The Cosmological Constant Problem and its Possible Solutions

Pisin Chen

LeCosPA, National Taiwan University/KIPAC, Stanford

COSMO/CosPA2010
Tokyo, Japan, Sept. 17- Oct. 1, 2010

Introduction

- Cosmological constant has long been a problem in theoretical physics during most of the 20th century.

- Since 1998, it has further become one of the most challenging issue in astrophysics in the new century.

- Several excellent review articles:
  S. Weinberg (1989), Carroll (2000), Sahni & Starobinsky (2000, 2006), Peebles & Ratra (2002), Padmanabhan (2003)…

More than 1000 papers in arXiv that has ‘cosmological constant’ in the title. Can’t possibly cover all ideas.

What is the Problem?

- History

After completing his formulation of general relativity (GR), Einstein (1917) introduced a cosmological constant (CC) to his eq. for the universe to be static:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}.$$

As is well-known, he gave up this term after Hubble's discovery of cosmic expansion.

Unfortunately, not so easy to drop it.

> In GR, anything that contributes to the energy density of the vacuum acts like a CC.

The Old CC Problem

The old (< 1998):

- Lorentz invariance, upon which QFT is based, tells us that in the vacuum the energy-momentum tensor must take the form

$$\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}.$$

This is equivalent to adding a term to CC:

$$\Lambda_{eff} = \Lambda + 8\pi G\langle\rho\rangle. \Longleftrightarrow \rho_V = \langle\rho\rangle + \Lambda/8\pi G = \Lambda_{eff}/8\pi G.$$

- Quantum vacuum (zero point) energies with cutoff at Planck scale gives

$$\rho_V \sim M_{Pl}^4 \sim 10^{112} eV^4.$$

- Astrophysics, however, demands that it must be smaller than the critical density of the universe:

$$\rho_V \leq \rho_{cr} \sim 10^{-12} eV^4.$$

This is 124 orders of magnitude in discrepancy!

- Evidently QVE should not gravitate. Otherwise our universe would not have survived until now.

- This conflict between GR and quantum theory is the essence of the longstanding CC problem, which clearly requires a resolution. In short,

$$\boxed{\text{Why doesn't quantum vacuum energy gravitate?}}$$

We shall call this the “old” CC problem.

The New CC Problem, or the Dark Energy Puzzle

The new (>1998)

- The dramatic discovery of the accelerating expansion of the universe ushers in a new chapter of the CC problem.

- The substance responsible for it is referred to as the dark energy (DE), described by its equation of state

