The CMB bispectrum from vector-mode perturbations induced by primordial magnetic fields

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Abstract

one of the studies to probe the nature of the early universe : constraint on Primordial Magnetic field (PMF) by the analysis of CMB bispectrum

PMF can become a source of CMB bispectra generated from scalar, vector and tensor-mode perturbations

So far, for simplicity, only the dependence on scalar magnetic mode has been considered, despite greater effect of vector one

We compute the contribution of vector mode and update bound on the magnitude of PMF

Primordial Non-Gaussinity (PNG)

- scalar mode

$$\Phi_{\rm NL}(\mathbf{x}) \equiv f_{\rm NL}[\Phi_{\rm L}(\mathbf{x})^2 - \langle\Phi_{\rm L}(\mathbf{x})^2\rangle]$$

$$\langle\Phi_{\rm L}(\mathbf{k_1})\Phi_{\rm L}(\mathbf{k_2})\Phi_{\rm NL}(\mathbf{k_3})\rangle = (2\pi)^3 P_S(k_1)P_S(k_2)2f_{NL}\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})$$

- from theories

$$f_{\rm NL}^{\rm local} = \frac{5}{12}(1-n_s) \simeq 0.015$$

in all single-field inflation models

[Creminelli & Zaldarriaga (2004)]

on the other hand, large PNG can be induced from specific models (multifield inflation, cosmic string, phase transition, ...)

CROSS CHECK

- from observation

$$-10 < f_{\rm NL}^{\rm local} < 74 \ (95\%{\rm CL})$$

from WMAP-7yr data

[Komatsu + (2010) : 1001.4538]

we can probe the nature of the early universe through constraining on $f_{\rm NL}$!

- contributions of tensor PNG are presented from

Primordial Magnetic Field (PMF) : one of sources of PNG on scalar, vector or tensor perturbation

✓ energy momentum tensor (EMT) of PMF :

$$T^0{}_0 = -\rho_B = -\frac{1}{8\pi a^4} B^2(\mathbf{x}) \equiv -\rho_\gamma \Delta_B$$

$$T^0{}_j = T^i{}_0 = 0 \ ,$$

$$T^i{}_j = \frac{1}{4\pi a^4}\left[\frac{B^2(\mathbf{x})}{2}\delta^i{}_j - B^i(\mathbf{x})B_j(\mathbf{x})\right]$$

$$\equiv \rho_\gamma\left(\Delta_B \delta^i{}_j + \Pi^i_{Bj}\right) \ ,$$

➡Its Fourier component :

$$T_{ij}(\mathbf{k},\tau) \ \equiv \ \frac{1}{4\pi a^2}\left[\frac{1}{2}\delta_{ij}\tilde{T}_{bb}(\mathbf{k}) - \tilde{T}_{ij}(\mathbf{k})\right]$$

$$\tilde{T}_{ij}(\mathbf{k}) \ = \ \int \frac{d^3\mathbf{k}'}{(2\pi)^3} B_i(\mathbf{k}')B_j(\mathbf{k}-\mathbf{k}'),$$

✦PMF is assumed as stochastic Gaussian field :

$$\langle B_i(\mathbf{k})B_j(\mathbf{p})\rangle = (2\pi)^3 \frac{\mathcal{P}_B(k)}{2} P_{ij}(\hat{\mathbf{k}})\delta^{(3)}(\mathbf{k}+\mathbf{p})$$

Here, $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i\hat{k}_j$ $\quad \mathcal{P}_B(k) = \frac{(2\pi)^{n_B+5}}{\Gamma(n_B/2+3/2)k^3_{1\mathrm{Mpc}}} B^2_{1\mathrm{Mpc}}\left(\frac{k}{k_{1\mathrm{Mpc}}}\right)^{n_B}$

➡EMF bispectrum ($\propto$ B^6) is finite value :

$$
\begin{aligned}
\mathcal{B}_{abcdef}(\mathbf{k},\mathbf{p},\mathbf{q}) &\equiv \left\langle \tilde{T}_{ab}(\mathbf{k})\tilde{T}_{cd}(\mathbf{p})\tilde{T}_{ef}(\mathbf{q}) \right\rangle \\
&= \frac{1}{8}\int d^3\mathbf{k}' \int d^3\mathbf{p}' \int d^3\mathbf{q}' \delta(\mathbf{k}-\mathbf{k}'+\mathbf{q}')\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}')\delta(\mathbf{q}-\mathbf{q}'+\mathbf{p}') \\
&\quad \times \mathcal{P}_B(k')\mathcal{P}_B(p')\mathcal{P}_B(q')[P_{ad}(\hat{k}')P_{be}(\hat{q}')P_{cf}(\hat{p}') + \{a \leftrightarrow b \text{ or } c \leftrightarrow d \text{ or } e \leftrightarrow f\}]
\end{aligned}
$$

➡finite B$_{abcdef}$ induces finite initial bispectrum on all modes as

◉bispectrum of vector anisotropic stress

$$
\langle \Pi_{Bv}^{(\lambda_k)}(\mathbf{k})\Pi_{Bv}^{(\lambda_p)}(\mathbf{p})\Pi_{Bv}^{(\lambda_q)}(\mathbf{q})\rangle = \mathcal{B}_{abcdef}(\mathbf{k},\mathbf{p},\mathbf{q}) \times \hat{k}_a \epsilon_b^{(-\lambda_k)} \hat{p}_c \epsilon_d^{(-\lambda_p)} \hat{q}_e \epsilon_f^{(-\lambda_q)} / (-4\pi\rho_{\gamma,0})^3 \ ,
$$

