Some Recent Developments in Leptogenesis

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Leptogenesis has been a very fruitful idea (Fukugita, Yanagida '86 ...); some titles during the past year:

leptogenesis from quantum gravity, aidnogenesis from leptogenesis, thermal leptogenesis, supersymmetric leptogenesis, quantum mechanics of leptogenesis, flavoured leptogenesis, soft leptogenesis, fermion triplet leptogenesis, nonthermal leptogenesis, flavoured soft leptogenesis, degenerate leptogenesis, low-scale leptogenesis, N_2 dominated leptogenesis, gauged B-L leptogenesis, resonant leptogenesis, Dirac leptogenesis, electromagnetic leptogenesis, radiatively generated leptogenesis, testable leptogenesis, colour octet leptogenesis, ...

corresponding list of authors ...

main areas of research: connection with flavour physics, QFT treatment of thermal leptogenesis, connection with inflation and dark energy, connection with collider physics, connection with dark matter

I. Nonequilibrium QFT for thermal LG

The seesaw mechanism explains smallness of the light neutrino masses by largeness of the heavy Majorana masses; mass eigenstates:

$$
\begin{aligned}
N &\simeq \nu_R + \nu_R^c \; : \qquad m_N \simeq M \; ; \\
\nu &\simeq \nu_L + \nu_L^c \; : \qquad m_\nu = -m_D^T \frac{1}{M} m_D \; .
\end{aligned}
$$

For third gneration Yukawa couplings $\mathcal{O}(1)$, one has

$$
M_3 \sim \Lambda_{GUT} \sim 10^{15} \text{ GeV} \; , \quad m_3 \sim \frac{v^2}{M_3} \sim 0.01 \text{ eV} \; .
$$

Lightest (heavy) Majorana neutrino N_1 ideal agent for baryogenesis: no SM gauge interactions (out-of-equilibrium condition !), decays to lepton-Higgs pair $\rightarrow$ lepton asymmetry $\langle L \rangle_T \neq 0$, partially converted to baryon asymmetry $\langle B \rangle_T \neq 0$ (sphaleron processes).

Generated baryon asymmetry is proportional to the CP asymmetry in N_1-decays (simplest case, rough estimate),

$$\epsilon_1 = \frac{\Gamma(N_1 \to l\phi) - \Gamma(N_1 \to \bar{l}\bar{\phi})}{\Gamma(N_1 \to l\phi) + \Gamma(N_1 \to \bar{l}\bar{\phi})}$$
$$\sim \frac{3}{16\pi}\frac{M_1 m_3}{v^2} \sim 0.1\ \frac{M_1}{M_3}\ .$$

Order of magnitude estimate for hierarchical heavy Majorana neutrinos, e.g. $M_1/M_3 \sim m_u/m_t \sim 10^{-5}$, $\epsilon_1 \sim 10^{-6}$; baryon asymmetry:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -d\ \epsilon_1\ \kappa_f \sim 10^{-10}\ ,$$

with dilution factor $d \sim 0.01$ (increase of photon number density), efficiency factor $\kappa_f \sim 10^{-2}$ (Boltzmann equations, competition between production and washout); baryogenesis temperature $T_B \sim M_1 \sim 10^{10}$ GeV; **OK!!**

Quantitative analysis: Boltzmann equations

Decays and inverse decays of heavy Majorana neutrinos sufficient for relevant range of neutrino masses; simplest case: hierarchical heavy neutrinos ($N_1 \equiv N$), "one-flavour" approximation; dynamics described by set of Boltzmann equations:

