# Direct measurement of Hubble parameter with gravitational waves

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# Abstract

- Standard candle

  Type-Ia supernova data provide important evidence about cosmic acceleration (dark energy). However, there are various systematic erorrs, e.g. uncertainty in the light curve, extinction by dust etc.

- Standard siren

  Gravitational waves from a binary object are often called standard siren, because they can be an unique tool to measure cosmological expansion with a good precision.

- Our work

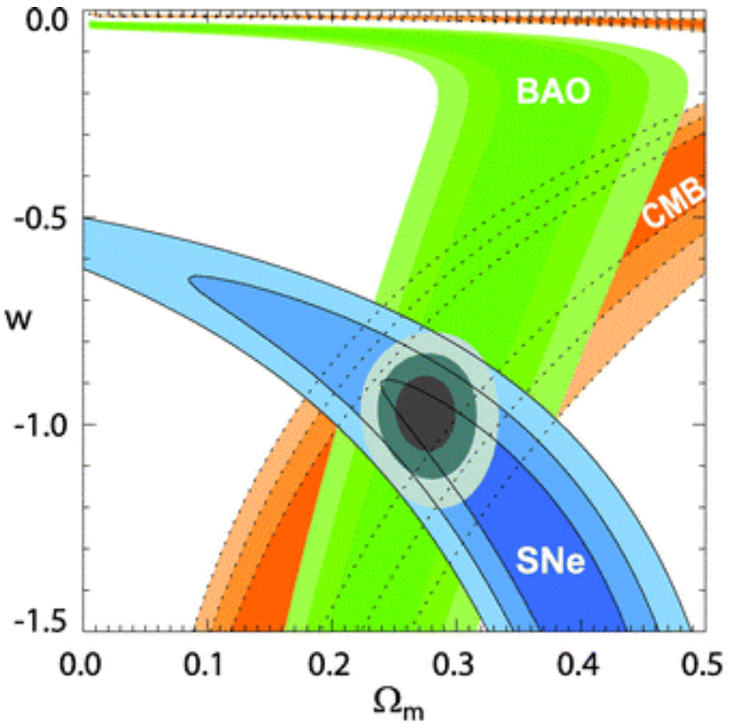
  We show that the Hubble parameter at each redshift, H(z), can be measured with the GW standard siren. Supposing future space-based GW detector DECIGO and neutron star binaries observed by DECIGO, we found that H(z) can be measured with 3-10% accuracy up to z=1.

# Constraints on dark energy

[ Kowalski+ 2008 ]

$p = w\rho$ ( $w$ : const.)

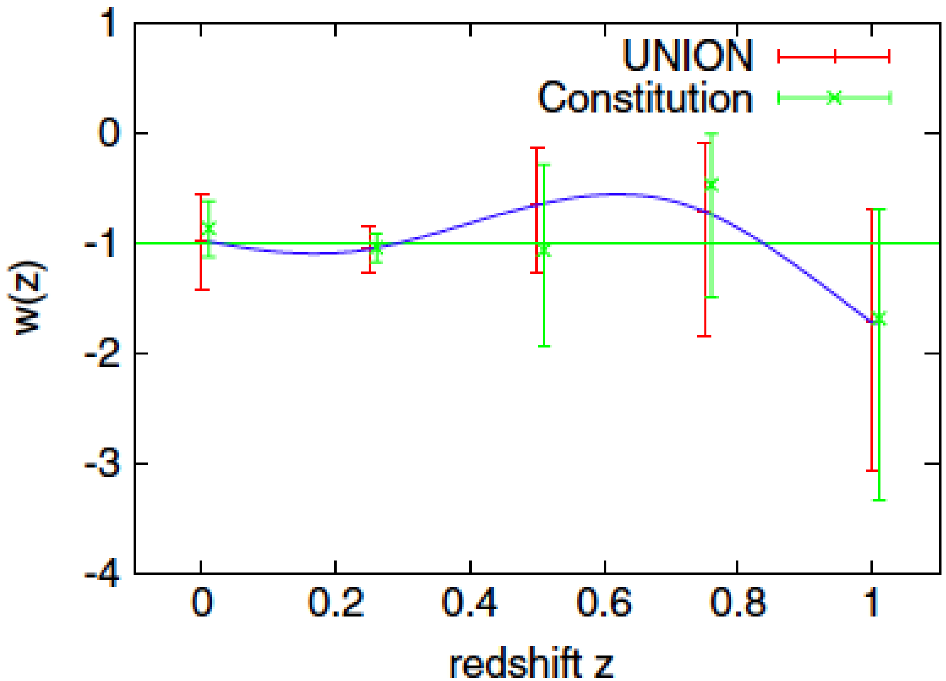
Assume flat universe.




