Are 3-point Correlations of ζ Constant Outside the Horizon ?

[Yes, but read on for more]

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NON-GAUSSIANITIES IN SINGLE FIELD INFLATION

NON-GAUSSIANITIES IN THE CURVATURE PERTURBATION ζ CAN ARISE FROM 1) SELF INTERACTIONS OF THE INFLATON, AND 2) NON-LINEARITIES IN COSMOLOGICAL PERTURBATION THEORY.

USUALLY THE SELF INTERACTION CONTRIBUTION IS IGNORED. FOR EXAMPLE, CONSIDER CUBIC SELF INTERACTIONS, $\mu\,\phi^3$.

THE SELF INTERACTION CONTRIBUTION TO < ζ^3 > IS PROPORTIONAL TO THE SLOW ROLL PARAMETER $\xi \sim V'''$, AND SO IS IGNORED COMPARED TO TERMS PROPORTIONAL TO SLOW ROLL PARAMETERS $\varepsilon \sim V'^2$ AND $\eta \sim V''$. BUT

1. FOR MANY MODELS (NEW INFLATION, SMALL FIELD NATURAL INFLATION AND RUNNING MASS INFLATION), $\xi >> \varepsilon$.
2. THE SELF INTERACTION CONTRIBUTION IS ACTUALLY ~ $\xi\, N_e$, WHICH IS COMPARABLE TO η.

N_e = THE NUMBER OF E-FOLDINGS SINCE HORIZON EXIT, AND IS 60 FOR OUR CURRENT HORIZON.

THEREFORE SELF INTERACTIONS SHOULD NOT BE IGNORED OUTRIGHT.

DO SELF INTERACTIONS IMPLY GROWTH OUTSIDE THE HORIZON ?

BUT $<\zeta(k)^3> \sim N_e$ IMPLIES THAT THE 3-POINT FUNCTION IS GROWING AFTER HORIZON EXIT ! THIS IS CONTRARY TO ONE'S EXPECTATIONS.

IS THE CALCULATION OF THE INFLATON SELF INTERACTION CONTRIBUTION TO $<\zeta^3>$ INCORRECT?

IT HAS BEEN DONE INDEPENDENTLY BY FALK ET AL (1993), ZALDARRIAGA (2004), BERNARDEAU ET AL (2004) AND SEERY ET AL (2008) IN DIFFERENT CONTEXTS.

THEN LET US RE-EXAMINE THE ARGUMENT THAT CORRELATIONS OF THE CURVATURE PERTURBATION ζ(k) ARE CONSTANT OUTSIDE THE HORIZON.

IN THE LITERATURE, ζ(k) IS SHOWN TO BE CONSTANT OUTSIDE THE HORIZON.

BUT THIS ONLY IMPLIES THAT THE 2-POINT FUNCTION, $<\zeta(k_1)\,\zeta(k_2)> = (2\pi)^3\, |\zeta(k_1)|^3\, \delta^3(\mathbf{k_1} - \mathbf{k_2})$ IS CONSTANT AFTER HORIZON EXIT.

WHAT ABOUT HIGHER POINT FUNCTIONS ?

CONSTANCY CONDITION FOR ζ(k) 3-POINT FUNCTION

$$\langle\hat{\zeta}(t)^3\rangle = i\int_{t_0}^{t} dt'\left\langle\left[\hat{H}_I(t'),\hat{\zeta}_I(t)^3\right]\right\rangle$$

(OPERATORS ON THE RHS ARE IN THE INTERACTION PICTURE)

FOR < $\zeta(t)^3$ > TO BE CONSTANT OUTSIDE THE HORIZON THE CONTRIBUTION TO THE INTEGRAL FOR t AFTER HORIZON EXIT SHOULD BE SUPPRESSED.

WEINBERG (2008) SHOWED THIS WAS SO, BUT FOR GAUSSIAN INFLATON FLUCTUATIONS, I.E., IGNORING SELF INTERACTIONS OF THE INFLATON.