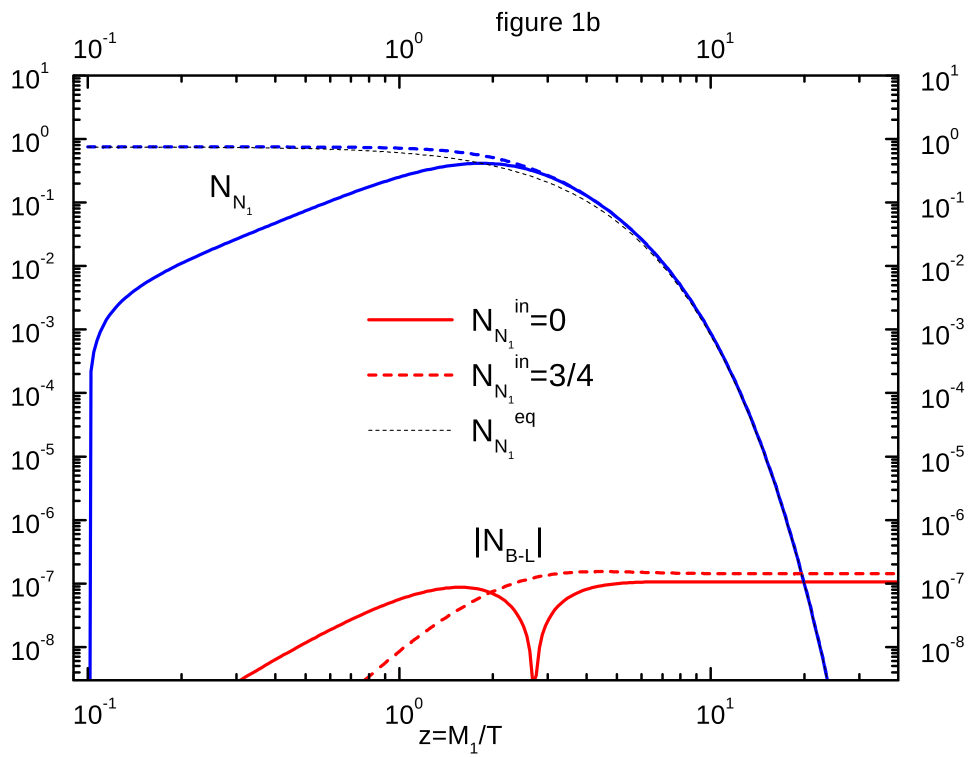
$$\begin{aligned} \frac{dn_N}{dt} + 3Hn_N &= -\left(n_N - n_N^{eq}\right)\ \Gamma_N\ , \\ \frac{dn_L}{dt} + 3Hn_L &= -\epsilon\left(n_N - n_N^{eq}\right)\ \Gamma_N + \text{washout}; \end{aligned}$$

number densities and distribution functions:

$$n_N(t) = \int \frac{d^3q}{(2\pi)^3}\ f_N(t,\omega)\ , \quad n_L(t) = \int \frac{d^3k}{(2\pi)^3}\ f_L(t,k)\ ;$$

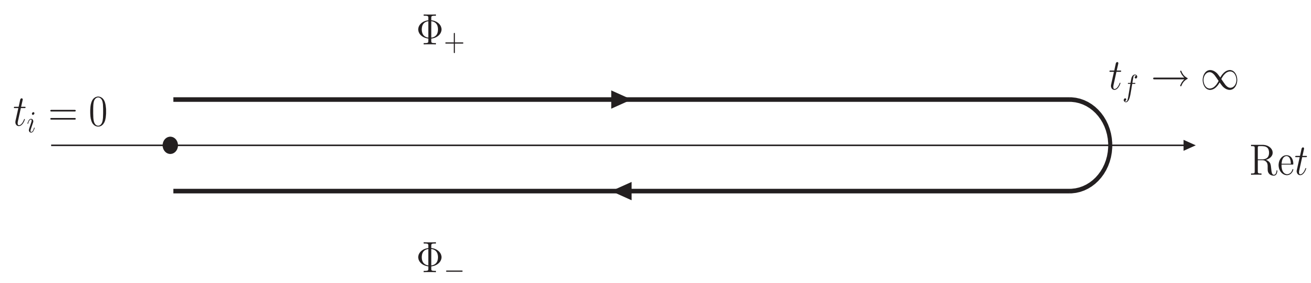
CP asymmetry ϵ: quantum interference; washout terms: tree level

Leptogenesis in expanding universe (WB, Di Bari, Plümacher '02)



For zero initial N-abundance: initial asymmetry has comparable magnitude as final asymmetry [later dicussion: initial asymmetry at fixed T]

Schwinger-Keldysh Formalism



Consider Green's function on contour C (Δ: N_1, lepton, Higgs),

$$(\Box_1 + m^2)\Delta_C(x_1, x_2) + \int_C d^4x'\Pi_C(x_1, x')\Delta_C(x', x_2) = -i\delta_C(x_1 - x_2) \; .$$

