Off-center CMB anisotropies in the local void model

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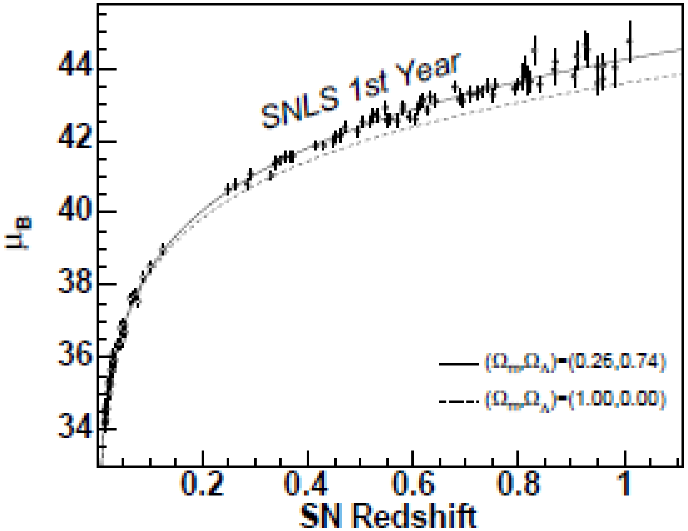
Reference: H. Kodama, K. Saito and A. Ishibashi,

Prog. Theor. Phys. **124**, 1 (2010), arXiv:1004.3089.

Introduction



P. Astier et al (2005)

Type Ia SN

→apparent accelerated expansion

⇩

- dark energy
- modified gravity
- local void model

Lemaitre-Tolman-Bondi (LTB) model

(spherically symmetric, dust dominant model)

G. Lemaitre (1933), R.C. Tolman (1934), H. Bondi (1947)

- metric $$ds^2 = -dt^2 + \frac{\{R'(t,r)\}^2}{1-k(r)r^2}dr^2 + R^2(t,r)d\Omega^2$$ $$\begin{cases} \dot{} \equiv \partial_t \\ ' \equiv \partial_r \end{cases}$$

- Einstein eq. $$\left(\frac{\dot{R}}{R}\right)^2 = \frac{2GM(r)}{R^3} - \frac{k(r)r^2}{R^2} \qquad 4\pi\rho(t,r) = \frac{M'}{R^2R'}$$

$$\begin{cases} k(r)>0: \ R(t,r) = \dfrac{M(r)}{k(r)r^2}(1-\cos\eta), \quad t-t_s(r) = \dfrac{M(r)}{\{k(r)r^2\}^{\frac{3}{2}}}(\eta-\sin\eta) \\ k(r)=0: \ R(t,r) = \left(\dfrac{9}{2}\right)^{\frac{1}{3}} M^{\frac{1}{3}}(r)\{t-t_s(r)\}^{\frac{2}{3}} \\ k(r)<0: \ R(t,r) = \dfrac{M(r)}{-k(r)r^2}(\cosh\eta-1), \quad t-t_s(r) = \dfrac{M(r)}{\{-k(r)r^2\}^{\frac{3}{2}}}(\sinh\eta-\eta) \end{cases}$$

⇨ The solutions admit two arbitrary functions

$$t_s(r), \ M(r) \ (\text{or } k(r))$$

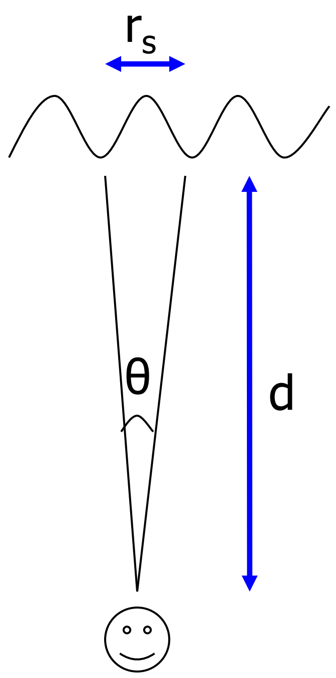
Big-bang time

Constraints from observations

① BAO

the wavelength of an oscillation of baryon after decoupling

sound horizon: $$r_s = \int_0^{\eta_{dec}} c_s(\eta)\mathrm{d}\eta \sim 108\mathrm{Mpc}/h$$



