4$ etc. and rearranging gives

$$\hat{R}_{\alpha\beta} = \partial_\lambda \Gamma^\lambda_{\alpha\beta} + \partial_4 \Gamma^4_{\alpha\beta} - \partial_\beta \Gamma^\lambda_{\alpha\lambda} - \partial_\beta \Gamma^4_{\alpha 4} + \Gamma^\lambda_{\alpha\beta}\Gamma^\mu_{\lambda\mu} + \Gamma^\lambda_{\alpha\beta}\Gamma^4_{\lambda 4} + \Gamma^4_{\alpha\beta}\Gamma^D_{4D} - \Gamma^\mu_{\alpha\lambda}\Gamma^\lambda_{\beta\mu} - \Gamma^4_{\alpha\lambda}\Gamma^\lambda_{\beta 4} - \Gamma^D_{\alpha 4}\Gamma^4_{\beta D}. \tag{5}$$

Part of this equation is the $4D$ Ricci tensor, so

$$\hat{R}_{\alpha\beta} = R_{\alpha\beta} + \partial_4 \Gamma^4_{\alpha\beta} - \partial_\beta \Gamma^4_{\alpha 4} + + \Gamma^\lambda_{\alpha\beta}\Gamma^4_{\lambda 4} + \Gamma^4_{\alpha\beta}\Gamma^D_{4D} - \Gamma^4_{\alpha\lambda}\Gamma^\lambda_{\beta 4} - \Gamma^D_{\alpha 4}\Gamma^4_{\beta D}, \tag{6}$$

where $\hat{}$ denotes the $4D$ part of the $5D$ quantities. Evaluating the Christoffel symbols gives

$$\hat{R}_{\alpha\beta} = R_{\alpha\beta} - \frac{\nabla_\alpha \nabla_\beta \Phi}{\Phi}. \tag{7}$$

Returning to Eq.(4), we put $A = 4, B = 4$ and expand with $C \to \lambda, 4$ to obtain

$$R_{44} = \partial_\lambda \Gamma^\lambda_{44} - \partial_4 \Gamma^\lambda_{4\lambda} + \Gamma^\lambda_{44}\Gamma^\mu_{\lambda\mu} + \Gamma^4_{44}\Gamma^\mu_{4\mu} - \Gamma^\lambda_{4\mu}\Gamma^\mu_{4\lambda} - \Gamma^4_{4\mu}\Gamma^\mu_{44}. \tag{8}$$

Putting the corresponding Christoffel symbols in Eq.(8) leads to

$$R_{44} = -\epsilon \Phi \Box \Phi. \tag{9}$$

We now construct the space-time components of the Einstein tensor

$$G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R_{(5)}.$$

In so doing, we first obtain the $5D$ Ricci scalar $R_{(5)}$ as

$$R_{(5)} = g^{AB}R_{AB} = \hat{g}^{\alpha\beta}\hat{R}_{\alpha\beta} + g^{44}R_{44} = g^{\alpha\beta}(R_{\alpha\beta} - \frac{\nabla_\alpha\nabla_\beta\Phi}{\Phi}) + \frac{\epsilon}{\Phi^2}(-\epsilon\Phi\Box\Phi)$$
$$= R - \frac{2}{\Phi}\Box\Phi, \tag{10}$$

where the $\alpha 4$ terms vanish and R is the $4D$ Ricci scalar. The space-time components of the Einstein tensor is written $\hat{G}_{\alpha\beta} = \hat{R}_{\alpha\beta} - \frac{1}{2}\hat{g}_{\alpha\beta}R_{(5)}$. Substituting $\hat{R}_{\alpha\beta}$ and $R_{(5)}$ into the space-time components of the Einstein tensor gives

$$\hat{G}_{\alpha\beta} = G_{\alpha\beta} + \frac{1}{\Phi}(g_{\alpha\beta}\Box\Phi - \nabla_\alpha\nabla_\beta). \tag{11}$$

In the same way, the 4-4 component is written $G_{44} = R_{44} - \frac{1}{2}g_{44}R_{(5)}$, and substituting R_{44}, $R_{(5)}$ into this component of the Einstein tensor gives

$$G_{44} = -\epsilon\Phi\Box\Phi - \frac{1}{2}\epsilon R\Phi^2 + \epsilon\Phi\Box\Phi, \tag{12}$$

where we keep the first and third terms for our next purpose.