Convenient quantities for nonequilibrium processes: spectral function and statistical propagator,

$$\Delta^+(x_1, x_2) = \frac{1}{2}\langle\{\Phi(x_1), \Phi(x_2)\}\rangle \; , \quad \Delta^-(x_1, x_2) \quad = i\langle[\Phi(x_1), \Phi(x_2)]\rangle \; .$$

Assume spacial homogeneity; Kadanoff-Baym equations:

$$(\partial_{t_1}^2 + \omega_{\mathbf{q}}^2)\Delta_{\mathbf{q}}^-(t_1,t_2) = -\int_{t_2}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1,t') \Delta_{\mathbf{q}}^-(t',t_2) \ ,$$

$$(\partial_{t_1}^2 + \omega_{\mathbf{q}}^2)\Delta_{\mathbf{q}}^+(t_1,t_2) = -\int_{t_i}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1,t') \Delta_{\mathbf{q}}^+(t',t_2) + \int_{t_i}^{t_2} dt' \Pi_{\mathbf{q}}^+(t_1,t') \Delta_{\mathbf{q}}^-(t',t_2) \ .$$

Neglect backreaction for large thermal bath ($\Pi_{\mathbf{q}}^{\pm}(t_1 - t')$); general solution: $\Delta_{\mathbf{q}}^-(t_1 - t_2)$ in terms of spectral function; statistical propagator ($\rightarrow$ distribution function):

$$\Delta_{\mathbf{q}}^+(t_1,t_2) = \ldots + \Delta_{\mathbf{q},\text{mem}}^+(t_1,t_2) \ ,$$

$$\Delta_{\mathbf{q},\text{mem}}^+(t_1,t_2) = \int_{t_i}^{t_1} dt' \int_{t_i}^{t_2} dt'' \Delta_{\mathbf{q}}^-(t_1 - t') \Pi_{\mathbf{q}}^+(t' - t'') \Delta_{\mathbf{q}}^-(t'' - t_2) \ .$$

First approximation: neglect memory effects, zeroth order in derivative expansion w.r.t. relative time,

$$\Delta^{+}_{\mathbf{q}}(t_1,t_2) \simeq \widetilde{\Delta}^{+}_{\mathbf{q}}(t) \propto f(t,q) \ , \quad t = \frac{t_1+t_2}{2} \ ,$$

$\longrightarrow$ Quantum Boltzmann Equations. Solution for lepton asymmetry $f_{Li} = f_{li} - f_{\bar{l}i}$, with initial condition $f_{Li}(0,k) = 0$ (Γ: N_1 decay width),

$$\begin{aligned} f_{Li}(t,k) &= -\epsilon_{ii}\frac{16\pi}{k}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' \\ &\quad \times(2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p) \\ &\quad \times f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega)\frac{1}{\Gamma}\left(1-e^{-\Gamma t}\right) \ ; \end{aligned}$$

$f_{l\phi}(k',q')$ additional quantum statistical factor compared to ordinary

Boltzmann equation,

$$\begin{aligned} f_{l\phi}(k',q') &= f_l(k')f_\phi(q') + (1-f_l(k'))(1+f_\phi(q')) \\ &= 1 - f_l(k') + f_\phi(q') \ , \end{aligned}$$

leads to effective temperature dependent CP-asymmetries $\langle\epsilon_{ii}^{\mathrm{th}}(\beta M_1)\rangle$, recently computed by several groups.