[ Serra+ 2009 ]

w(z) in each bin is uncorrelated.

WMAP+BAO+SNe




Determination of time variation of w is difficult,
but less than 10% accuracy is needed to discriminate
various theoretical models.

# Standard siren

[ Schutz 1986 ]

Gravitational waves from a compact binary

chirp mass

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\dot{f}(t) \propto [(1+z)M_c]^{5/3} f^{11/3},$$

GW waveform

$$h_{+\times}(t) \propto \frac{[(1+z)M_c]^{5/3} f^{2/3}}{D_L},$$

From observational data,

$$h_{+\times}, f, \dot{f},$$

⇩

redshifted chirp mass

$$M_z \equiv (1+z)M_c$$

⇩

luminosity distance

$$D_L:$$

Luminosity distance is determined by GW observation.

# Standard siren

The redshift can NOT determined only by GW observation.

Assuming the redshift is determined by EM observation
of the host galaxy,

$M_c$ can be determined.
z-dL relation can be obtained.

⇩

Measurement of cosmological expansion by GWs

No need of distance ladder.
Consistency test of SNe observation.

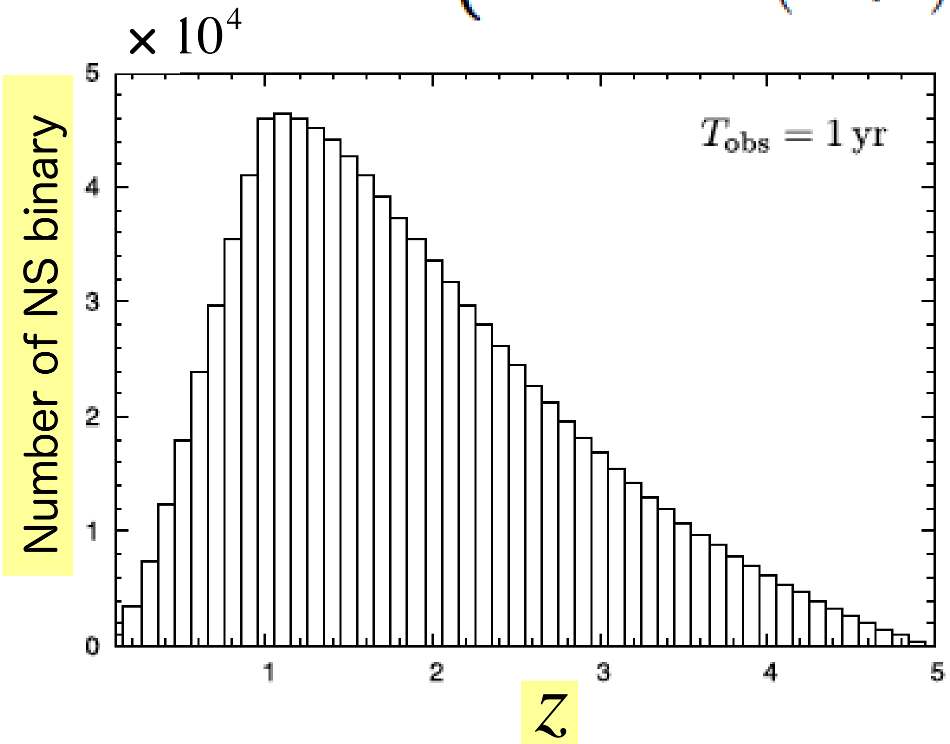
# z distribution of NS binary

Linear fitting of the estimation by Schneider+ 2001
[ Cutler & Holz 2009 ]