$$d = \frac{r_s}{\theta}$$

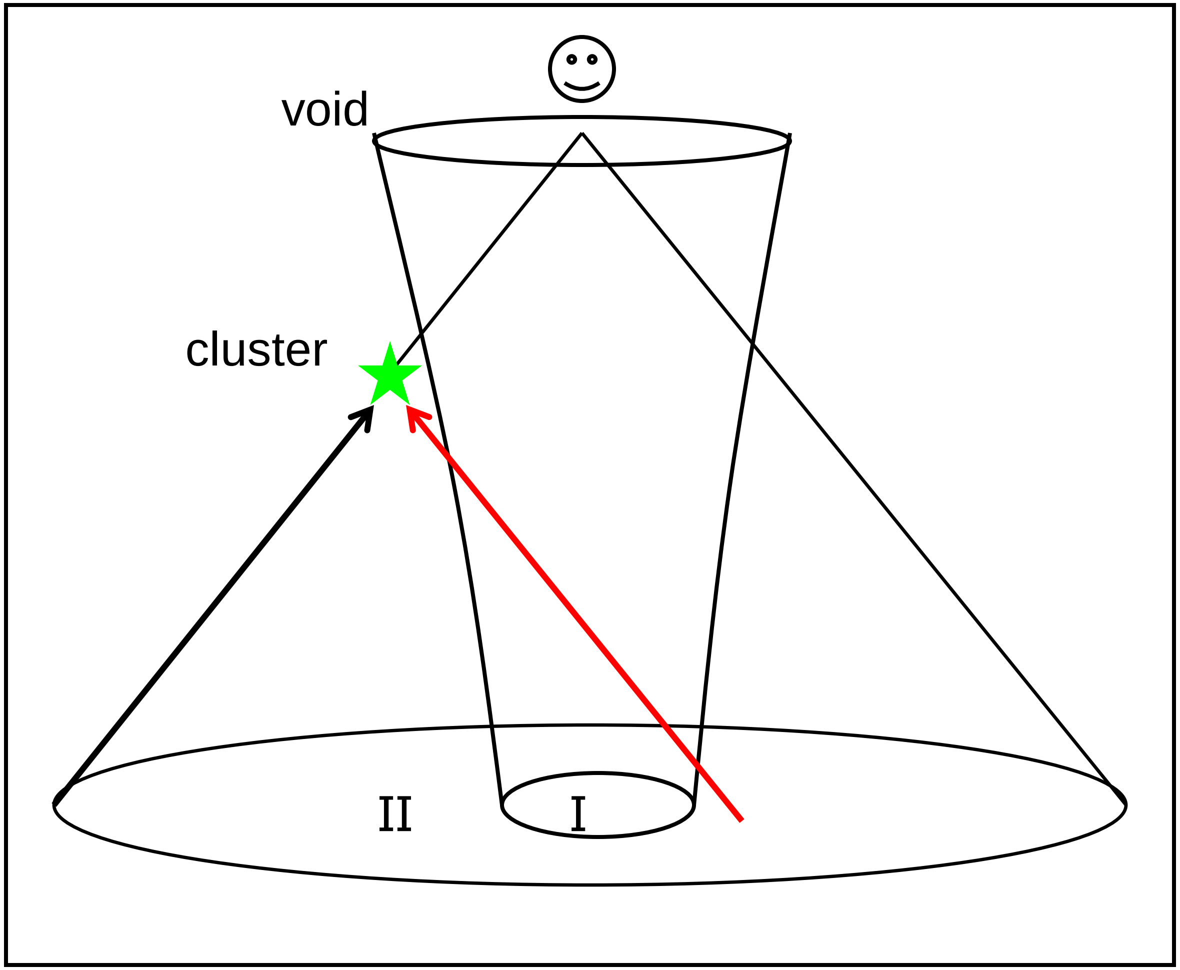
the epoch z~0.35 must belong to the inner underdense region

⇓

$$r_0 \sim \mathrm{Gpc}$$

W.J. Percival et al. (2006)