$$p = w\rho, \qquad p\text{: pressure, } \rho\text{: density}$$

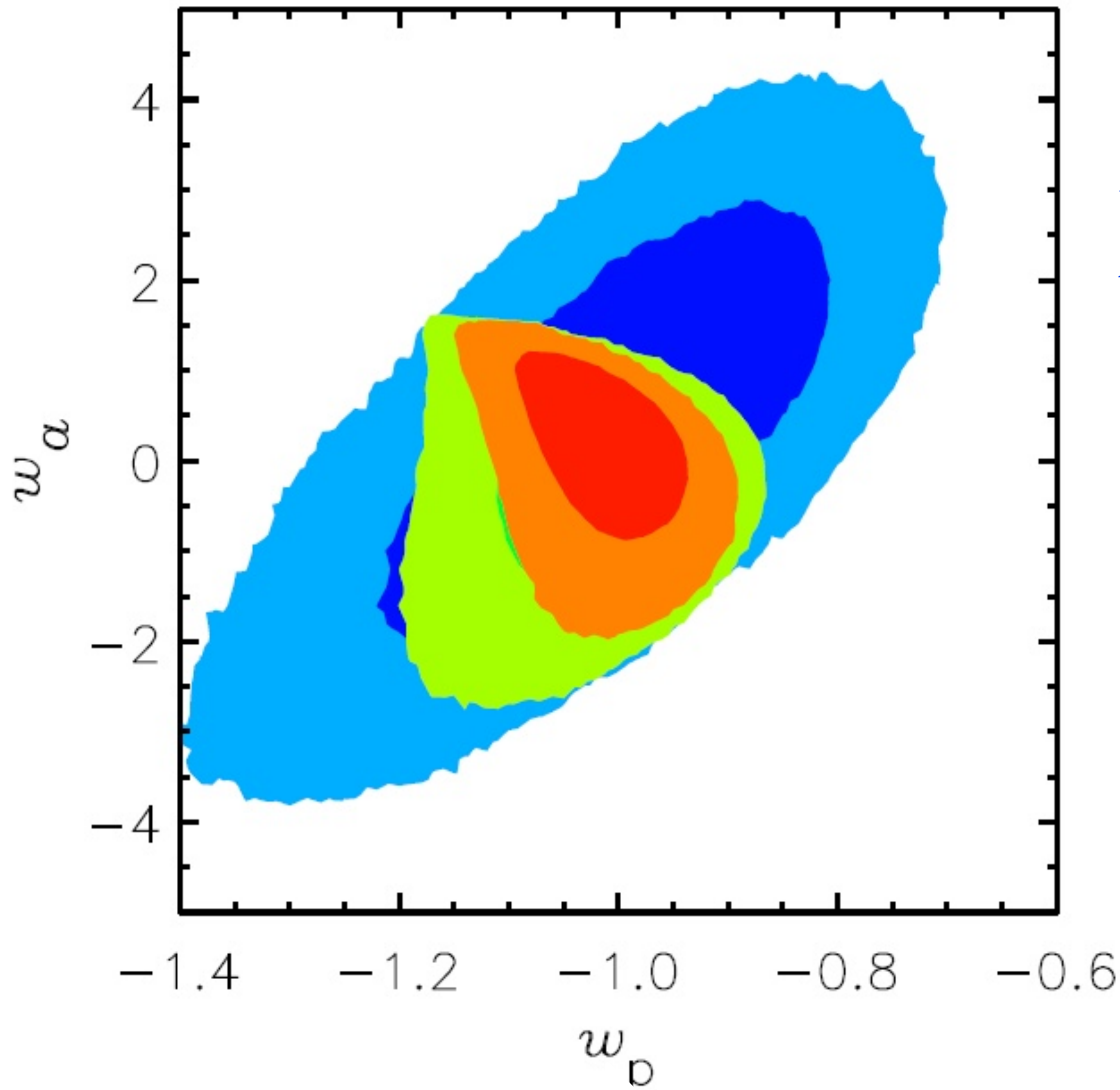
- According to GR, accelerating expansion can happen if $w < -1/3$. Einstein's CC corresponds to $w = -1$.

- DE=CC remains the simplest and most likely answer.
- New challenge: after finding a way, hopefully, to cancel the CC to 124 decimal points, how do we reinstate 1 to the last digit and keep it tiny? That is,

Why is CC nonzero but tiny?

- We shall call this the “new” CC problem, or the DE puzzle.

Observations show that w ~ -1
⇨ Dark Energy = CC?



$$p = w\rho$$

$$w = w_0 + w_a(1-a)$$

2009 data:

$$w_0 = -0.97^{+0.12}_{-0.07}$$

$$w_a = 0.03^{+0.26}_{-0.75}$$

Possible Solutions

- Fundamental Physics
  - SUSY (Zumino 1975)
    In 4D global field theories, SUSY, if unbroken, would imply a vanishing vacuum energy.
    Unfortunately SUSY is broken at low energies.

  - Extra Dimensions (Rubakov-Shaposhnikov 1983)
    In 4+2D pure gravity with a Λ-term, a classical solution was found with $\Lambda = 0$.
    However, there also exist nonvanishing solutions and the case of $\Lambda = 0$ not preferred.

- Fundamental Physics (continued)
  - Wormhole Solutions (Hawking, Coleman, Giddings, Strominger, 80s)

    Wormhole solutions in Euclidean formulation of quantum gravity do not lead to a loss of quantum coherence. $\Longrightarrow$ CC=0 at late times.

    However, Unruh (1989) showed that wormhole solutions do lead to quantum decoherence.

  - Superstring/Brane World (Rohm, Polchinski, Moore)

    Atkin-Lehner symmetry in some 2D compactifications makes the sum of total vacuum energy vanish.

    However, it’s generally hard to make effective CC small.

- Superstring/Brane World

  If bare CC can be made zero, then the quan. correction to CC (Casimir energy) induced by a bulk fermion in RS model can be naturally small without fine-tuning (S-H Shao & PC, 2010).

  However, this does not actually solve the CC problem.

- QCD Condensate

  * QCD condensates give zero contribution to CC, since all of the gravitational effects of the in-hadron condensates are already included in the normal contribution from hadron masses. (Brodsky-Shrock 2010)
  * Applicable to Higgs vev if electroweak symmetry breaking occurs via a technicolor-type mechanism.