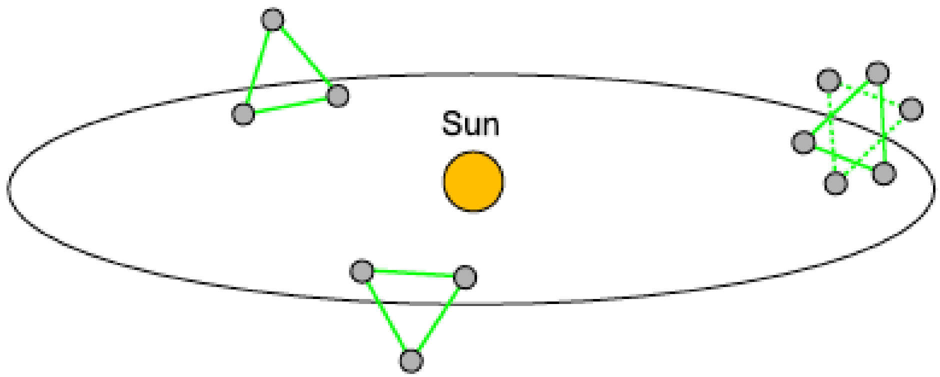
NS binary
merger rate
[ 1/Mpc^3 /yr ]

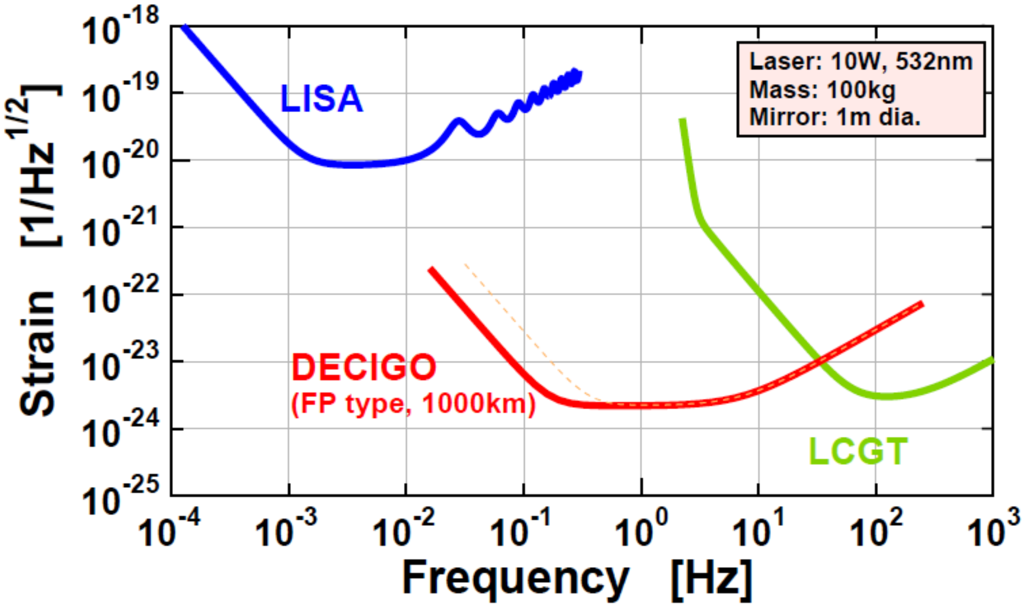
$$\dot{n}(z) = \dot{n}_0 \, r(z) \,,$$

$$r(z) = \begin{cases} 1 + 2z & (z \le 1) \\ \frac{3}{4}(5 - z) & (1 < z \le 5) \\ 0 & (5 < z) \end{cases}$$