◉bispectrum of scalar isotropic stress

$$
\langle \Delta_B(\mathbf{k})\Delta_B(\mathbf{p})\Delta_B(\mathbf{q})\rangle = \frac{\mathcal{B}_{abcdef}(\mathbf{k},\mathbf{p},\mathbf{q})}{(8\pi\rho_{\gamma,0})^3}\delta_{ab}\delta_{cd}\delta_{ef}
$$

$\epsilon^{(-\lambda)}$: divergenceless polarization vector

CMB bispectrum generated from all-mode perturbations

- 1-point function sourced from vector anisotropic stress

- 1-point function sourced from scalar isotropic stress

$$a^{(V)}_{X,\ell m} = (-i)^\ell \int \frac{k^2 dk}{2\pi^2} \mathcal{T}^{(V)}_{X,\ell}(k) \sum_\lambda [\mathrm{sgn}(\lambda)]^{\lambda+x} \, \Pi^{(\lambda)}_{Bv,\ell m}(k)$$

$$\Pi^{(\lambda)}_{Bv,\ell m}(k) \equiv \int d^2\hat{\mathbf{k}} \Pi^{(\lambda)}_{Bv}(\mathbf{k})_{-\lambda} Y^*_{\ell m}(\hat{\mathbf{k}})$$

$$a^{(S)}_{X,\ell m} = (-i)^\ell \int \frac{k^2 dk}{2\pi^2} \mathcal{T}^{(S)}_{X,\ell}(k) \Delta_{B,\ell m}(k)$$

$$\Delta_{B,\ell m}(k) \equiv \int d^2\hat{\mathbf{k}} \Delta_B(\mathbf{k}) Y^*_{\ell m}(\hat{\mathbf{k}})$$

λ = ±1, x = 0 (for Intensity and E-mode pol : X ≡ I,E),
x = 1 (for B-mode pol : X ≡ B), $T_{X,l}$: transfer function

details in Appendix

The CMB angle-averaged bispectrum of vector PMF mode is calculated as

performing angular integrals and summation of Wigner symbols...

$$B^{(VVV)}_{X_1X_2X_3,\ell_1\ell_2\ell_3} \equiv \sum_{m_1m_2m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a^{(V)}_{X_1,\ell_1m_1} a^{(V)}_{X_2,\ell_2m_2} a^{(V)}_{X_3,\ell_3m_3} \right\rangle$$

$$= \left[\prod_{i=1}^{3} (-i)^{\ell_i} \int_0^\infty \frac{k_i^2 dk_i}{2\pi^2} \mathcal{T}^{(V)}_{X_i,\ell_i}(k_i)\right] \left(\frac{-1}{4\pi\rho_{\gamma,0}}\right)^3 \tilde{B}^{(VVV)}_{x_1x_2x_3,\ell_1\ell_2\ell_3}(k_1,k_2,k_3)$$

$$\begin{aligned}
\tilde{B}^{(VVV)}_{x_1x_2x_3,\ell_1\ell_2\ell_3}(k,p,q) = & -8^3 \left(\frac{4\pi}{3}\right)^6 \sum_{\substack{L_1L_2L_3 \\ L_1'L_2'L_3' \\ L_1''L_2''L_3''}} \sum_{\substack{L \\ L' \\ L''}} \sum_{\substack{L_k \\ L_p \\ L_q}} (-1)^{\frac{L_1+L_2+L_3+L_1'+L_2'+L_3'+L_1''+L_2''+L_3''}{2}} I^{0\,0\,0}_{L_1L_2L_3} I^{0\,0\,0}_{L_1'L_2'L_3'} I^{0\,0\,0}_{L_1''L_2''L_3''} \\
& \times (-1)^{\ell_1+\ell_2+\ell_3} \begin{Bmatrix} \ell_1 & \ell_2 & \ell_3 \\ L' & L'' & L \end{Bmatrix} \begin{Bmatrix} L'' & L & \ell_1 \\ L_3 & L_2 & L_1 \\ 1 & 1 & L_k \end{Bmatrix} \begin{Bmatrix} L & L' & \ell_2 \\ L_3' & L_2' & L_1' \\ 1 & 1 & L_p \end{Bmatrix} \begin{Bmatrix} L' & L'' & \ell_3 \\ L_3'' & L_2'' & L_1'' \\ 1 & 1 & L_q \end{Bmatrix} \\
& \times \int_0^\infty A^2 dA\, j_{L_1}(kA) \int_0^\infty B^2 dB\, j_{L_1'}(pB) \int_0^\infty C^2 dC\, j_{L_1''}(qC) \\
& \times \int_0^\infty k'^2 dk' \mathcal{P}_B(k') j_{L_2}(k'A) j_{L_3'}(k'B) \int_0^\infty p'^2 dp' \mathcal{P}_B(p') j_{L_2'}(p'B) j_{L_3''}(p'C) \\
& \times \int_0^\infty q'^2 dq' \mathcal{P}_B(q') j_{L_2''}(q'C) j_{L_3}(q'A) \\
& \times \sum_{SS'S''} (-1)^{L_2+L_2'+L_2''+L_3+L_3'+L_3''} I^{0S-S}_{L_3'1L} I^{0S-S}_{L_21L} I^{0S'-S'}_{L_3''1L'} I^{0S'-S'}_{L_2'1L'} I^{0S''-S''}_{L_31L''} I^{0S''-S''}_{L_2''1L''} \\
& \times \sum_{\lambda_1\lambda_2\lambda_3} \frac{1}{8} \left[1+(-1)^{L_k}\right] \left[1+(-1)^{L_q}\right] \left[1+(-1)^{L_p}\right] \\
& \times \mathrm{sgn}(\lambda_1)^{\lambda_1+x_1} \mathrm{sgn}(\lambda_2)^{\lambda_2+x_2} \mathrm{sgn}(\lambda_3)^{\lambda_3+x_3} \lambda_1\lambda_2\lambda_3 \\
& \times I^{0\lambda_1-\lambda_1}_{L_1\ell_1L_k} I^{0\lambda_1-\lambda_1}_{11L_k} I^{0\lambda_2-\lambda_2}_{L_1'\ell_2L_p} I^{0\lambda_2-\lambda_2}_{11L_p} I^{0\lambda_3-\lambda_3}_{L_1''\ell_3L_q} I^{0\lambda_3-\lambda_3}_{11L_q}
\end{aligned}$$