  But there are still other fields whose eve contri. to CC.

Possible Solutions

- Adjustment Mechanism
  - (Dolgov, Wilczek-Zee, Peccei et al., Barr-Hochberg…)

  Suppose a scalar field whose source is proportional to the trace of the energy-momentum tensor:

  $$\Box^2 \phi \propto T^{\mu}_{\mu}(\phi) \propto R.$$

  Suppose in addition that $T^{\mu}_{\mu}(\phi)$ vanishes at some equilibrium value ϕ_0. Then Einstein field eq. have flat-space solution, i.e., $\Lambda = 0$.

  Unfortunately, one still cannot avoid vev from massless fields.

  - Quintessence (Steinhardt…)

  Approaches CC-like state at late times.

Possible Solutions

- Changing Gravity
  - CC as Constant of Integration (Unimodular Gravity)
    (van der Bij 82, Weinberg 83, Wilzcek-Zee 83, Buchmüller-Dragon 88, Ng-van Dam 99, Smolin 09, Cook 09, PC 10)

If the determinant g is not dynamical, then only the traceless part of the Einstein eq. needs to vanish:

$$R^{\mu\nu} - \frac{1}{4} g^{\mu\nu} R = -8\pi G \left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha} \right).$$

* Conservation law and Bianchi identities still hold:

$$T^{\mu\nu}_{\ ;\mu} = 0; \qquad \left(R^{\mu\nu} - (1/2) g^{\mu\nu} R \right)_{;\mu} = 0.$$

Taking covariant derivatives, the traceless Einstein eq.

$$\frac{1}{4} \partial_\mu \left(R - 8\pi G T^{\alpha}_{\ \alpha} \right) = 0.$$

Einstein eq. recovered: $= -4\Lambda$ (constant)

- Changing Gravity (continued)

CC not any more a ‘substance’, e.g., vacuum energy, but a const. of integration dependent on spacetime boundary conditions.

Key Q: Can any quantum theory of gravity give rise to this formulation as its classical limit?

- Jacobi’s Action for GR (Brown-York, 1989)

* In Jacobi’s form of action principle energy, not t, is fixed.

* Using Jacobi’s action, the time-dependent Wheeler -DeWitt eq. can be derived in Schrödinger form, where CC plays the role of energy as a constant of motion.

- Gauge Theory of Gravity with de Sitter Symmetry (PC, arXiv:1002.4275)

1. Gauge theory of gravity to substitute Einstein's GR as the fundamental theory of gravity;
2. The universe obeys de Sitter symmetry.

- Neither of these two ideas are new.

A Brief History

- Gauge theory of gravity (GG)
  - C. N. Yang first formulated integral formalism of GG in 74
    3^{rd} order differential field eq. of the metric
  - Further investigated by many authors:
    Thompson (75), Nie (75),Szczyrba (87), Gronwald-Hehl (96), etc.
  - Post-DE era, R. Cook (08):
    q. vacuum energy cannot be a source of gravity in GG;
    DE associated with constant of integration

- de Sitter (dS) symmetry of the universe
  - Luigi Fantappie (54), Bacry-Levy-Leblond (68)
  - Post-DE era, H.Y. Guo et al. (04), Aldrovandi et al. (07), Cacciatori et al. (08), Zee (10)

- Motivations for Gauge theory of gravity (GG)
  - To reformulate gravity as a gauge theory
  - To hopefully quantize gravity theory
  - To substantiate the ‘constant of integration’ approach as a means to solve the CC problem.

C. N. Yang (1983): “In [ ] I proposed that the gravitational equation should be changed to a third order equation. I believe today, even more than 1974, that this is a promising idea, because the third order equation is more natural than the second order one and because quantization of Einstein’s theory leads to difficulties.”

Separately these two solutions to CC are incomplete

- Gauge theory of gravity (GG)

  DE = CC-like constant of integration

  - So long as this is still a $T_{\mu\nu}$-like substance, one is obliged to address its microscopic, or QM origin.
  - Without a symmetry principle to protect it, the smallness of DE or CC may be difficult to preserve.
- de Sitter (dS) symmetry of the universe
  - The drawback of it alone is obvious; it simply does not address the old CC problem.

Here we suggest that the fusion of these two ideas may solve the CC problem, old and new, more aptly.