# DECIGO



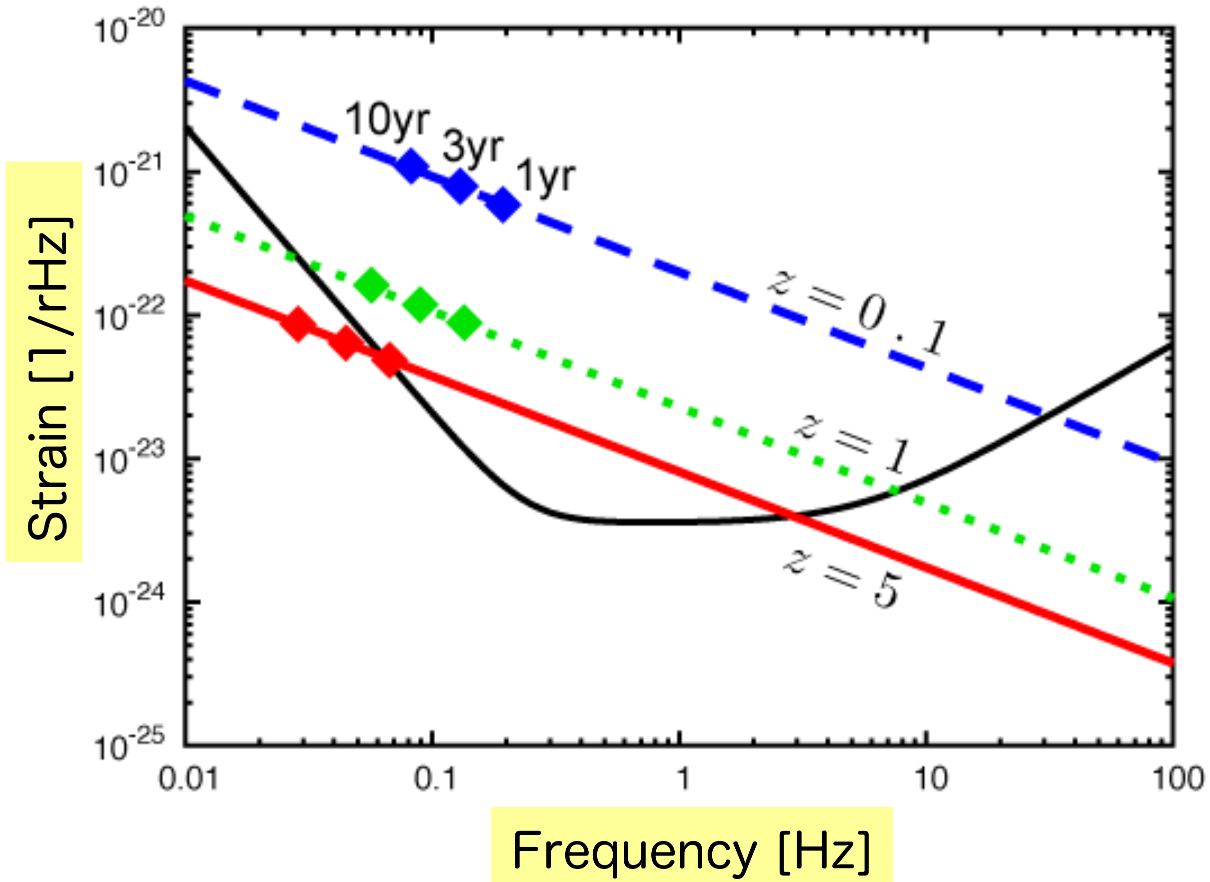

Deci-hertz Interferometer
Gravitational wave
Observatory

- Launch 2027-
- 4 clusters
- independent
  8 interferometers
- arm length： 1000 km
- Fabry-Perot cavity
- finnese: 10
- targeted sources
  inflationary GWB,
  IMBH binaries,
  NS binaries

# NS binaries seen by DECIGO

| | | |
|---|---|---|
| z=0.1 | | S/N~700 |
| z=1 | ⇨ | ~80 |
| z=10 | | ~20 |

$\sim 10^6$ NS binaries
up to z~5 are observable
with high SNR.



# Why do we need to measure H(z)?

- Luminosity distance is an integrated quantity of H(z)

$$d_L^{(0)}(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

It's likely to average out small structures on cosmic expansion.