We now consider the $5D$ energy momentum tensor in the form of perfect fluid

$$T_{AB} = (\rho + p)_{(5)}U_A U_B - p_{(5)}g_{AB}, \tag{13}$$

with $U_A = (1, 0, 0, 0, 0)$. The $4D$ part of the $5D$ energy momentum tensor is given

$$\hat{T}_{\alpha\beta} = (\rho + p)_{(4)}\hat{U}_\alpha\hat{U}_\beta - p_{(4)}\hat{g}_{\alpha\beta}$$
$$= (\rho + p)u_\alpha u_\beta - pg_{\alpha\beta}, \tag{14}$$

where ρ and p are the conventional density and pressure of perfect fluid in the $4D$ standard cosmology. Here, we make an important assumption that the dark energy can exist in higher dimensional sector of T_{AB} through (different) constant density $\bar{\rho} > 0$ and constant pressure $\bar{p}$ with the equation of state $\bar{p} = \bar{\omega}\bar{\rho}$. This may be fulfilled by taking

$$T_{44} = (\bar{\rho} + \bar{p})U_4 U_4 - \bar{p} g_{44} = -\epsilon \bar{p} \Phi^2. \tag{15}$$

Substituting the energy momentum components (14), (15) in front of the $4D$ and extra dimensional part of Einstein tensors (11) and (12), respectively, we obtain[1]

$$G_{\alpha\beta} = 8\pi G[(\rho + p)u_\alpha u_\beta - p g_{\alpha\beta}] + \frac{1}{\Phi}\left[\nabla_\alpha \nabla_\beta \Phi - \Box\Phi g_{\alpha\beta}\right], \tag{16}$$

and

$$-\epsilon\Phi\Box\Phi - \frac{1}{2}\epsilon R\Phi^2 + \epsilon\Phi\Box\Phi = -8\pi G\epsilon\bar{p}\Phi^2. \tag{17}$$

The last equation, by eliminating the first and third terms, says that the $4D$ constant curvature R is set to $16\pi G\bar{p}$ or $16\pi G\bar{\omega}\bar{\rho}$, and it means that the dark energy penetrated to higher dimension determines the curvature of the $4D$ universe. On the other hand, with no loss of generality one can assume

$$\Box\Phi - \frac{1}{2}(R + 16\pi G\bar{\rho})\Phi = 0, \tag{18}$$

which, using (17), leads to the result

$$\Box\Phi = 8\pi G(\bar{\omega} + 1)\bar{\rho}\Phi. \tag{19}$$

Equation (18) gives the scalar field potential

$$V(\Phi) = -\frac{1}{4}(R + 16\pi G\bar{\rho})\Phi^2, \tag{20}$$

[1] The $\alpha 4$ components of Einstein equation (3) result in

$$R_{\alpha 4} = 0,$$

which is an identity with no useful information.

whose minimum occurs at $\Phi = 0$, where the equations (16), (17) reduce to describe a usual $4D$ FRW universe filled with ordinary matter. In other words, our $4D$ universe without dark energy corresponds to the ground state of the scalar field Φ. It is then appealing to assume that the scalar field excitations around this minimum may contribute to the dark energy as follows

$$\frac{1}{\Phi}\nabla_\alpha\nabla_\beta\Phi = 8\pi G(\bar{\rho} + \bar{p})u_\alpha u_\beta, \tag{21}$$

$$\frac{1}{\Phi}\Box\Phi = 8\pi G(\bar{p} + \bar{\rho}), \tag{22}$$

where the latter is the same equation (19). Contracting Eq.(21) by $g^{\alpha\beta}$ we obtain (22); so both assumptions (21), (22) are consistent with each other. Using these equations in the r.h.s of Eq.(16) we obtain

$$G_{\alpha\beta} = 8\pi G\left[[(\rho + p)u_\alpha u_\beta - p g_{\alpha\beta}] + [(\bar{\rho} + \bar{p})u_\alpha u_\beta - (\bar{\rho} + \bar{p})g_{\alpha\beta}]\right], \tag{23}$$

where we interpret the second part in the r.h.s as the *effective* energy momentum tensor of dark energy induced on the $4D$ universe. As long as only states of the universe differing not too widely from the present one are considered we may take $p = 0$. Therefore, Eq.(23) reduces to

$$\begin{aligned} G_{\alpha\beta} &= 8\pi G[(\rho + \bar{\rho} + \bar{p})u_\alpha u_\beta - (\bar{\rho} + \bar{p})g_{\alpha\beta}] \\ &= 8\pi G\left[(\rho + \bar{\rho}(1 + \bar{\omega}))u_\alpha u_\beta - \bar{\rho}(1 + \bar{\omega})g_{\alpha\beta}\right] \\ &= 8\pi G\left[(\rho + \tilde{p})u_\alpha u_\beta - \tilde{p} g_{\alpha\beta}\right]. \end{aligned} \tag{24}$$

where $\tilde{p} = \bar{\rho}(\bar{\omega} + 1)$ is the redefined pressure of dark energy. This energy momentum tensor effectively describes a perfect fluid with density ρ and pressure $\tilde{p}$. The matter and dark energy contributions come in such a way that $p = 0$ for the matter and $\tilde{\rho} = 0$ for the dark energy.