Here's how it goes

- In GG, the gauge potential (affine connection) is the dynamical variable, which determines the curvature tensor

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\tau\mu}\Gamma^{\tau}_{\beta\nu} + \Gamma^{\alpha}_{\tau\nu}\Gamma^{\tau}_{\beta\mu}.$$

- In close analogy with Maxwell theory, the action for gravity reads (Cook 09)

$$S_G = \kappa \int dx^4 \sqrt{-g}\left(R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} + 16\pi J^{\mu}_{\alpha\beta}\Gamma^{\alpha\beta}_{\mu}\right),$$

where the "gravitational current" (∇_α = covariant deriv.)

$$J^{\mu}_{\alpha\beta} = \frac{2G}{c^4}\left[\nabla_{\alpha}\overline{T}^{\mu}_{\beta} - \nabla_{\beta}\overline{T}^{\mu}_{\alpha}\right],$$

and

$$\overline{T}^{\mu}_{\beta} = T^{\mu}_{\beta} - \frac{1}{2}\delta^{\mu}_{\beta}T, \qquad T = T^{\mu}_{\mu}.$$

Field Equations

- Varying S_G against $\Gamma^{\mu}_{\alpha\beta}$, we arrive at the field eq.

$$\nabla_{\nu} R^{\mu\nu}_{\alpha\beta} = -4\pi J^{\mu}_{\alpha\beta}.$$

This and the Bianchi identity,

$$\nabla_{\lambda} R_{\alpha\beta\mu\nu} + \nabla_{\nu} R_{\alpha\beta\lambda\mu} + \nabla_{\mu} R_{\alpha\beta\nu\lambda} = 0,$$

together determine the curvature tensor.

- Now we recall that

$$\Gamma_{\alpha\mu\nu} = \frac{1}{2}\left[\partial_{\nu} g_{\alpha\mu} + \partial_{\mu} g_{\alpha\nu} - \partial_{\alpha} g_{\mu\nu}\right],$$

and that covariant divergence of $g_{\mu\nu}$ is identically 0. Therefore the field eq. of GG removes the CC term by construction.

Pure Space

- A pure space that is empty of stress energy-momentum satisfies the condition (Yang 74)

$$\nabla_\gamma R_{\alpha\beta} - \nabla_\beta R_{\alpha\gamma} = 0.$$

This reduces the Bianchi identity to

$$\nabla_\alpha R^\alpha_{\beta\mu\nu} = 0.$$

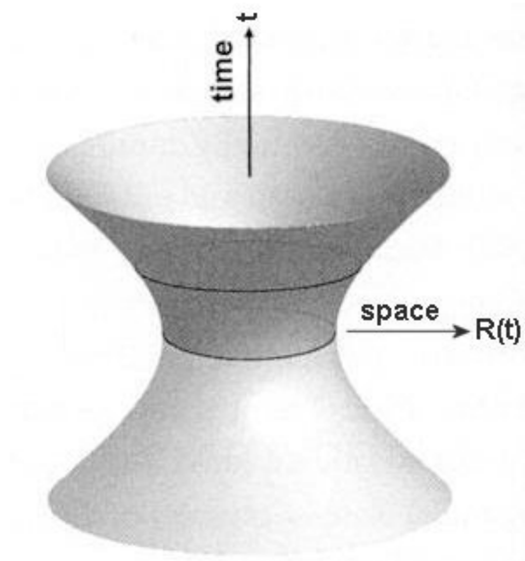
- Integrating this eq. once, we recover the Einstein eq. with a constant of integration which is associated with the boundary condition of the universe.

de Sitter Universe as Asymptotic Limit of Hubble Expansion

- Now we invoke our second assumption, that the universe is inherently de Sitter, where the 4-spacetime is a hyperboloid of a 5-d Minkowski space with the constraint

$$-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = l_{dS}^2$$

where l_{dS} is the radius of curvature of dS.



- dS universe as asymptotic limit of Hubble expansion.

- Observation gives $\Omega_{DE} = \rho_{DE} / \rho_{cr} \simeq 0.75,$
  where $\rho_{cr} = 3H_0^2 / 8\pi G = 1.88 \times 10^{-29} h^2 [g / cm^2].$
  We have

$$l_{dS} \simeq 1.33 H_0 \sim 1.5 \times 10^{28} \, cm.$$

de Sitter Universe

- Identifying the constant of integration as $3/l_{dS}^2$, we have

$$G_{\mu\nu} = -\frac{3}{l_{dS}^2} g_{\mu\nu},$$

The only nontrivial solution to it is $R = \dfrac{12}{l_{dS}^2}$.