- Parameterization of dark energy

$$H(z) = H_0 \left\{ \Omega_m (1+z)^3 + (1-\Omega_m) \exp\left[ 3 \int_0^z dz' \frac{1+w(z')}{1+z'} \right] \right\}^{1/2}$$

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

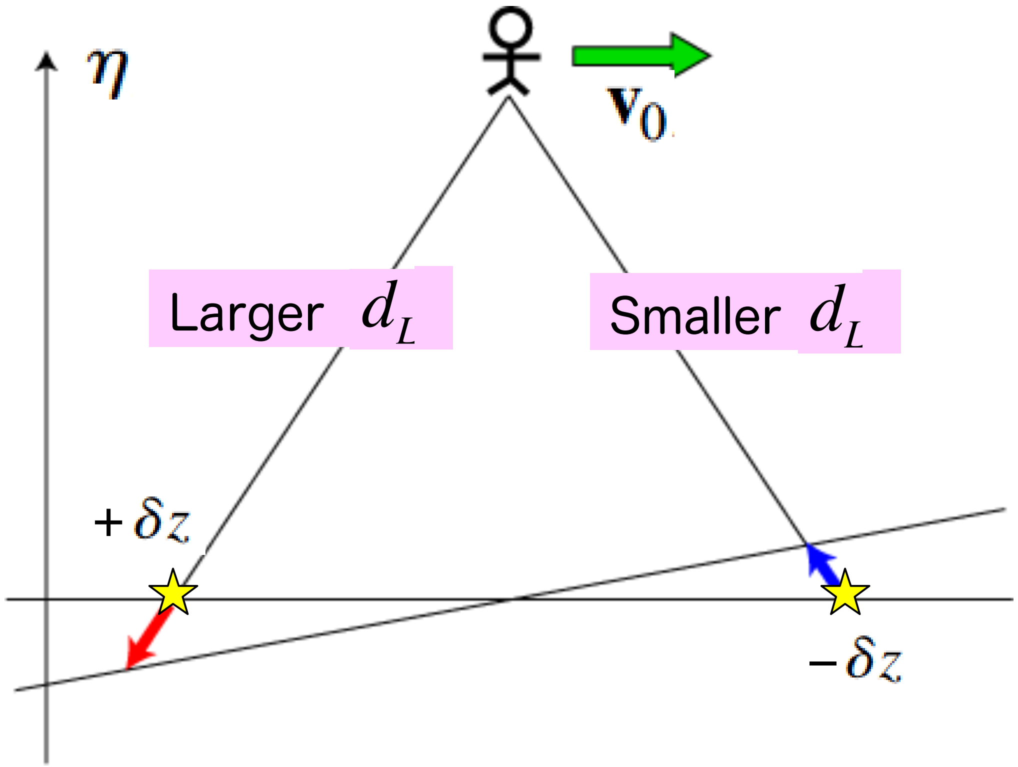
Measurement accuracy of cosmological parameters strongly depends on the parameterization,
which sometimes makes the observational errors amplified.

To avoid those issues, we need to directly measure H(z).

# Dipole of luminosity distance

[ Bonvin+ 2006 ]

Method was first proposed
in the context of a supernova.




Observer is at rest
relative to CMB frame.
(we see only monopole)

Observer is moving
with respect to the CMB frame

⇩

Luminosity of a binary
is Doppler-shifted.

⇩

Dipole of the luminosity
Distance is induced.