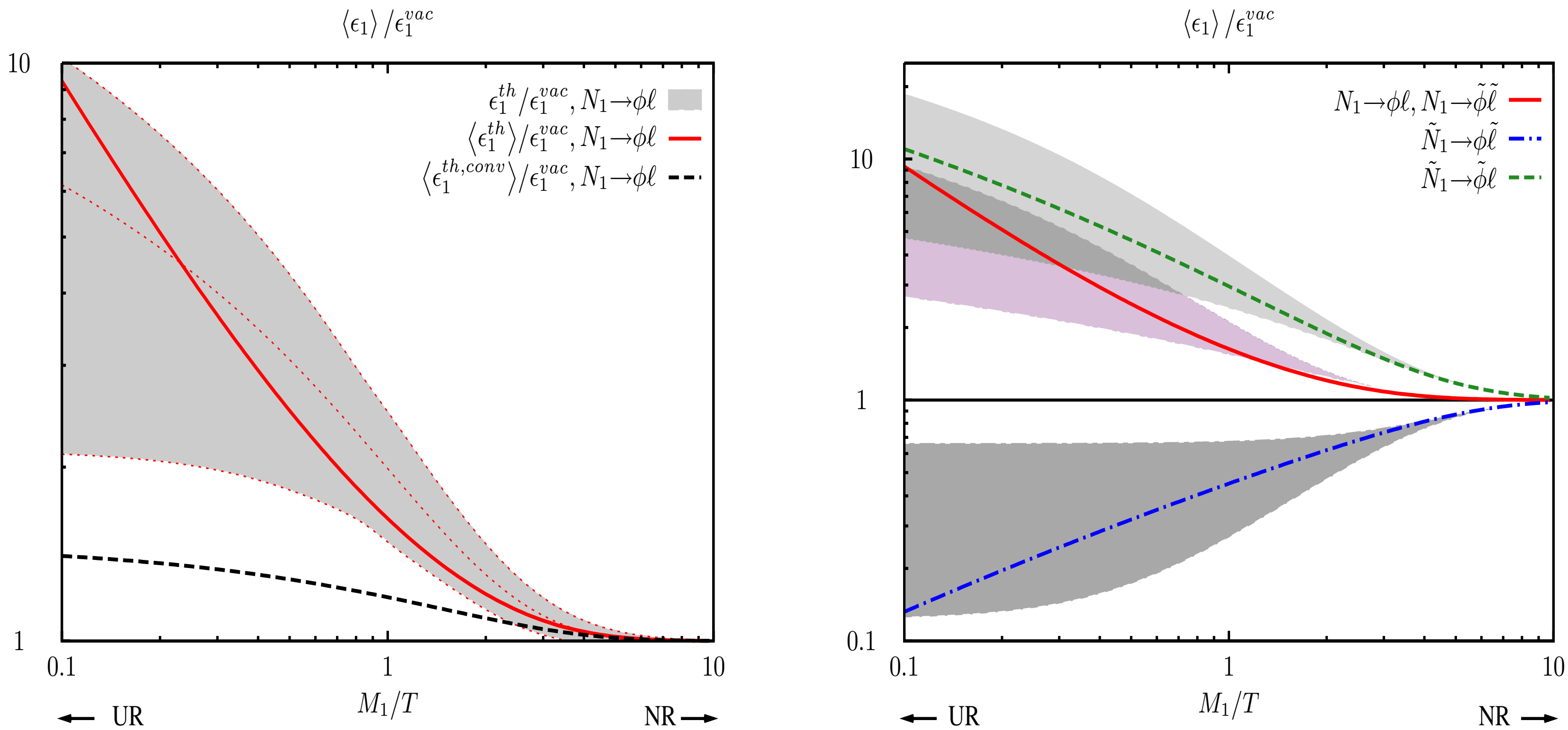
Kadanoff-Baym equations for lepton Green's functions are matrix equations in flavour space, natural starting point for investigation of flavoured leptogenesis; matrix of lepton asymmetries:

$$L_{ij}(t) = L_{ji}(t)^* \ , \quad Y_{ij} \propto \frac{L_{ij}(t)}{g_* T^3} \ .$$

Asymmetry matrix is affected by flavour dependent interactions, has been computed as function of τ-Yukawa coupling.