- The local structure is then characterized by

$$R_{\alpha\beta\mu\nu} = \frac{1}{12}\left[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}\right]R,$$

and the Kretschmann scalar is a const. in dS space,

$$R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{1}{6}R^2 = \frac{24}{l_{dS}^2}.$$

Overall Factor of the Action

- Overall factor in classical action does not affect the dynamics and is therefore irrelevant.
- In GR the overall factor of the EH action is fixed by demanding that its nonrelativistic limit reduces to Newtonian gravity, and thus the factor $1/16\pi G$.
- It does matter, however, if one considers quantum fluctuations around the classical minimum (e.g. think in terms of path integral).
- Since gravity is weak, and therefore $1/G$ is large, quantum fluctuations of the curvature is tiny at scales much larger than the Planck length $(l_{Pl}^2 = hG/2\pi c^3)$.

Overall Factor of the Action

- Dimensionally,

$$R \sim R_{\mu\nu} \sim R_{\mu\nu\alpha\beta} \sim [L]^{-2}.$$

- So for the GG action

$$\int dx^4 \sqrt{-g} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \sim [L]^0. \qquad \text{Dimension zero}$$

- A natural choice for the overall factor κ would be $\hbar$. But that would make the quantum fluctuations of the classical minimum of the order unity at ALL scales.
- Alternative: introduce a length parameter so that

$$\kappa \sim L^2 / G.$$

Demanding that GG and GR approach the same dS limit, we find

$$\kappa \sim \frac{l_{dS}^2}{96\pi G}.$$

Quantum Instability of de Sitter

- dS spacetime is classically stable (Abbott-Deser 82).
- It is, however, QM unstable (Ford 85, Antoniadis et al. 86).
- Coupling of the field with the dS background would induce a term in the energy-momentum tensor,

$$\langle T_{\mu\nu} \rangle \propto g_{\mu\nu} H^4 (Ht).$$

- Clearly, the decay time is $\tau \sim H^{-1}$.
- In our case this means it is of the order of the dS radius of curvature:

$$\tau \sim l_{dS} \simeq 1.33 H_0^{-1}.$$

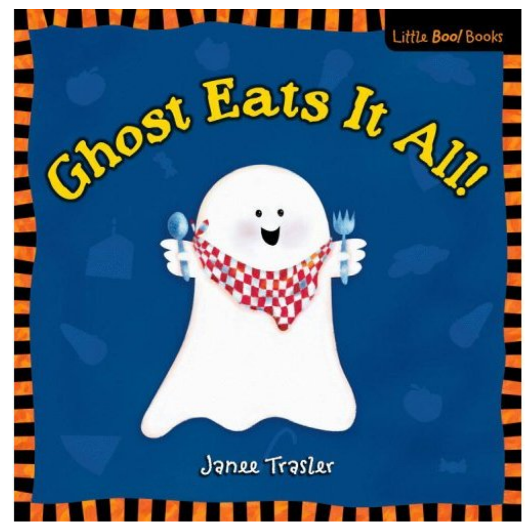
So we are safe to still observe the accelerating expansion.

Ghost!

- Another major challenge is the ghost problem generally associated with higher order QFTs.
- Ghost states are quantum states having negative norms. $\Rightarrow$ Negative probability $\Rightarrow$ unacceptable
- Since our action is quadratic in R, the theory is conformal invariant. Analogous to the scalar FT, this symmetry can be spontaneously broken and induce a mass, in our case an Einstein term R.
- So the graviton propagator goes like

$$G(k) = \frac{1}{k^4 + ak^2 + b}.$$

$\Rightarrow$ One of the poles must be negative.



Ghost!

- However since such theory is scale invariant, we are free to choose the scale such that the ghost is pushed to the Planck scale. (Kleinert)
- Although this does not completely expel the ghost, it should become harmless.
- After all in QED, for example, there is the Landau pole that appears when the energy goes to $m_e e^{1/\alpha}$.

However, this approach still does not solve the CC problem!
What determines the value of the CC, or dS radius?

Possible Solutions

- Anthropic Principle

  (Carter 74, Linde 86, Weinberg 87, Rees, Susskind, Wilzcek, Tegmark, Vilenkin,…)

  Fundamental constants have the values they have not for fundamental physical reasons, but rather because such values are necessary for life.

  - Anthropic bound of CC (Weinberg 87,89):

    Λ must be small enough to allow for the formation of sufficiently large gravitational condensations.