# Measurement of H(z)

$$d_L(z,\mathbf{n}) = \sum_{\ell,m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n})$$

Approximated by taking up to the dipole term

$$d_L(z,\mathbf{n}) = d_L^{(0)}(z) + d_L^{(1)}(z)(\mathbf{n}\cdot\mathbf{e}).$$

monopole $$d_L^{(0)}(z) = \frac{1}{4\pi}\int d\Omega_{\mathbf{n}} d_L(z,\mathbf{n}) = (1+z)\int_0^z \frac{dz'}{H(z')},$$

dipole $$d_L^{(1)}(z) = \frac{3}{4\pi}\int d\Omega_{\mathbf{n}} (\mathbf{n}\cdot\mathbf{e}) d_L(z,\mathbf{n}). = \frac{|\mathbf{v}_0|(1+z)^2}{H(z)},$$

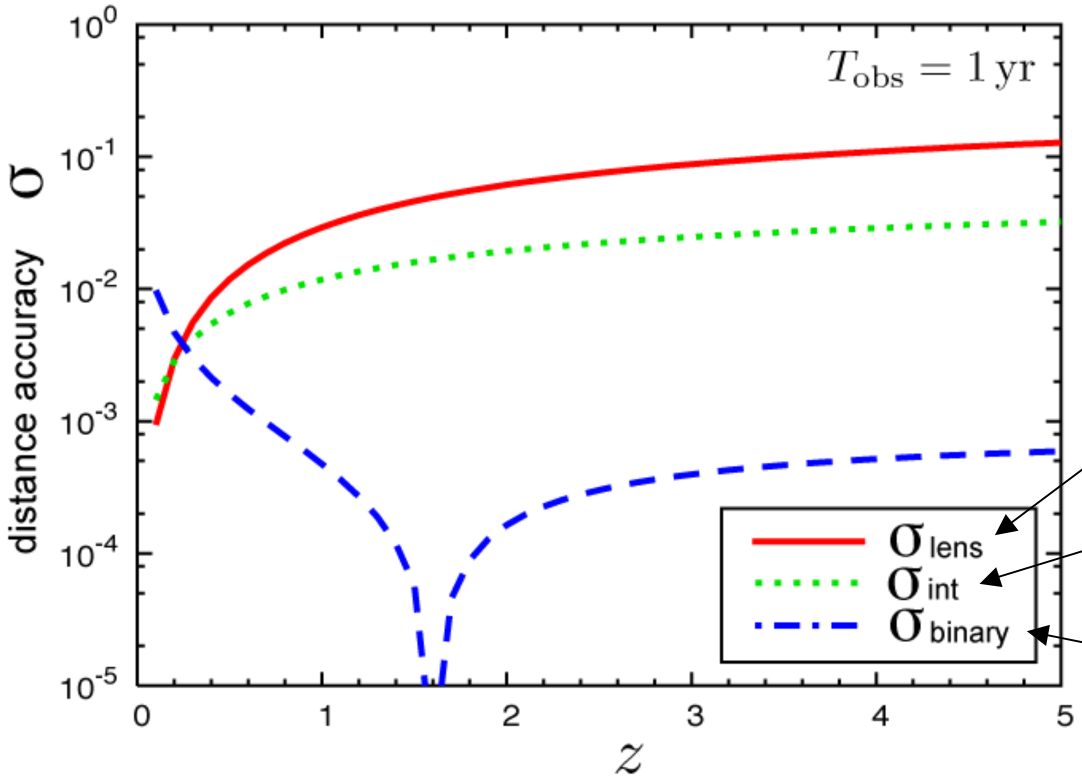
If $v_0$ is known from CMB measurement,
H(z) can be directly measured.

$$\frac{\Delta H(z)}{H(z)} = \frac{\Delta d_L^{(1)}(z)}{d_L^{(1)}(z)} = \sqrt{3}\left[\frac{d_L^{(1)}(z)}{d_L^{(0)}(z)}\right]^{-1}\left[\frac{\Delta d_L^{(0)}(z)}{d_L^{(0)}(z)}\right]$$

# Measurement accuracy of $d_L^{(0)}(z)$

For a single binary

$$\left[\frac{\Delta d_L^{(0)}(z)}{d_L^{(0)}(z)}\right]^2 = \sigma_{\rm int}^2(z) + \sigma_{\rm lens}^2(z) + \sigma_{\rm binary}^2(z)$$




Accuracy is limited by lensing error at almost all z.

Gravitational lensing error

Intrinsic error (no systematic error)

Binary peculiar velocity error (300km/s)

Random errors are reducible by observing many binaries.