Effective temperature dependent CP-asymmetries

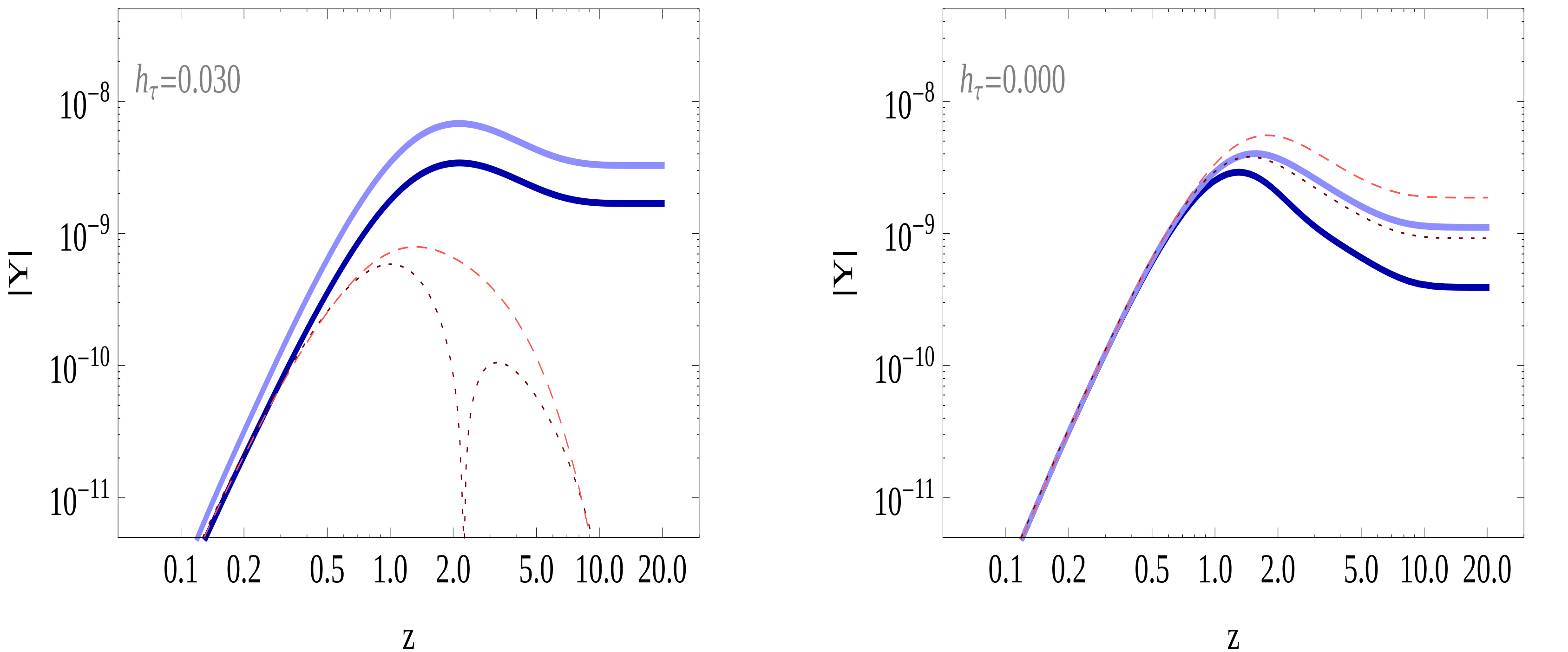
(Garny, Hohenegger, Kartavtsev '10)



Left: SM; new results [red] compared with previous results; shaded region: dependence on lepton momentum. *Right:* MSSM.

Lepton asymmetry density matrix

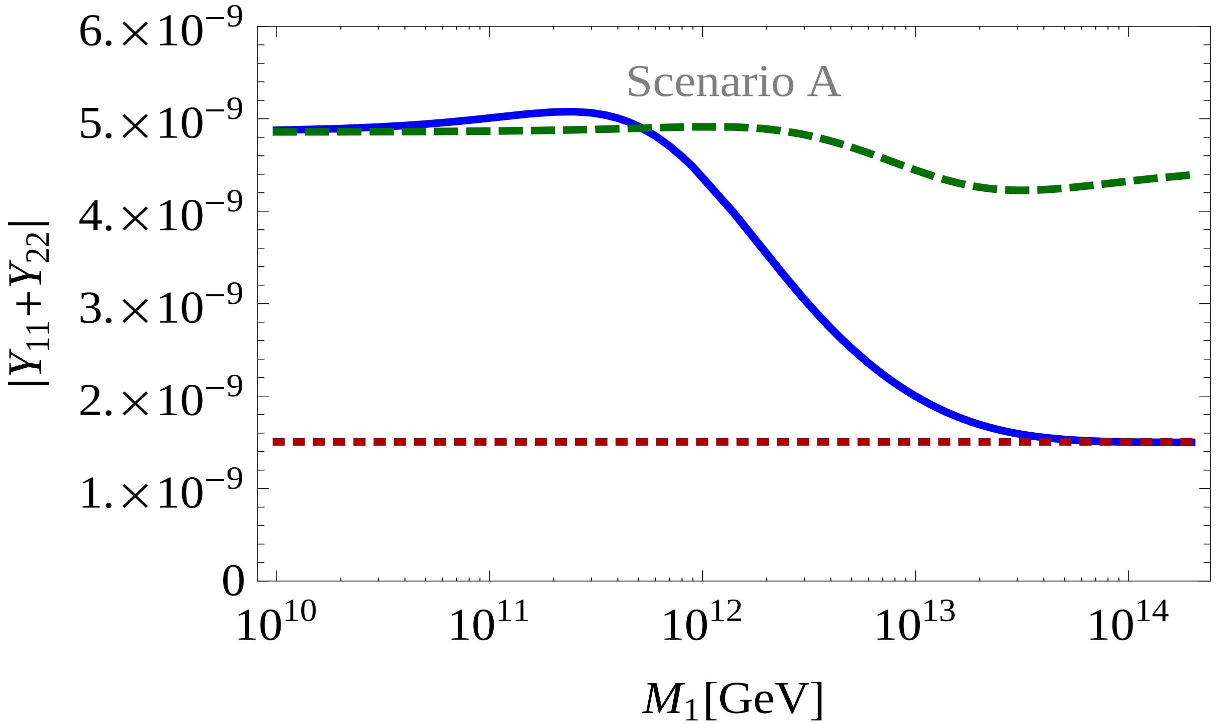
(Beneke,Garbrecht, Fidler,Herranen,Schwaller '10)