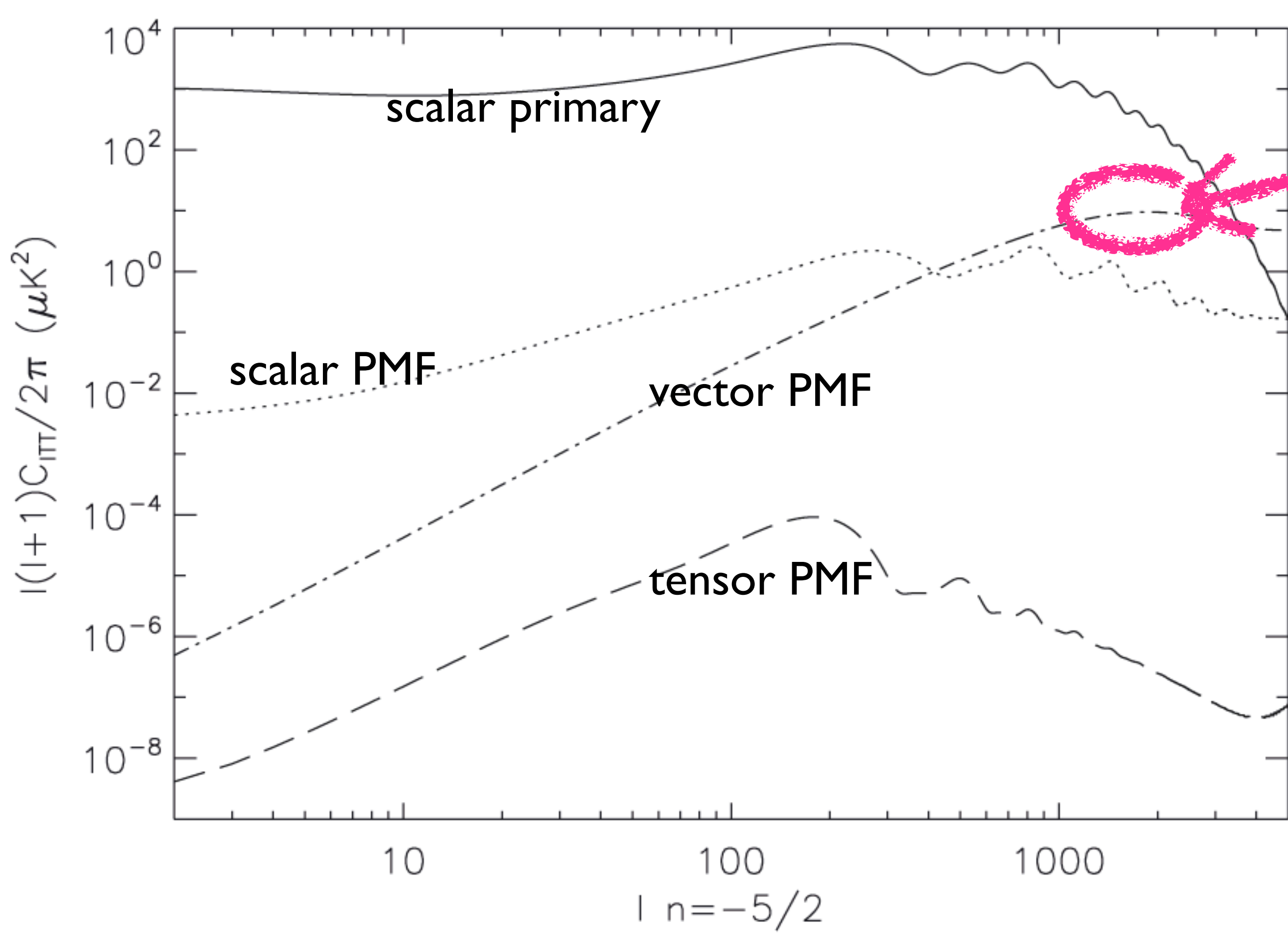
$$I^{s_1s_2s_3}_{\ell_1\ell_2\ell_3} \equiv \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

That of scalar mode is derived in the same manner

Previous works

from the analysis of bispectrum generated from scalar isotropic stress, the magnitude of PMF was constrained as

- $B_0 < 35$nG [T. R. Seshadri & K. Subramanian (2009) : 0902.4006]
- $B_0 < O(10)$nG [C. Caprini + (2009) : 0903.1420]