② kSZ (kinematic Sunyaev-Zeldovich) effect



$\rho_I < \rho_{\mathrm{II}}$

$H_{\mathrm{I}} > H_{\mathrm{II}}$

⇓

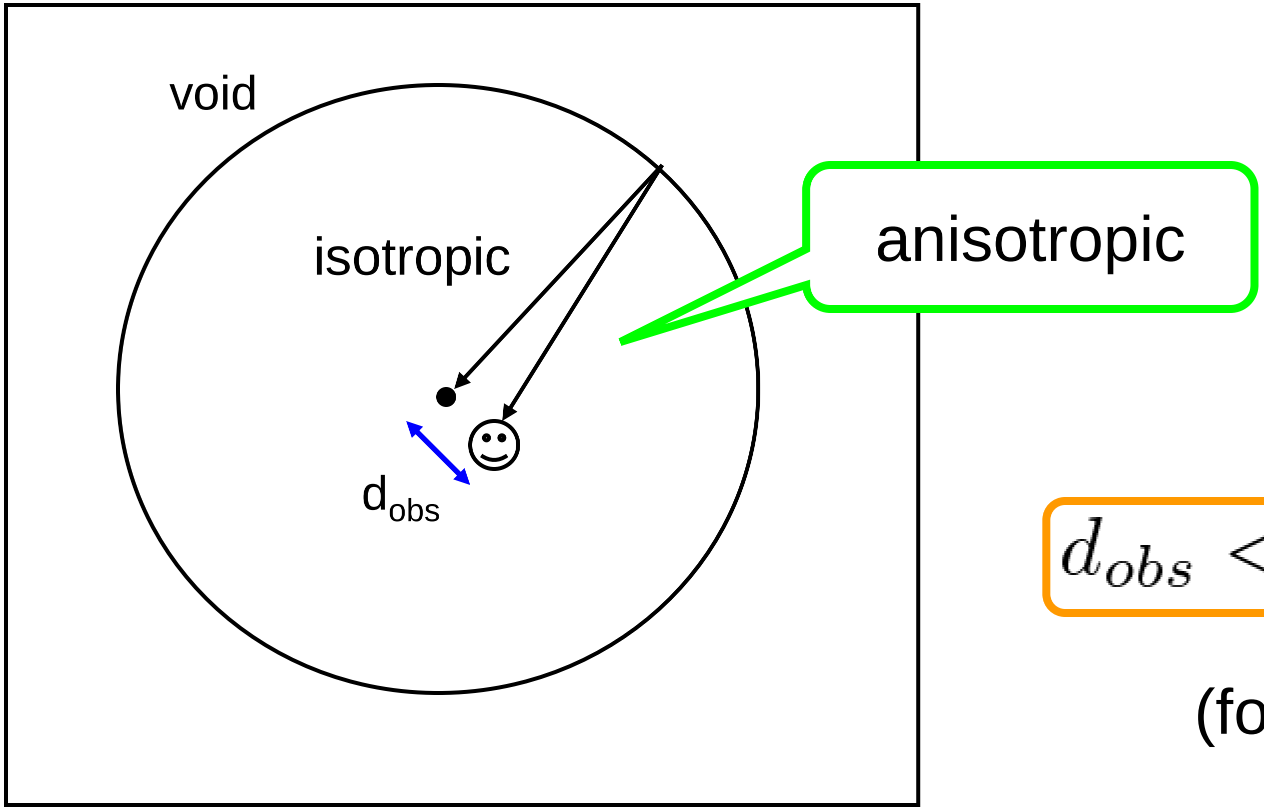
the cluster receives dipole

⇓

$r_0 < 1.5\mathrm{Gpc}$

J. Garcia-Bellido, T. Haugbolle (2008)

③ CMB dipole



$$d_{obs} < 15\mathrm{Mpc}$$

(for r_0~1.5Gpc)

H. Alnes, M. Amarzguioui (2006)

Constraints from observations

- BAO → the size of void ～ Gpc
  （Baryon Acoustic Oscillation）
  S. Alexander et al. (2007)

- kSZ effect → the size of void ＜ 1.5Gpc
  （kinematic Sunyaev-Zeldovich）
  J. Garcia-Bellido, T. Haugbolle (2008)

- CMB dipole → **the distance from the observer to the center ＜ 15Mpc**
  H. Alnes, M. Amarzguioui (2006)

we derive the analytic formulae for the CMB multipole.