Diagonal (full) and off-diagonal (dashed) lepton asymmetry densities as functions of $z = M_1/T$ for different values of the τ-lepton Yukawa coupling.

Total lepton asymmetry density

(Beneke,Garbrecht, Fidler,Herranen,Schwaller '10)



Total lepton asymmetry $\mathrm{Tr}[Y] = Y_{11} + Y_{22}$ as function of $z = M_1/T$. The solution [blue; $h_\tau = 0.007$] interpolates between the fully flavoured approximation [green] and the unflavoured approximation [red].

Beyond Quantum Boltzmann Equations

(Anisimov, WB, Drewes, Mendizabal '10)

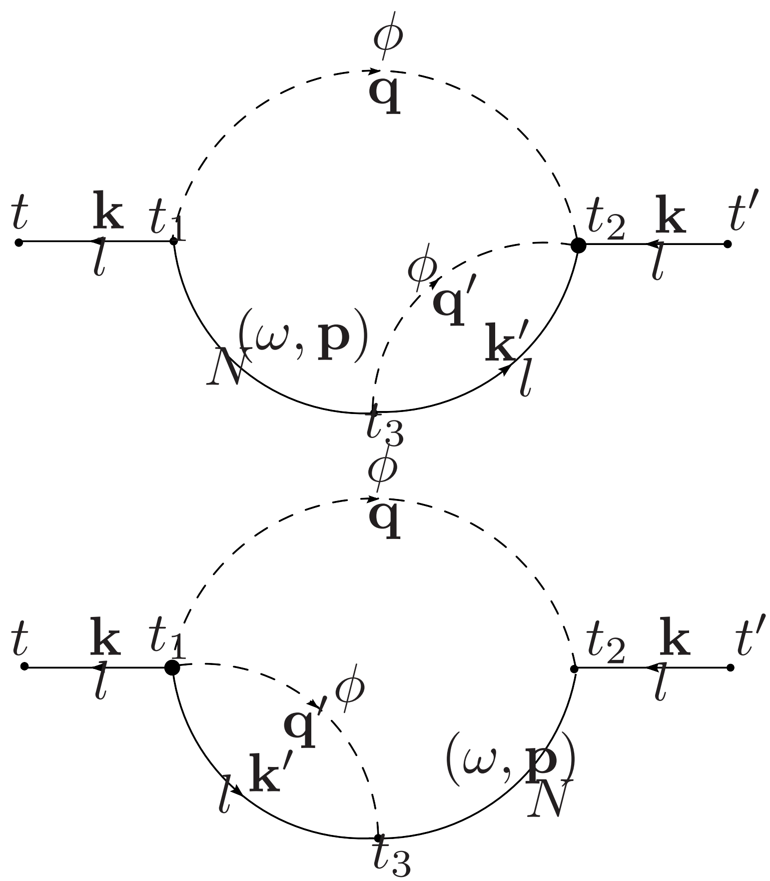
Solve Kadanoff-Baym equations for heavy neutrino propagator (Breit-Wigner approximation; $y = t_1 - t_2$, $t = (t_1 + t_2)/2$),

$$G^-_{\mathbf{p}}(y) = \left(i\gamma_0 \cos(\omega y) + \frac{M - \mathbf{p}\gamma}{\omega} \sin(\omega y)\right) \times e^{-\Gamma|y|/2} C^{-1} \, ,$$