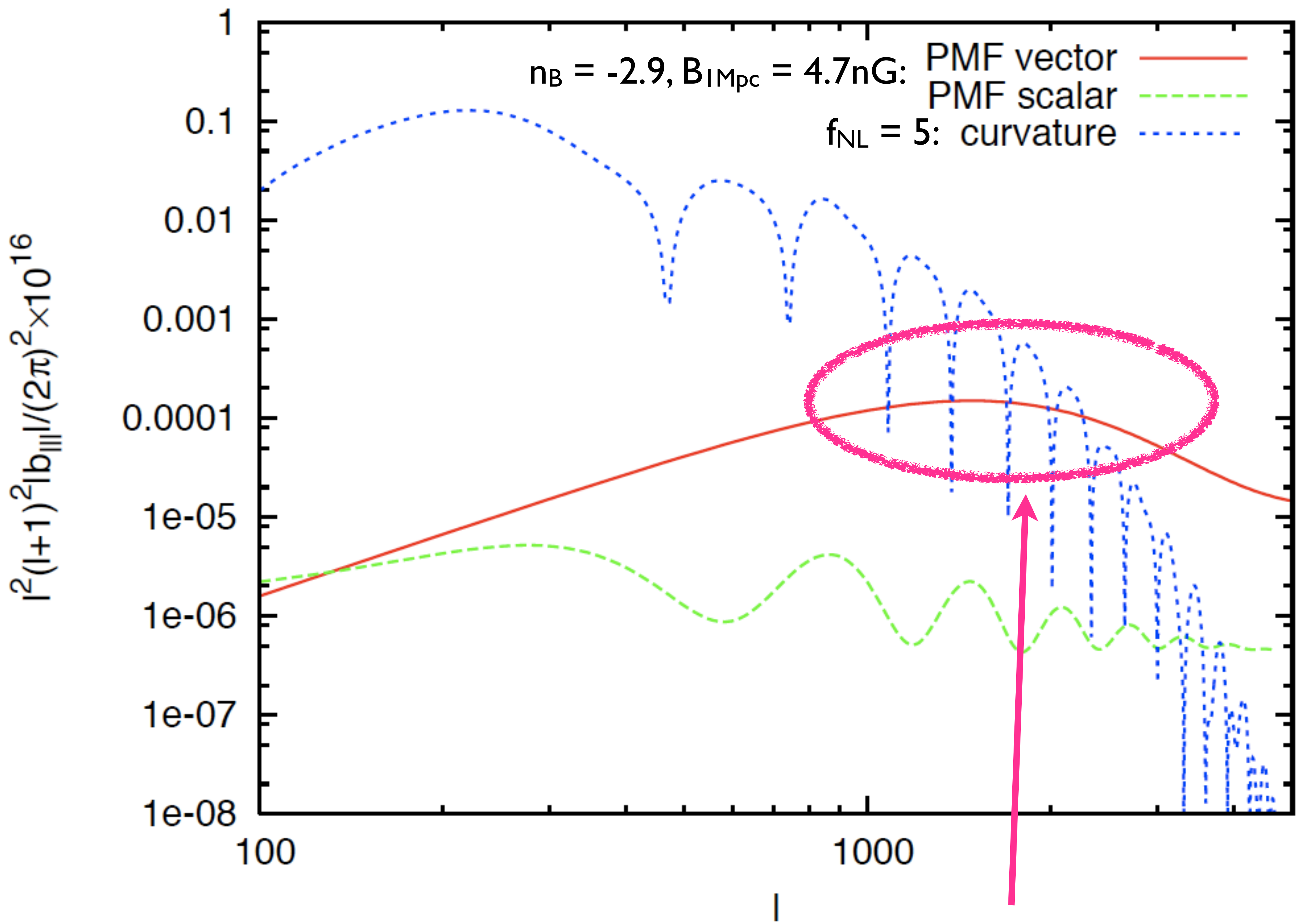
- [C. Caprini + (2009) : 0811.0230]

vector Cl dominates for l ~ 1000

☞ vector bispectrum is also expected the dominance in small scale

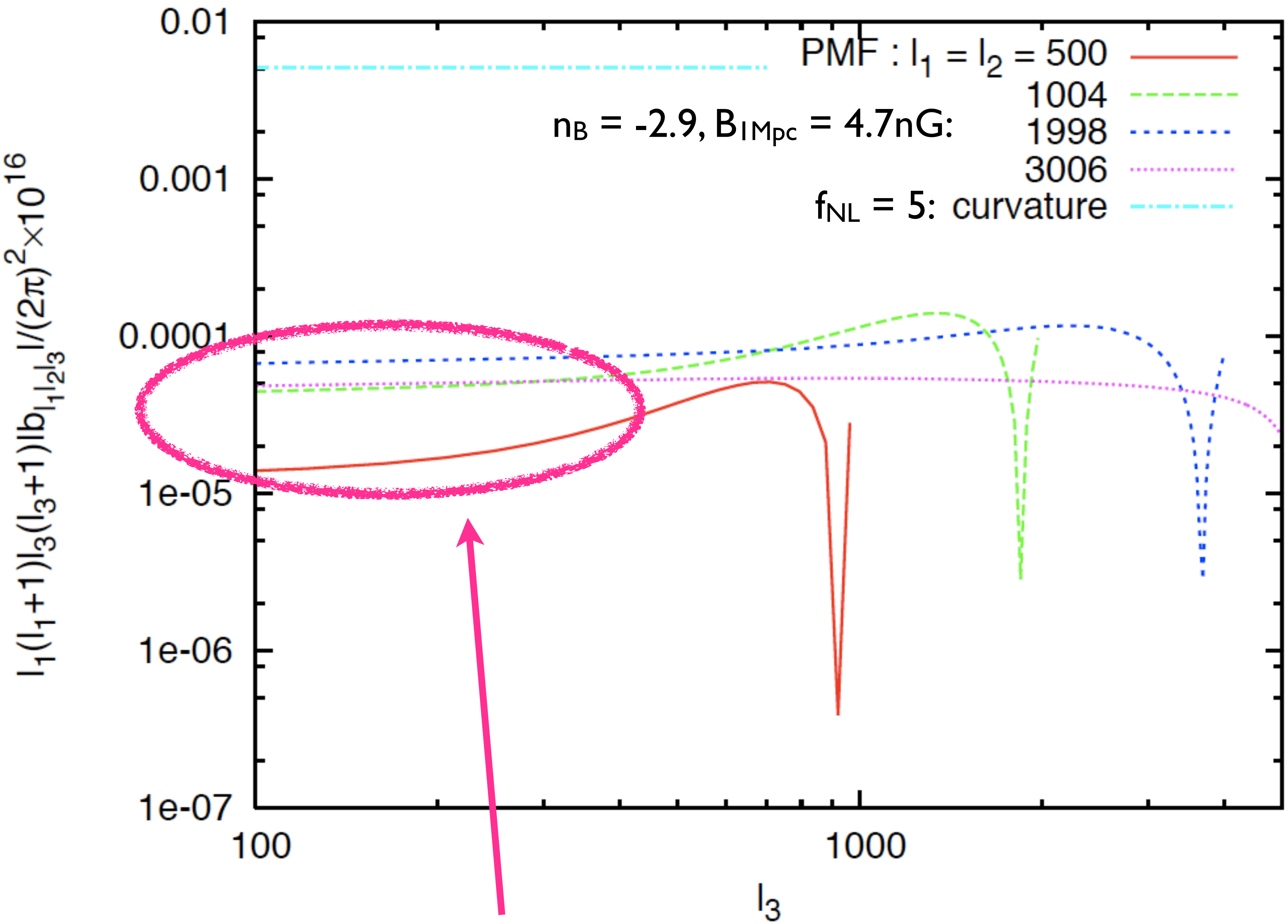
in order to obtain more precise bound on PMF, we should use vector bispectrum