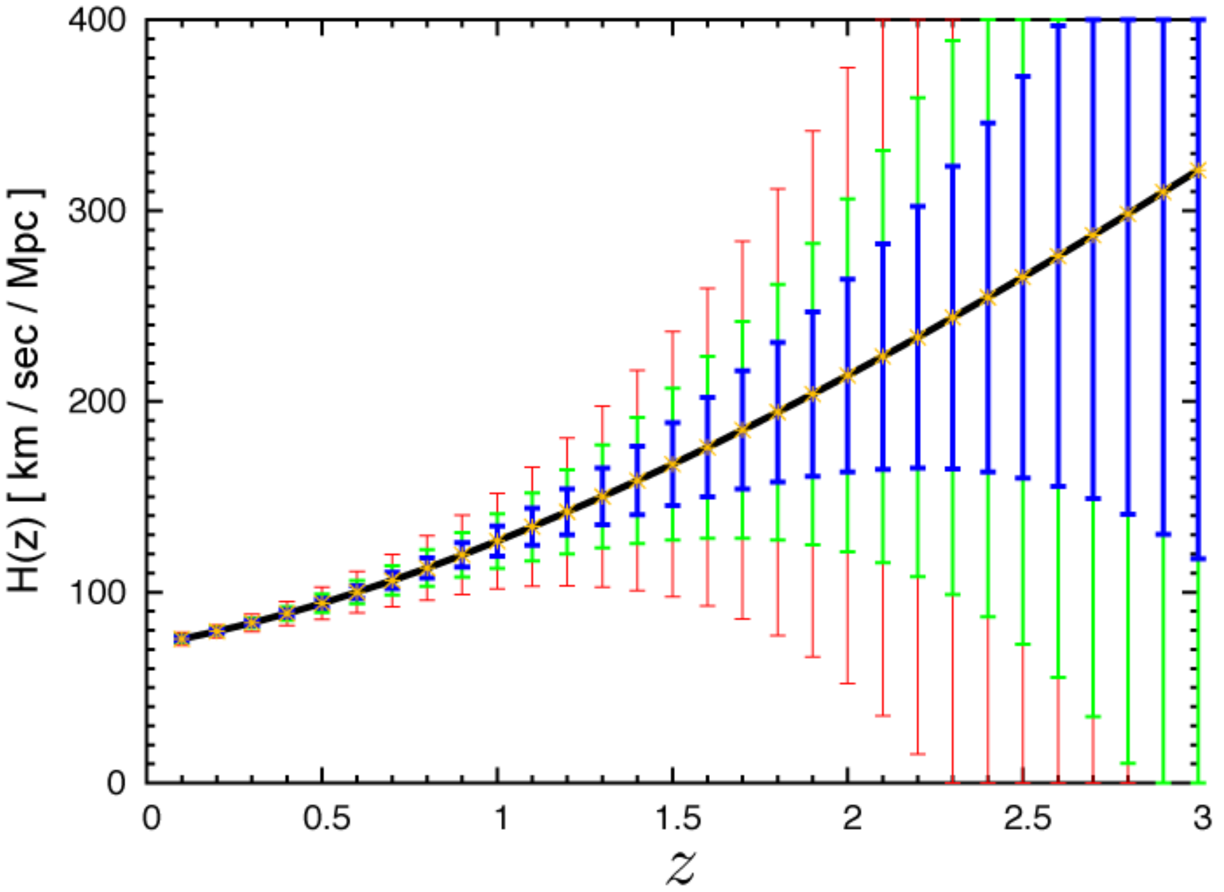
# Measurement error of H(z)