    Assume that the gravitational condensate began at z_c .

    Anthropic principle requires that

$$\rho_M(z_c) = \rho_{M0}(1+z_c)^3 \geq \rho_V, \implies 100\rho_{M0} \geq \rho_V \text{ for } z_c \geq 4.$$

Possible Solutions

- Anthropic Principle

By assuming $k = 0$ (flat spacetime) and insisting

$$\Omega = \Omega_\Lambda + \Omega_{M0} = 0,$$

Weinberg further found, based on $\Omega_{M0} \sim 0.1 - 0.2$ from dynamics of clusters, that

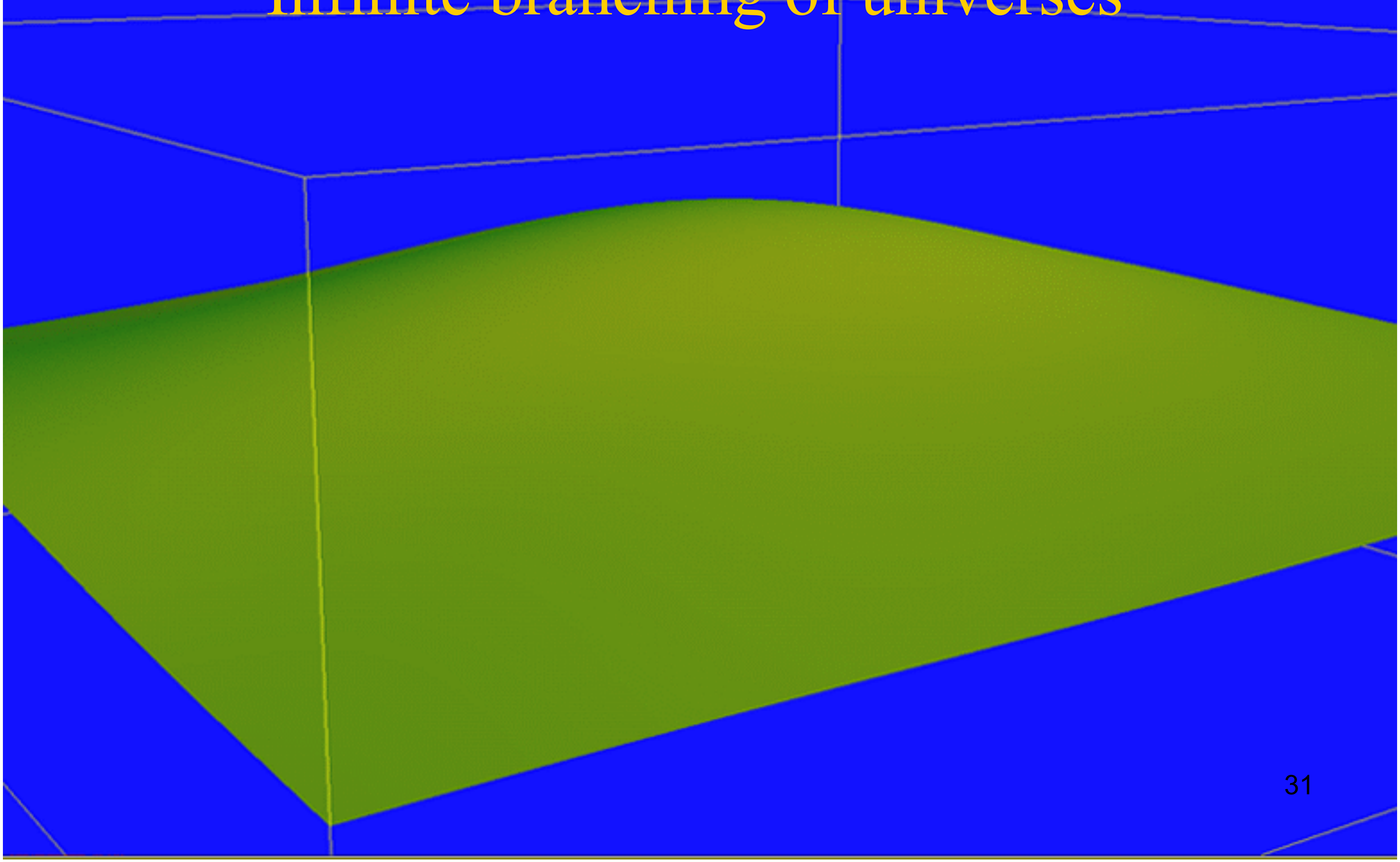
$$\rho_V / \rho_{M0} \sim 4 - 9.$$

This is amazingly close to our current measure of DE!