CMB reduced bispectrum $b_{\ell_1\ell_2\ell_3} \equiv \sqrt{\frac{(4\pi)}{(2\ell_1+1)(2\ell_1+2)(2\ell_1+3)}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^{-1} B_{\ell_1\ell_2\ell_3}$ of intensity mode for $l_1 = l_2 = l_3$



vector component of b_{lll} dominates at small scale

CMB reduced bispectrum of intensity mode for $l_1 = l_2 \neq l_3$



the shape of vector bispectrum for $n_B = -2.9$ is close to the local-type configuration due to $b_{lll} \propto l_3^{-2}$

Constraints

- our bispectrum of magnetic vector mode (n_B = -2.9)

$$\ell_1(\ell_1+1)\ell_3(\ell_3+1)b_{\ell_1\ell_2\ell_3} \sim -2\times 10^{-19}\left(\frac{B_{1\mathrm{Mpc}}}{4.7\mathrm{nG}}\right)^6$$

VS

- bispectrum from PNG in the curvature perturbations

$$\ell_1(\ell_1+1)\ell_3(\ell_3+1)b_{\ell_1\ell_2\ell_3} \sim 4\times 10^{-18} f_{\rm NL}^{\rm local}$$

$$\left(\frac{B_{1\mathrm{Mpc}}}{1\mathrm{nG}}\right) \sim 7.74\,|f_{\rm NL}^{\rm local}|^{1/6} \quad (\text{for } \mathrm{n_B} = -2.9)$$

tighter by factor of two
than Seshadri + (2009)!!

- WMAP : $|f_{NL}| < 100$ ☞ $B_{1Mpc} <$ 17nG
- PLANCK : $|f_{NL}| < 5$ ☞ $B_{1Mpc} <$ 10nG

Summary

- we present the CMB bispectrum induced from the vector mode of PMFs by taking into account the full angular dependence of the bispectrum of magnetic fields
- we find that
  - magnetic vector mode dominates at small scale
  - the bispectrum has significant signals on the squeezed limit if PMF spectrum is nearly scale invariant
- we obtain the constraint : $B_{1Mpc} < 10nG$ from the expected observational data of PLANCK experiment

(Appendix) Use of All-sky Formalism for the CMB Bispectrum Sourced from Vector or Tensor non-Gaussianity

based on

- MS + (2010) : 1003.2096
- MS + in prep.

"We demonstrate how to compute the CMB vector or tensor bispectrum by considering Maldacena's issue about the primordial correlation between two scalars and a graviton as a sample calculation."

PNG in vector and tensor perturbation

- Decomposing scalar, vector and tensor perturbations ζ, ω_a, and γ_{ab} into each helicity state λ

$$\zeta = \xi^{(0)}, \quad \omega_a = \sum_{\lambda=\pm1} \xi^{(\lambda)} \epsilon_a^{(\lambda)}, \quad \gamma_{ab} = \sum_{\lambda=\pm2} \xi^{(\lambda)} e_{ab}^{(\lambda)},$$

where ε_a and e_{ab} are polarization vector and tensor defined by

$$\begin{aligned} \epsilon_a^{(\pm1)*}(\hat{n}) &= \epsilon_a^{(\mp1)}(\hat{n}) = \epsilon_a^{(\pm1)}(-\hat{n}) \ , \\ \epsilon_a^{(\pm1)}(\hat{n})\epsilon_a^{(\mp1)}(\hat{n}) &= 1 \ , \quad \epsilon_a^{(\pm1)}(\hat{n})\epsilon_a^{(\pm1)}(\hat{n}) = 0 \\ e_{ab}^{(\pm2)}(\hat{n}) &= \sqrt{2}\epsilon_a^{(\pm1)}(\hat{n})\epsilon_b^{(\pm1)}(\hat{n}) \end{aligned}$$