WE INVESTIGATE WHETHER THE 3-POINT FUNCTION IS CONSTANT OUTSIDE THE HORIZON, IN LIGHT OF THE TIME DEPENDENT CONTRIBUTION PROPORTIONAL TO N_e FROM INFLATON SELF INTERACTIONS,

CALCULATION OF THE 3-POINT FUNCTION OF ζ(k) AND f_{NL}

WE CALCULATE < $(\delta\phi)^3$ > IN THE $\delta\phi \neq 0$ GAUGE, USING THE CANONICAL FORMALISM FOR CUBIC NEW INFLATION $V(\phi) = V_0 - \mu\,\phi^3$.
WE THEN RELATE ζ(k,t) TO δϕ(k,t). AND CALCULATE < ζ^3 >, AND THE NON-GAUSSIANITY PARAMETER f_{NL}. BELOW t IS ARBITRARY, UNLIKE IN THE δN FORMALISM WHERE t ~ TIME OF HORIZON EXIT.

$$\hat{\zeta}(\mathbf{k},t) = -\frac{1}{\sqrt{2\epsilon}}\delta\hat{\phi}(\mathbf{k},t) + \frac{1}{2}\left(1-\frac{\eta}{2\epsilon}\right)\int\frac{d^3q}{(2\pi)^3}\,\delta\hat{\phi}(\mathbf{k}_1-\mathbf{q},t)\delta\hat{\phi}(\mathbf{q},t)+\cdots$$

$$\langle\hat{\zeta}(\mathbf{k}_1,t)\hat{\zeta}(\mathbf{k}_2,t)\hat{\zeta}(\mathbf{k}_3,t)\rangle \equiv (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{6}{5}f_{NL}\sum_{i<j}P_\zeta(k_i)P_\zeta(k_j) \qquad i,j=1,2,3,\ k_i=|\mathbf{k}_i|$$

$$\frac{6}{5}f_{\rm NL}(k_1,k_2,k_3,t) = \underline{\xi}\left[\frac{1}{3}+\gamma-\underline{\mathbf{N_e}}+\frac{3}{\sum_i k_i^3}\left(k_t\sum_{i<j}k_ik_j-\frac{4}{9}k_t^3\right)\right]+\frac{3}{2}\epsilon-\underline{\eta}+\frac{\epsilon}{\sum_i k_i^3}\left(\frac{4}{k_t}\sum_{i<j}k_i^2k_j^2+\frac{1}{2}\sum_{i\neq j}k_ik_j^2\right)$$

$k_t = \sum_i k_i$, k_i APPROX. EQUAL

FOR n_s = 0,96, **η = - 0,02**, $\xi = 0.5\,\eta^2$, $\varepsilon << \xi$. **$\xi\, N_e$ = 0.012**. SO THE SELF INTERACTION CONTRIBUTION ~ ξ SHOULD NOT BE IGNORED OUTRIGHT.

CONCLUSION: d f_{NL} /dt = 0

f_{NL} IS A FUNCTION OF ε(t), η(t), ξ(t) AND $N_e = H\,(t - t_{exit})$. NOW $d\epsilon/dt \simeq [\,4\epsilon^2 - 2\eta\epsilon\,]\,H,\ d\eta/dt \simeq [\,2\epsilon\eta - \xi\,]\,H,\ d\xi/dt \simeq [\,4\epsilon\xi - \eta\xi\,]\,H$

$$df_{\rm NL}/dt \approx (5/6)\,d[-\xi N_e - \eta]/dt = (5/6)\,[-\xi H + \xi H]\ = 0$$

TIME EVOLUTION OF f_{NL} DUE TO SELF INTERACTIONS IS CANCELLED BY CONTRIBUTION OF OTHER TERMS FROM COSM. PERT. THEORY.

THUS THE 3-POINT FUNCTION OF ζ DOES NOT GROW OUTSIDE THE HORIZON.