$$G^+_{\mathbf{p}}(t, y) = -\left(i\gamma_0 \sin(\omega y) - \frac{M - \mathbf{p}\gamma}{\omega} \cos(\omega y)\right) \times \left[\frac{\tanh\left(\frac{\beta\omega}{2}\right)}{2} e^{-\Gamma|y|/2} + f_N^{eq}(\omega) e^{-\Gamma t}\right] C^{-1} \, ,$$

with vacuum initial condition $G^+_{\mathbf{p}}(0,0) = G^{+vac}_{\mathbf{p}}(0)$, analogue of initial condition $f_N(0,\omega) = 0$ for distribution function; satisfies KMS condition for $t \to \infty$, becomes vacuum solution for $\beta \to \infty$.

Generation of lepton asymmetries



direct calculation of nonlocal lepton asymmetry matrix:

$$L_{\mathbf{k}ij}(t,t') = -\mathrm{tr}[\gamma^0 S^+_{\mathbf{k}ij}(t,t')] \; ;$$

free fields in equilibrium:

$$L_{\mathbf{k}ii}(t,t) = f_{li}(k) - \bar{f}_{li}(k) \; .$$

Result for lepton asymmetries to leading order in $l\phi N_1$-coupling λ:

$$
\begin{aligned}
L_{\mathbf{k}ii}(t,t) \quad \supset \quad & -\epsilon_{ii}\, 8\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{kk'\omega} \\
& \times \frac{\frac{1}{2}\Gamma}{((\omega-k-q)^2+\frac{\Gamma^2}{4})((\omega-k'-q')^2+\frac{\Gamma^2}{4})} \\
& \times f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega) \\
& \times \Big(\cos[(k+q-k'-q')t] + e^{-\Gamma t} \\
& -(\cos[(\omega-k-q)t] + \cos[(\omega-k'-q')t]) e^{-\frac{\Gamma t}{2}}\Big) ,
\end{aligned}
$$

with $\mathbf{p} = \mathbf{q} + \mathbf{k} = \mathbf{q}' + \mathbf{k}'$; logarithmic divergence $\mathcal{O}(\lambda^4)$; dominant contribution from momenta $k+q \sim k'+q' \sim \omega$; integrand $\mathcal{O}(1/\Gamma)$, lepton asymmetries $\mathcal{O}(\lambda^2)$; expected finite width corrections and important oscillating factors, representing memory effects.

Important effect of thermal damping rates of lepton and Higgs fields, much larger than N_1 decay width: $\Gamma_l \sim \Gamma_\phi \sim g^2 T \gg \lambda^2 M$ for $M \lesssim T$, qualitatively more important than thermal masses which, ($\Gamma_{l\phi} = \Gamma_l + \Gamma_\phi$):