Expanding in terms of spin spherical harmonic functions

$$\begin{aligned} \hat{\epsilon}_a^{(\pm1)}(\hat{n}) &= \mp\alpha_a^m {}_{\pm1}Y_{1m}(\hat{n}) \\ \alpha_a^m \alpha_a^{m'*} &= \frac{4\pi}{3}\delta_{m,m'} \end{aligned}$$

- Bispecta are given: $\left\langle \xi^{(\lambda_1)}(\mathbf{k_1})\xi^{(\lambda_2)}(\mathbf{k_2})\xi^{(\lambda_3)}(\mathbf{k_3}) \right\rangle \equiv (2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})F^{\lambda_1\lambda_2\lambda_3}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3})$

- Performing the integral of angular dependencies, we obtain:

$$\begin{aligned} \mathcal{F}^{\lambda_1\lambda_2\lambda_3}_{\ell_1\ell_2\ell_3}(k_1,k_2,k_3) \ = \ & \sum_{m_1m_2m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \int d^2\hat{k_1} \int d^2\hat{k_2} \int d^2\hat{k_3} {}_{-\lambda_1}Y^*_{\ell_1m_1}(\hat{k_1}) {}_{-\lambda_2}Y^*_{\ell_2m_2}(\hat{k_2}) {}_{-\lambda_3}Y^*_{\ell_3m_3}(\hat{k_3}) \\ & \times\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})F^{\lambda_1\lambda_2\lambda_3}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}) \end{aligned}$$

Formulae of the CMB bispectra

- Expand CMB temperature anisotropy and E,B-mode of the polarization as

$$\frac{\Delta T(\hat{\mathbf{n}})}{T} = \sum_{lm}\sum_{Z} a^{(Z)}_{I,lm} Y_{lm}(\hat{\mathbf{n}}), \quad E(\hat{\mathbf{n}}) = \sum_{lm}\sum_{Z} a^{(Z)}_{E,lm} Y_{lm}(\hat{\mathbf{n}}), \quad B(\hat{\mathbf{n}}) = \sum_{lm}\sum_{Z} a^{(Z)}_{B,lm} Y_{lm}(\hat{\mathbf{n}}),$$

where Z = S (: scalar), = V (: vector), and = T (: tensor)

- The coefficients are given by $a^{(Z)}_{X,\ell m} = 4\pi(-i)^{\ell} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_{\lambda} [\mathrm{sgn}(\lambda)]^{\lambda+x} {}_{-\lambda}Y^{*}_{\ell m}(\hat{k}) \xi^{(\lambda)}(\mathbf{k}) \mathcal{T}^{(Z)}_{X,\ell}(k)$

- *We find the formulae of CMB bispectra*

$$\begin{aligned} B^{(Z_1 Z_2 Z_3)}_{X_1 X_2 X_3, \ell_1, \ell_2, \ell_3} &\equiv \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \left\langle a^{(Z_1)}_{X_1,\ell_1 m_1} a^{(Z_2)}_{X_2,\ell_2 m_2} a^{(Z_3)}_{X_3,\ell_3 m_3} \right\rangle \\ &= \left[\prod_{i=1}^{3} 4\pi(-i)^{\ell_i} \int \frac{k_i^2 dk_i}{(2\pi)^3} \mathcal{T}^{(Z_i)}_{X_i,\ell_i}(k_i) \right] \\ &\quad \times (2\pi)^3 \sum_{\lambda_1 \lambda_2 \lambda_3} \mathrm{sgn}(\lambda_1)^{\lambda_1 + x_1} \mathrm{sgn}(\lambda_2)^{\lambda_2 + x_2} \mathrm{sgn}(\lambda_3)^{\lambda_3 + x_3} \mathcal{F}^{\lambda_1 \lambda_2 \lambda_3}_{\ell_1 \ell_2 \ell_3}(k_1, k_2, k_3) \end{aligned}$$

As a sample calculation: two scalars - one graviton interaction

✤Single field inflation model (Maldacena 2003)

metric:	$ds^2 = -N^2 dt^2 + a^2 e^{2\zeta} e^{\gamma_{ij}} (dx^i + N^i dt)(dx^j + N^j dt)$
Lagrangian:	$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$
gauge:	$\phi(t, \mathbf{x}) = \phi(t)$

Dividing the action into free parts and interaction parts;
S=S2+S3+…

✤We especially take into account ζζ-γ interaction: $S_{\rm int} \supset \int d^4x \, a g_{tss} \gamma_{ab} \partial_a \zeta \partial_b \zeta$

✤In Maldacena's original paper, coupling constant $g_{tss}=\varepsilon$.
➡Let us consider g is free parameter as a general case.

Shape of bispectrum

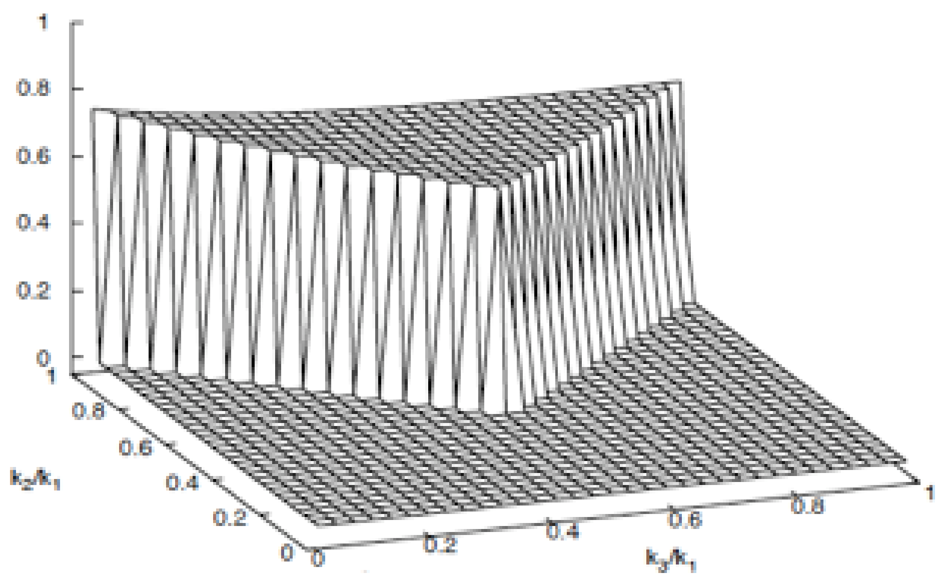
✤Primordial bispectrum has been calculated by Maldacena 2003