CMB multipole

In order to test the LTB cosmology, we have to analyze perturbations such as metric perturbation.

However, in the LTB cosmology, it is not easy since modes of the perturbation couple each other.

⇒ Here I derive analytic formulae for the CMB dipole and quadrupole by calculating **Taylor series for the distribution function of CMB photon at r=0** in the local void model.

CMB temperature anisotropies

(1) CMB dipole

Distribution function of CMB photon

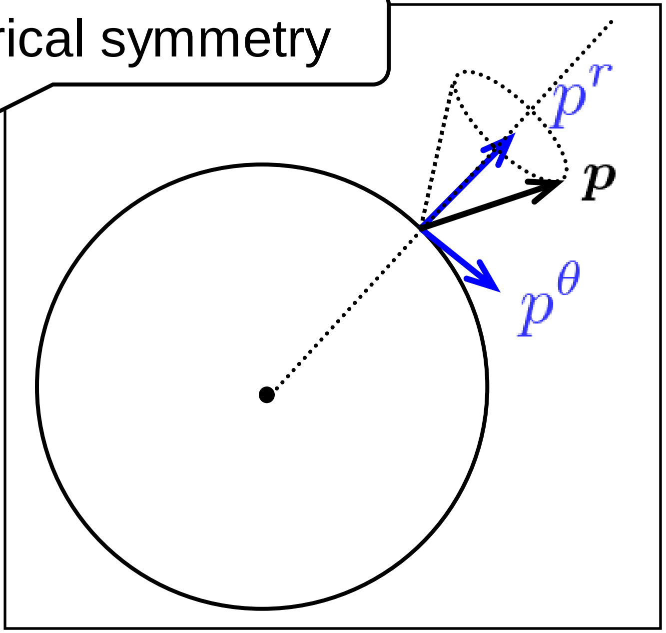
$$F(x,p,p^i) = F_0(t,r,\omega,\mu)$$

massless

$$\omega = p$$

$$\mu \equiv \frac{R'}{\sqrt{1-kr^2}}\frac{p^r}{p}$$

spherical symmetry



- Taylor series at r=0 → $\delta F = \delta x^i (\partial_i F_0)_{t=t_0, r=0}$ (present time)
- the universe is locally in thermal equilibrium at LSS → $\delta F = -\frac{\delta T}{T}\omega\partial_\omega F_0$

CMB dipole $\Longrightarrow$ $$\frac{\delta T}{T} = -\frac{\delta x^i (\partial_i F_0)_{t=t_0, r=0}}{\omega \partial_\omega F_0}$$

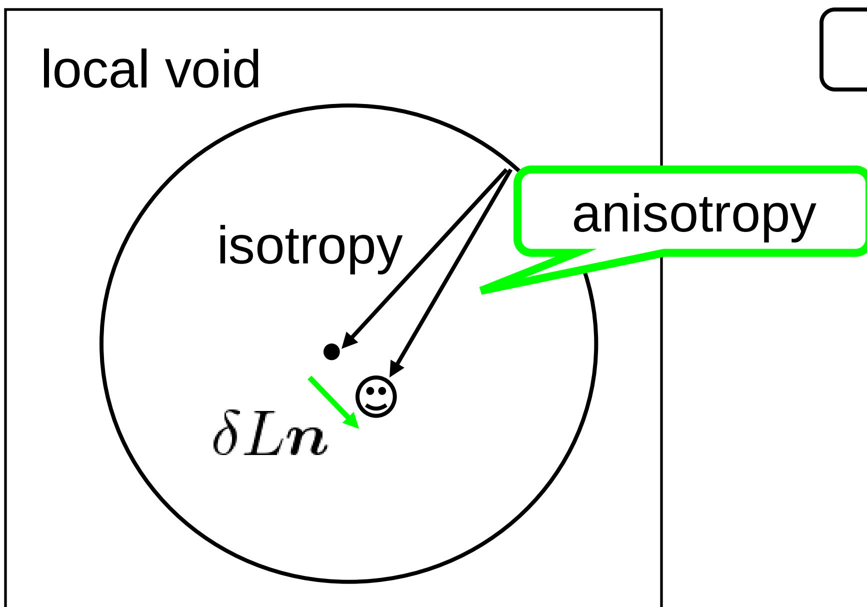
Boltzmann eq.