Therefore, this model describe effectively a universe filled with *dust* matter $\rho > 0$ and *dark* pressure $\tilde{p} < 0$.

The field equations lead to two independent equations

$$3\frac{\dot{R}^2 + k}{R^2} = 8\pi G\rho, \tag{25}$$

$$\frac{2R\ddot{R} + \dot{R}^2 + k}{R^2} = -8\pi G\tilde{p}. \tag{26}$$

Differentiating (25) and combining with (26) we obtain the fluid equation

$$\frac{d}{dt}(\rho R^3) + \tilde{p}\frac{d}{dt}(R^3) = 0, \tag{27}$$

or

$$\frac{d}{dt}[(\rho + \tilde{p})R^3] = 0, \tag{28}$$

resulting in

$$(\rho + \tilde{p}) \sim R^{-3}. \tag{29}$$

The equations (25) and (27) can be used to derive the acceleration equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3\tilde{p}). \tag{30}$$

For a universe to be accelerated we require $\ddot{R} > 0$ which means

$$\rho < -3\bar{p}(1 + \bar{\omega}). \tag{31}$$

In order the energy condition for matter $\rho \geq 0$ is satisfied we have to take

$$\bar{\omega} < -1. \tag{32}$$

A negative $\bar{\omega}$, in the first place, will lead to negative pressure $\tilde{p}$ resulting in the violation of strong energy condition $(\rho + 3\tilde{p}) < 0$ for the total perfect fluid (24) together with the acceleration of the universe ($\ddot{R} > 0$). In the second place, it results in an open universe since $R = 16\pi G\bar{p} < 0$. One may then choose an appropriate value for $\bar{\omega}$ so as to make a good agreement with the current observations on the ratio of matter to dark energy densities. Taking, for instance, $\bar{\omega} = -\frac{8}{7}$ gives

$$\frac{\rho}{\bar{\rho}} \approx \frac{\%30}{\%70}, \tag{33}$$

which is in good agreement with the current observations.

The reason for the current acceleration in this model is simple. If we keep $p \neq 0$ in the matter perfect fluid in (23) and the following equations, we obtain the modified acceleration equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}[\rho + 3(\tilde{p} + p)], \tag{34}$$

and acceleration condition

$$\rho \leq -3[\bar{\rho}(1 + \bar{\omega}) + p]. \tag{35}$$

Since the density $\bar{\rho}$ of dark energy as the vacuum energy is assumed to be very small, as long as we have a considerable positive pressure p such that $[\rho + 3(\tilde{p} + p)] > 0$ the r.h.s of (34) becomes negative. For some smaller positive pressure for which $p = -\frac{1}{3}(\rho + 3\tilde{p})$ the r.h.s becomes zero. So, for these values of p we do not expect acceleration. But, when $p = 0$ we certainly predict an acceleration according to (31), (32).

Conclusion

In this paper, we have studied a $(4+1)$-dimensional metric subject to a $(4+1)$ dimensional energy momentum tensor. The $4D$ part of the metric is taken to be Robertson-Walker one subject to the conventional perfect fluid with density ρ and pressure p, and the extra-dimensional part endowed by a scalar field is subject to a perfect fluid, violating strong energy condition, with constant density $\bar{\rho}$ and constant pressure $\bar{p}$ different from conventional ones. By writing down the reduced $4D$ and extra-dimensional components of $5D$ Einstein equations, and interpreting the scalar field excitations as contributing terms to dark energy in the form of perfect fluid with $\bar{\rho}$, $\bar{p}$ related by equation of state $\bar{p} = \bar{\omega}\bar{\rho}$, we obtained an accelerating $4D$ universe where the ratio of matter to dark energy densities is in good agreement with observations. This result is independent of the signature ϵ by which the higher dimension takes part in the $5D$ metric.

This model also offers a solution to the coincidence scandal: the reason why we observe the acceleration, when the matter density is comparable to dark energy one, is that we live in the epoch of the universe's history during which the galaxies and life are formed in the era $p = 0$.

Since no compactification of Kaluza-Klein or Brane world type is considered in this model, a question arises that: why we perceive the 4 dimensions of space-time and apparently do not see the fifth dimension? The answer is simple: the usual matter, of which all bodies in the universe are made, is coupled to space-time and we cane probe the space-time because we can probe the matter and its inertial properties. However, the extra dimension is coupled to a perfect fluid violating the strong energy condition and since we can not probe this type of matter directly we can not probe the extra dimension directly, as well. We may just see the indirect effects of the presence of extra-dimension, namely the acceleration and the curvature of our $4D$ universe. The very small dark energy (vacuum energy) density $\bar{\rho}$, according to $R = 16\pi G\bar{\omega}\bar{\rho}$, will result in an almost flat universe which is also in good agreement with the current observations.

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