$$F^{\pm 200}(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}) = f^{(TSS)}(k_1, k_2, k_3) e^{(\mp 2)}_{ab}(\hat{\mathbf{k_1}}) \hat{k}_{2a} \hat{k}_{3b} ,$$

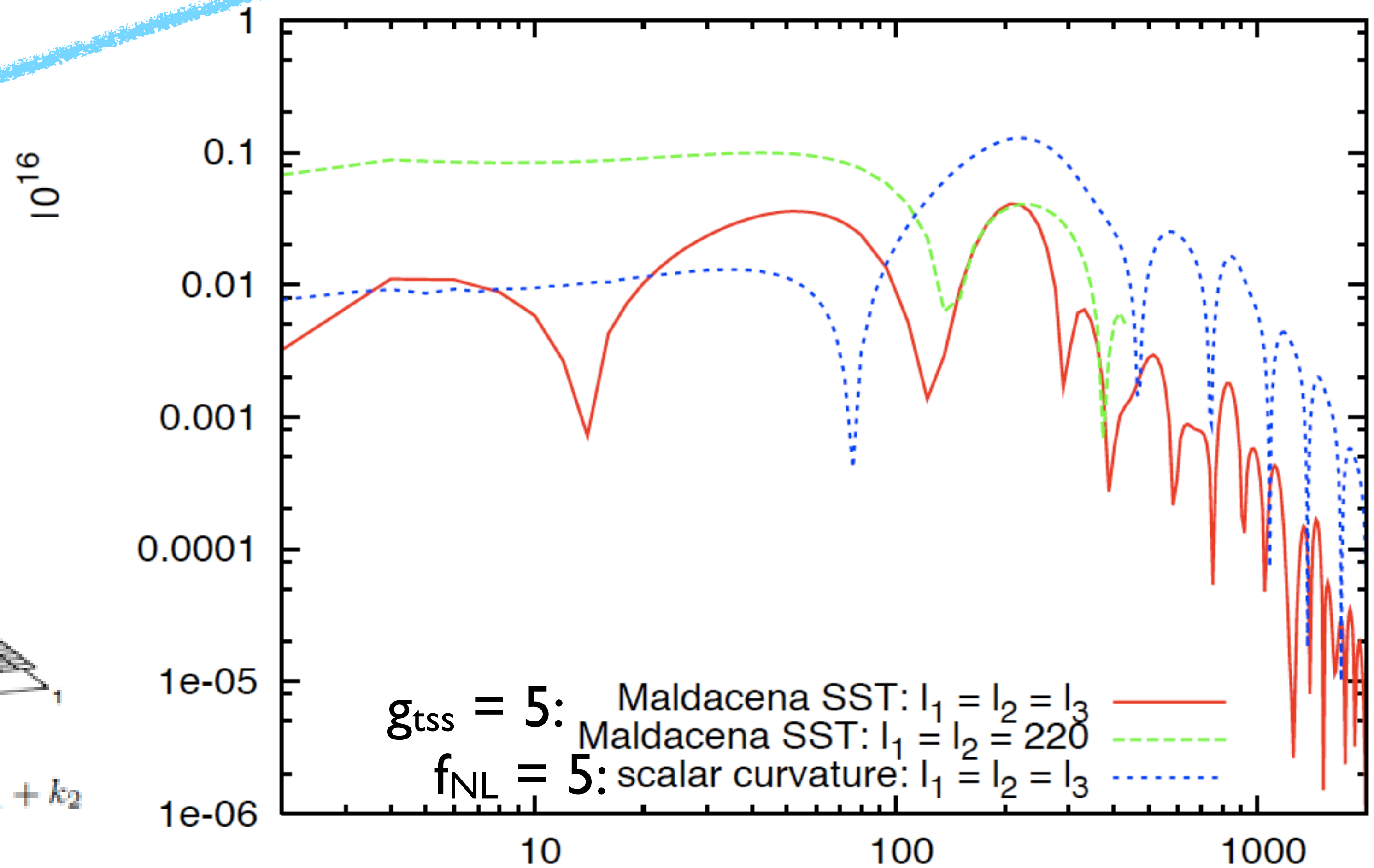
$$f^{(TSS)}(k_1, k_2, k_3) \equiv \frac{4 g_{tss}}{k_1^2 k_2^2 k_3^2} \boxed{\frac{I(k_1, k_2, k_3)}{k_t}} \frac{k_t}{k_1} k_1^3 P_S(k_1) k_2^3 P_S(k_2)$$