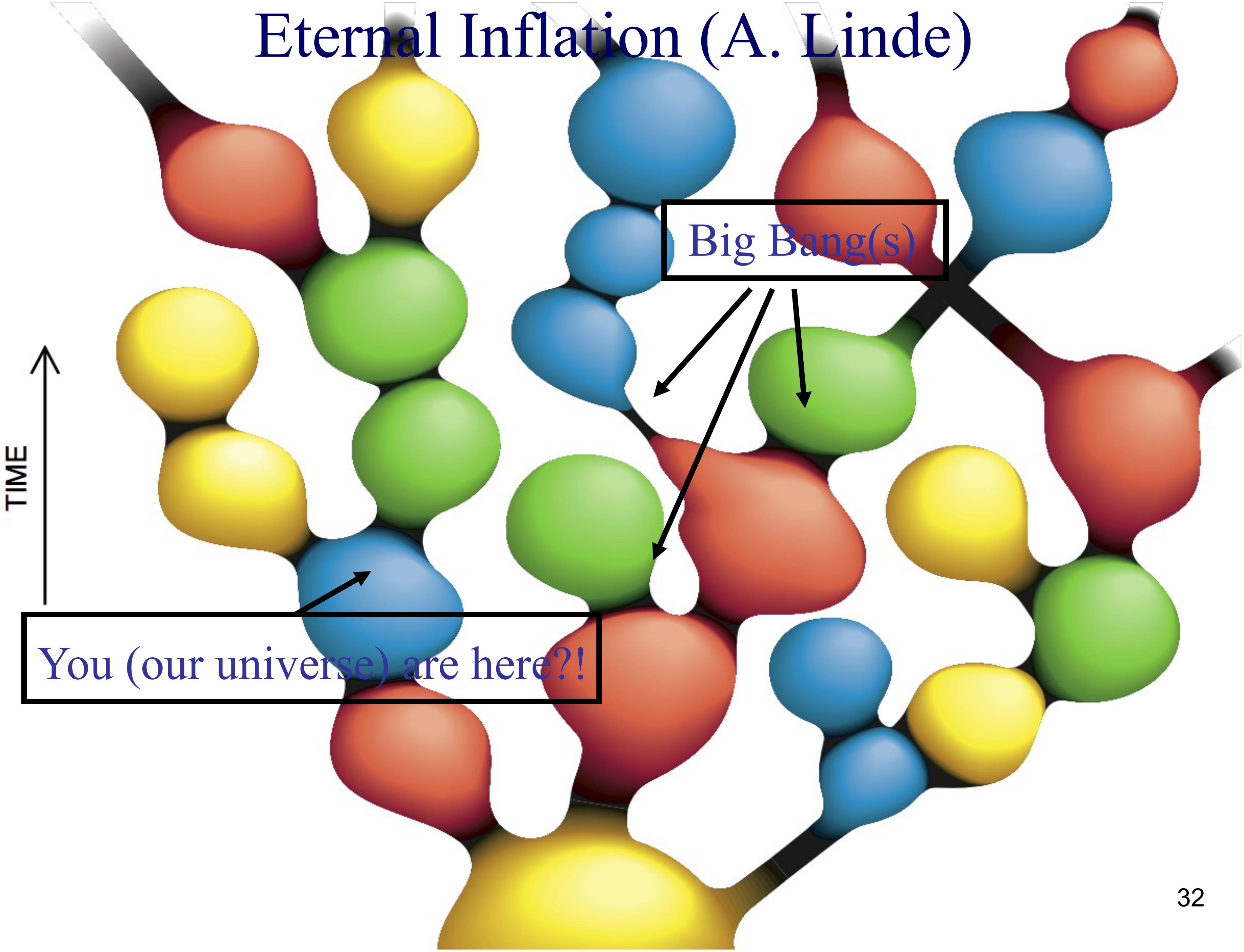
* But what physics would provide us the multiple choices for the anthropic selection?