$$\frac{d}{dt}F_0 = \partial_t F_0 + \dot{r}\partial_r F_0 + \dot{\omega}\partial_\omega F_0 + \dot{\mu}\partial_\mu F_0 = 0$$

CMB dipole formula

$$\left(\frac{\delta T}{T}\right)^{(1)} = \delta L \boldsymbol{n} \cdot \boldsymbol{\Omega} \left\{ \frac{\xi_i}{a_0} e^{-\tilde{P}(t_0,t_i)} \left(\frac{\partial_r F_0}{\omega \partial_\omega F_0}\right)_i + \int_0^{r_i} dr H'_{//} \exp\left[\int_{t_0}^{t} dt_1 H_{//}(t_1)\right] \right\}$$

Initial condition



$$\Omega^i = \frac{x^i}{r} \qquad a_0 = (R')_0$$

$$\xi = \sqrt{1 - kr^2} \qquad H_{//} \equiv \frac{\dot{R}'}{R'}$$

$$\tilde{P}(t_0, t) = \int_0^r dr_1 \left(\frac{R''}{R'}\right)_{r_1}$$

(2) CMB quadrupole formula

$$\left(\frac{\delta T}{T}\right)^{(2)} = -\frac{\delta x^i \delta x^j}{2\omega\partial_\omega F_0}\left[2(\delta_{ij}-\Omega_i\Omega_j)(f_2)_0 + \Omega_i\Omega_j(\partial_r^2 F_0)_0 + \left\{\frac{a_\perp''}{a_\perp}\delta_{ij} + a_\perp\left(S''-a_\perp''\right)\frac{\Omega_i\Omega_j}{S^2}\right\}_0 (\omega\partial_\omega F_0)_i\right]$$
$$+\frac{1}{2}\left\{\left(\frac{\delta T}{T}\right)^{(1)}\right\}^2 \frac{(\omega\partial_\omega)^2 F_0}{\omega\partial_\omega F_0}$$

$$(f_2)_0 = \frac{a_0^2}{2}e^{-\tilde{Q}(t_0,t_i)}\left(\frac{\partial_\mu F_0}{R^2}\right)_i - \frac{a_0^2}{a_{\perp i}^2}\frac{e^{-\tilde{Q}(t_0,t_i)}}{2r_i}\left(-\partial_r F_0 + \frac{R'}{\xi}B\omega\partial_\omega F_0\right)_i$$
$$-\frac{a_0^2}{2}\int_{t_i}^{t_0}\frac{dt}{r}\left[-\left\{2\left(H_{//}-H_\perp+\frac{\xi}{R'}\frac{a_\perp'}{a_\perp}\right)+\left(\frac{\xi}{R'}\right)'\right\}\partial_r F_0 + \left\{-H_{//}' + 2H_{//}\frac{R'}{\xi}B + a_\perp^2\frac{d}{dt}\left(\frac{R'B}{a_\perp^2\xi}\right)\right\}(\omega\partial_\omega F_0)_i\right]\frac{e^{-\tilde{Q}(t_0,t)}}{a_\perp^2}$$