$k_t = k_1 + k_2 + k_3$

nearly flat ~ -0.65



Plot range: $k_3 \le k_2 \le k_1$ and $|k_1 - k_2| \le k_3 \le k_1 + k_2$



✤We find the CMB bispectrum :

$$B^{(TSS)}_{X_1 X_2 X_3 \ell_1 \ell_2 \ell_3} = \frac{(8\pi)^{3/2}}{3} \sum_{L_1 L_2 L_3} (-1)^{\frac{L_1 + L_2 + L_3}{2}} I^{000}_{L_1 L_2 L_3} I^{20-2}_{\ell_1 L_1 2} I^{000}_{\ell_2 L_2 1} I^{000}_{\ell_3 L_3 1} \left\{ \begin{matrix} \ell_1 & \ell_2 & \ell_3 \\ L_1 & L_2 & L_3 \\ 2 & 1 & 1 \end{matrix} \right\}$$

$$\times \int_0^\infty y^2 dy \left[\prod_{i=1}^3 \frac{2}{\pi} (-i)^{\ell_i} \int_0^\infty k_i^2 dk_i T^{(Z_i)}_{X_i, \ell_i} j_{L_i}(k_i y) \right] f^{(TSS)}(k_1, k_2, k_3) ,$$

Signal-to-noise ratio

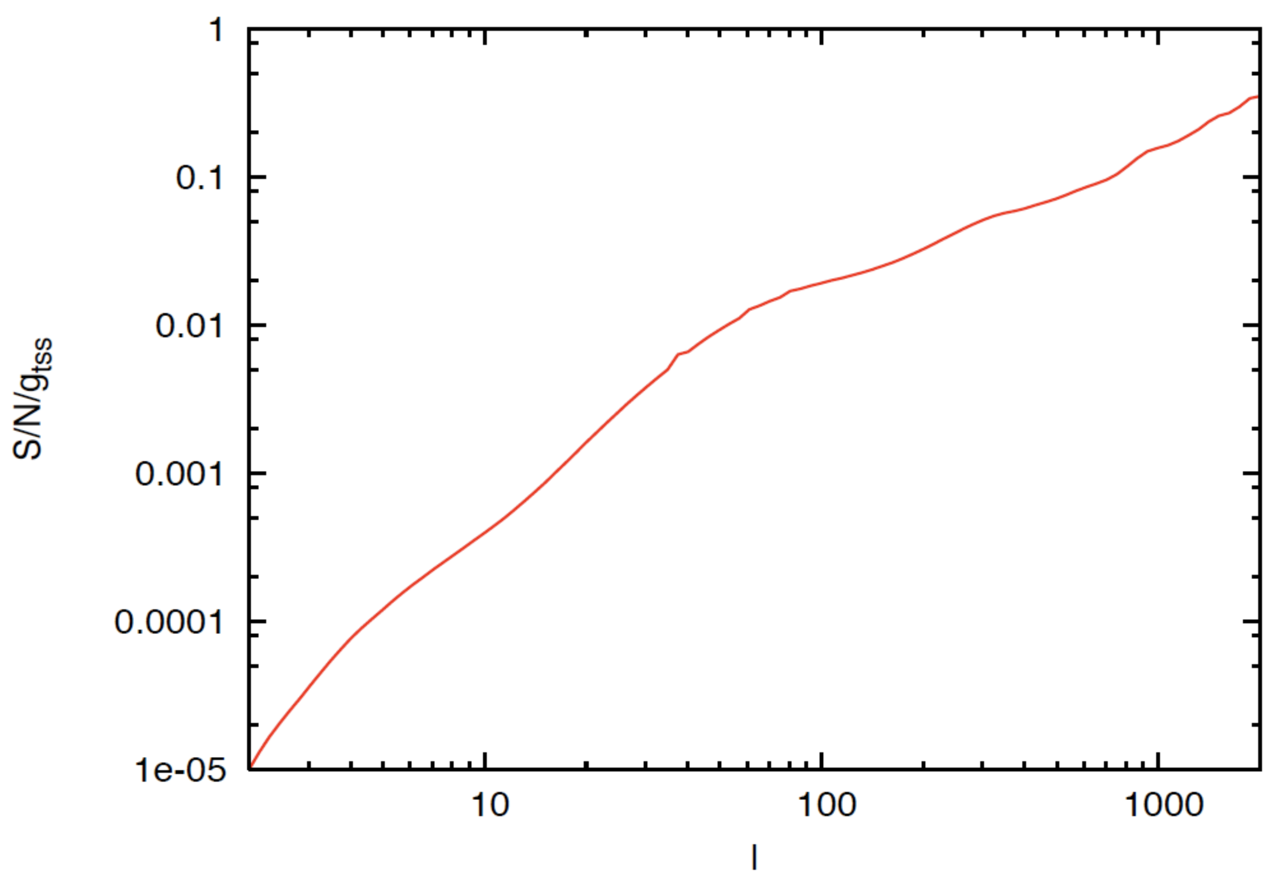
✤For ideal experiment (zero noise)

$$\left(\frac{S}{N}\right)^2 = \sum_{2\le\ell_1\le\ell_2\le\ell_3\le\ell} \frac{\left(B^{(TSS)}_{III\ell_1\ell_2\ell_3} + B^{(STS)}_{III\ell_1\ell_2\ell_3} + B^{(SST)}_{III\ell_1\ell_2\ell_3}\right)^2}{\sigma^2_{\ell_1\ell_2\ell_3}}$$

$$\sim\left(\ell \times 5.4 \times 10^{-5}|g_{tss}|\right)^2$$

cosmic variance

$\sim C_{l_1} C_{l_2} C_{l_3}$



If S/N > 1, it is needed that $|g_{tss}| > 6$ for l ~ 1000 [expected PLANCK resolution]

By using our formalism, one can estimate all-mode bispectra and S/N!!