- Eternal inflation: Quan. fluctuations during inflation (Linde)
- String Landscape: Huge amount of string vacuua (Susskind)

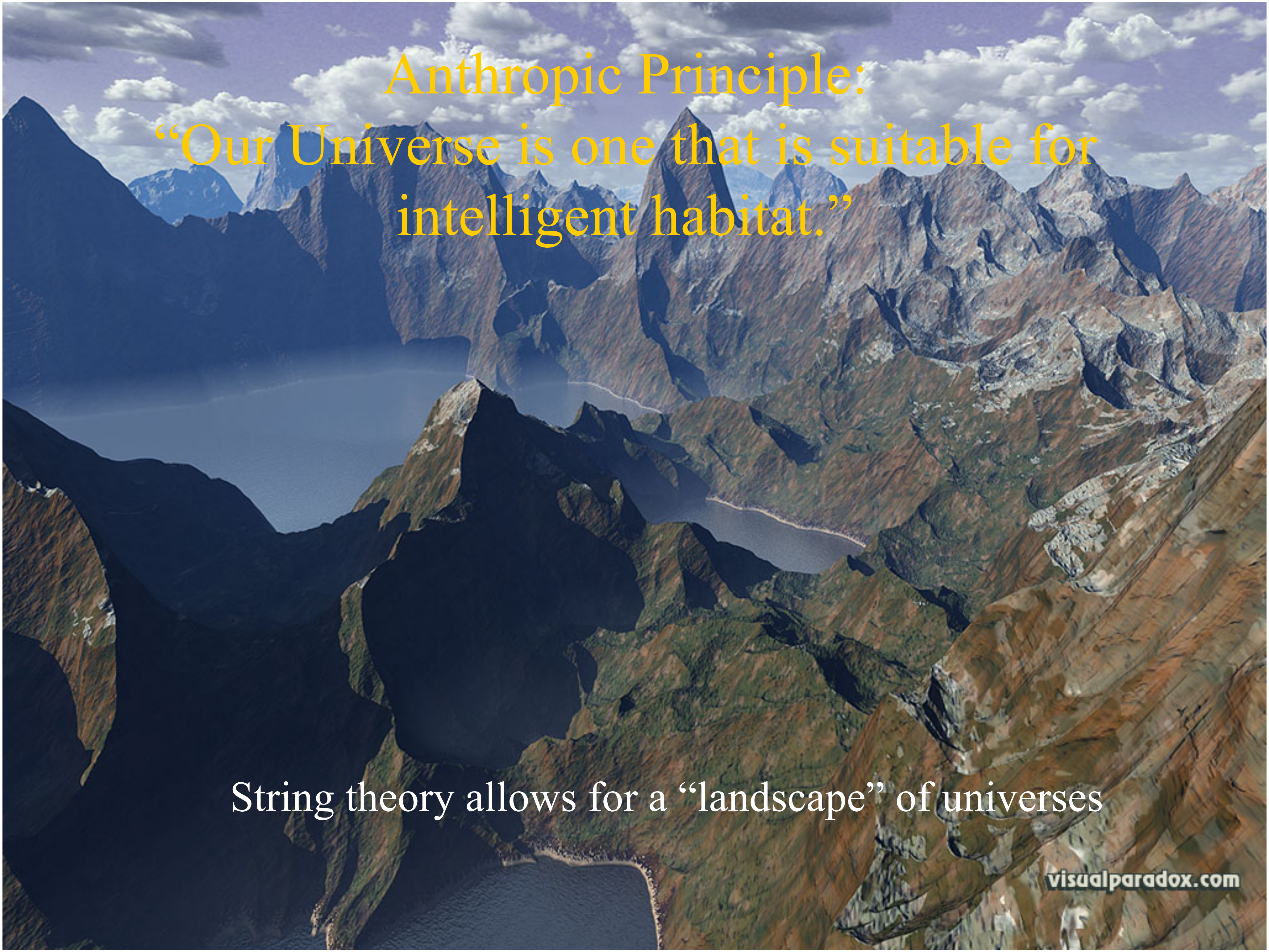
Eternal Inflation Model:
Infinite branching of universes



Eternal Inflation (A. Linde)



Anthropic Principle:
“Our Universe is one that is suitable for intelligent habitat.”



String theory allows for a “landscape” of universes

SUMMARY

- We have made a very partial overview of the CC problem and its possible solutions.

- Constant of integration/unimodular gravity solution attractive, but requires a deeper foundation.

- The GG+dS solution may provide such a theoretical basis.

- Difficulties with this approach when quantum effects are considered: ghost problem & instability of dS space. Both maybe circumvented.

- Anthropic principle needed to fix the const. of integration.