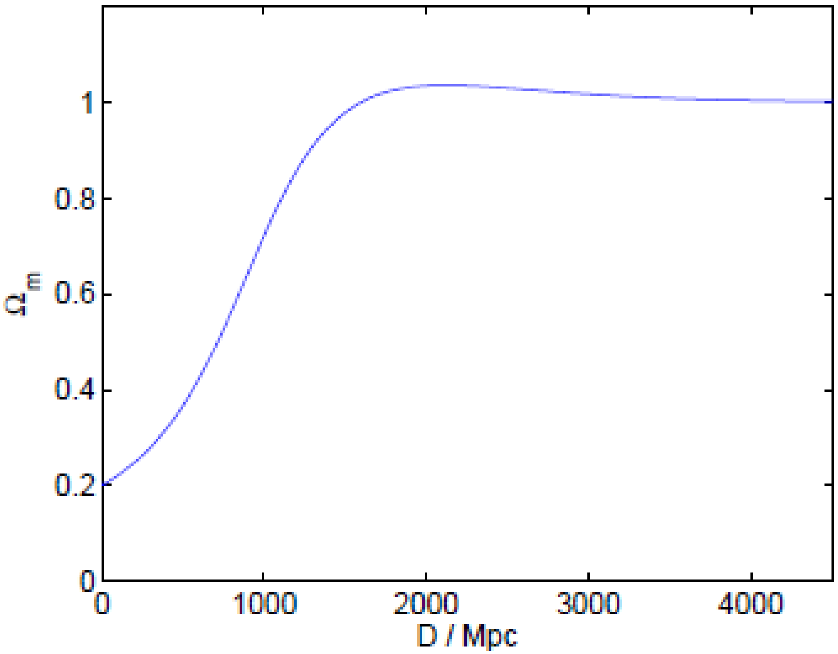
$$(\partial_r^2 F_0)_0 = \xi_i^2 e^{-2\tilde{P}(t_0,t_i)}(\partial_r^2 F_0)_i + \int_{t_i}^{t_0} dt\xi^2 e^{-2\tilde{P}(t_0,t)}\left\{2H_{//}'\omega\partial_\omega\partial_r F_0 + H_{//}''(\omega\partial_\omega F_0)_i + \left(\frac{\xi}{R'}\right)''\partial_r F_0\right\}$$

$$\partial_r F_0 = \frac{\xi_i}{\xi}e^{-\tilde{P}(t,t_i)}(\partial_r F_0)_i + \frac{(\omega\partial_\omega F_0)_i}{\xi}\int_{t_i}^{t} dt_1\xi(t_1)e^{-\tilde{P}(t,t_1)}H_{//}(t_1)$$

$$\omega\partial_\omega\partial_r F_0 = \frac{\xi_i}{\xi}e^{-\tilde{P}(t,t_i)}(\omega\partial_\omega\partial_r F_0)_i + \frac{\{(\omega\partial_\omega)^2 F_0\}_i}{\xi}\int_{t_i}^{t} dt_1\xi(t_1)e^{-\tilde{P}(t,t_1)}H_{//}(t_1)$$

$$H_\perp \equiv \frac{\dot{R}}{R} \qquad B = -2(H_{//}-H_\perp) \qquad a_\perp = \frac{R}{r}$$

Constraints on LTB models



density distribution at present time

$$M(r) = \frac{1}{2}H_\perp^2(t_0, r_{\rm out})r^3\left[1 - \Delta\alpha\left(\frac{1}{2} - \frac{1}{2}\tanh\frac{r-r_0}{2\Delta r}\right)\right]$$

$$k(r) = H_\perp^2(t_0, r_{\rm out})\Delta\beta\left(\frac{1}{2} - \frac{1}{2}\tanh\frac{r-r_0}{2\Delta r}\right)$$

$$t_s(r) = 0$$

$$H_\perp(t_0, r_{\rm out}) = 51\ \mathrm{km/s/Mpc}, \quad \Delta\alpha = 0.90,$$
$$r_0 = 1.34\ \mathrm{Gpc}, \quad \Delta r/r_0 = 0.4, \quad \Delta\beta = -\Delta\alpha = -0.90$$

$a_{10} < 2.52 \times 10^{-3}$ (G. Hinshaw et al (2008))

The distance from observers to the center $\delta L \lesssim 33\,\mathrm{Mpc}$

⟹ We can check the consistency with the numerical studies previously made (H. Alnes, M. Amarzguioui (2006))

Summary

- We derived analytic formulae for the CMB dipole and quadrupole by calculating Taylor series for the distribution function of CMB photon at r=0 in the general LTB cosmology.

- We place the constraints concerning the location of observers from the off-center CMB anisotropy.