For all observed binaries

Solid curve : fiducial cosmological model

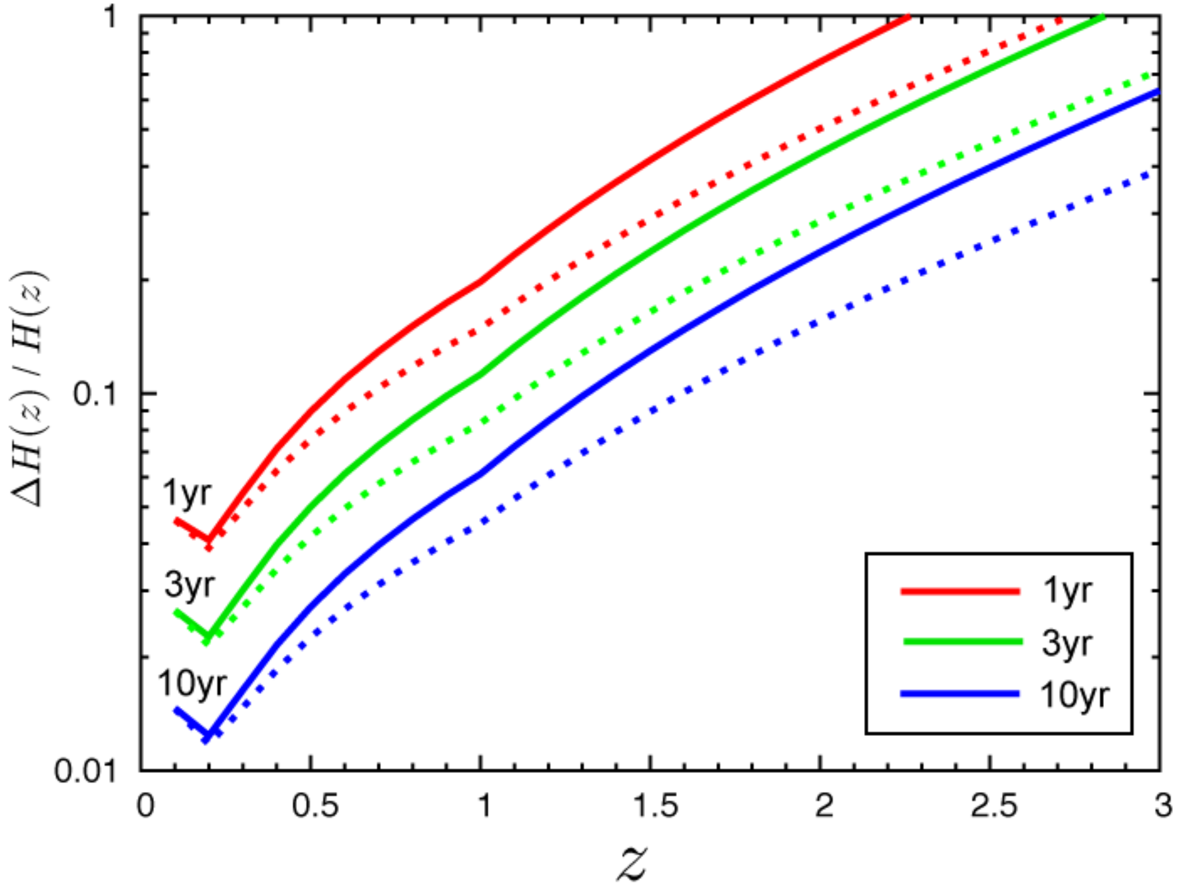
( $w_0 = -1,\ w_a = 0,\ \Omega_m = 0.3,\ H_0 = 72\,\mathrm{km/sec/Mpc}$ )

Error bars: red (observation time = 1yr), green (3yr), blue (10yr)



# Measurement accuracy of H(z)

Solid curves: with lensing error,
Dotted curves: no lensing error (somehow subtracted)




Assuming 3yr observation,
H(z) can be constrained with 3-10% accuracy up to z=1.

# Summary

We proposed the method to directly measure H(z) with a space-based GW detector, DECIGO and BBO.

- from measurement of the dipole of luminosity distance: DECIGO can measure H(z) at 3-10% level below z=1 for 3-yr observation.

- Comparison with other methods

SNe : To measure H(z) with 3% at z=0.1-0.5, $10^5 - 10^6$ samples are needed. This is unrealistic.

BAO : Future project can measure H(z) with accuracy 1% at z=1 and above. It's complementary to our method, because the sensitive z-range is different.