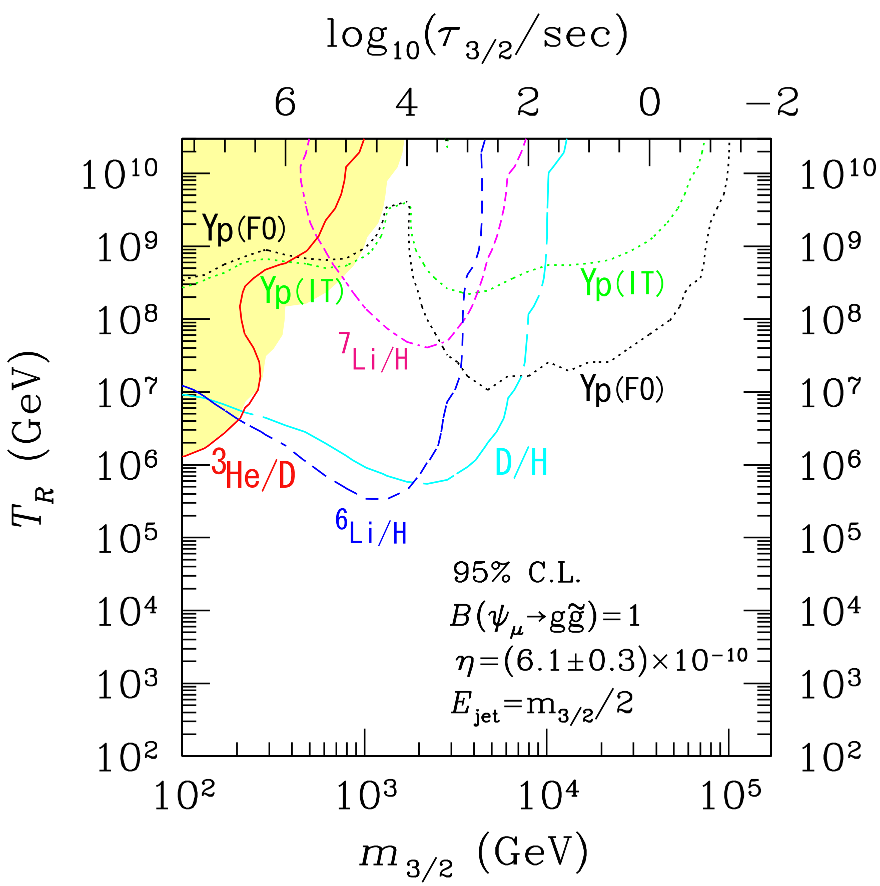
$$\begin{aligned}
\tilde{L}_{\mathbf{k}ij}(t,t) &= -\epsilon_{ij}\, 16\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k \cdot k'}{kk'\omega} \\
&\quad \times \frac{\frac{1}{4}\Gamma_{l\phi}\Gamma_\phi}{((\omega - k - q)^2 + \frac{1}{4}\Gamma_{l\phi}^2)((\omega - k' - q')^2 + \frac{1}{4}\Gamma_\phi^2)} \\
&\quad \times f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega) \\
&\quad \times \frac{1}{\Gamma}\left(1 - e^{-\Gamma t}\right);
\end{aligned}$$

thermal widths make leptogenesis local in time; in the "zero-width limit" result of Boltzmann equations except for statistical factor $f_{l\phi}(k',q')$.

Are we back to Boltzmann equations as best approximation for leptogenesis? Hope: YES, with some corrections and controllable errors ...

II. Common Origin of Matter and Dark Matter

(WB, Schmitz, Vertongen '10)



Leptogenesis requires high high reheating temperature $\longrightarrow$ "gravitino problem"

Most stringent upper bound on T_R (Kawasaki, Kohri, Moroi '05):

$$T_R < \mathcal{O}(1) \times 10^5 \text{ GeV},$$

hence standard mSUGRA with neutralino LSP incompatible with thermal leptogenesis !!

Possible way out: Gravitino LSP & DM, various options for NLSP ...

"Gravitino virtue": for typical leptogenesis temperatures and superparticle masses, thermal gravitino production can explain observed amount of DM,

$$\Omega_{\tilde{G}}h^2 = C\left(\frac{T_R}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{100\,\mathrm{GeV}}{m_{\tilde{G}}}\right)\left(\frac{m_{\tilde{g}}}{1\,\mathrm{TeV}}\right)^2 ,$$

with $C \sim 0.5$; $\Omega_{\mathrm{DM}}h^2 \sim 0.1$ is natural value. BUT WHY $T_L \sim T_R$?

Heavy neutrino decay width is determined by neutrino masses,

$$\Gamma^0_{N_1} = \frac{\tilde{m}_1}{8\pi}\left(\frac{M_1}{v_{\mathrm{EW}}}\right)^2 \sim 10^3\ \mathrm{GeV}\ , \quad \widetilde{m}_1 = 0.01\ \mathrm{eV}\ .$$

Reheating temperature for decaying gas of heavy neutrinos

$$T_R \sim 0.2 \cdot \sqrt{\Gamma^0_{N_1} M_P} \sim 10^{10}\ \mathrm{GeV}\ ,$$

wanted temperature for gravitino dark matter, misleading coincidence?

Spontaneous B-L breaking and false vacuum decay

SUSY Standard Model with right-handed neutrinos ($SU(5)$ notation),

$$W_M = h^u_{ij}\mathbf{10}_i\mathbf{10}_j H_u + h^d_{ij}\mathbf{5}^*_i\mathbf{10}_j H_d + h^\nu_{ij}\mathbf{5}^*_i n^c_j H_u + h^n_i n^c_i n^c_i S \ ,$$

with symmetry breaking: $\langle H_u\rangle = v_u$, $\langle H_d\rangle = v_d$, $\langle S\rangle = v_{B-L}$.

Yukawa couplings from FN $U(1)$ flavour symmetry (WB, Yanagida '99),

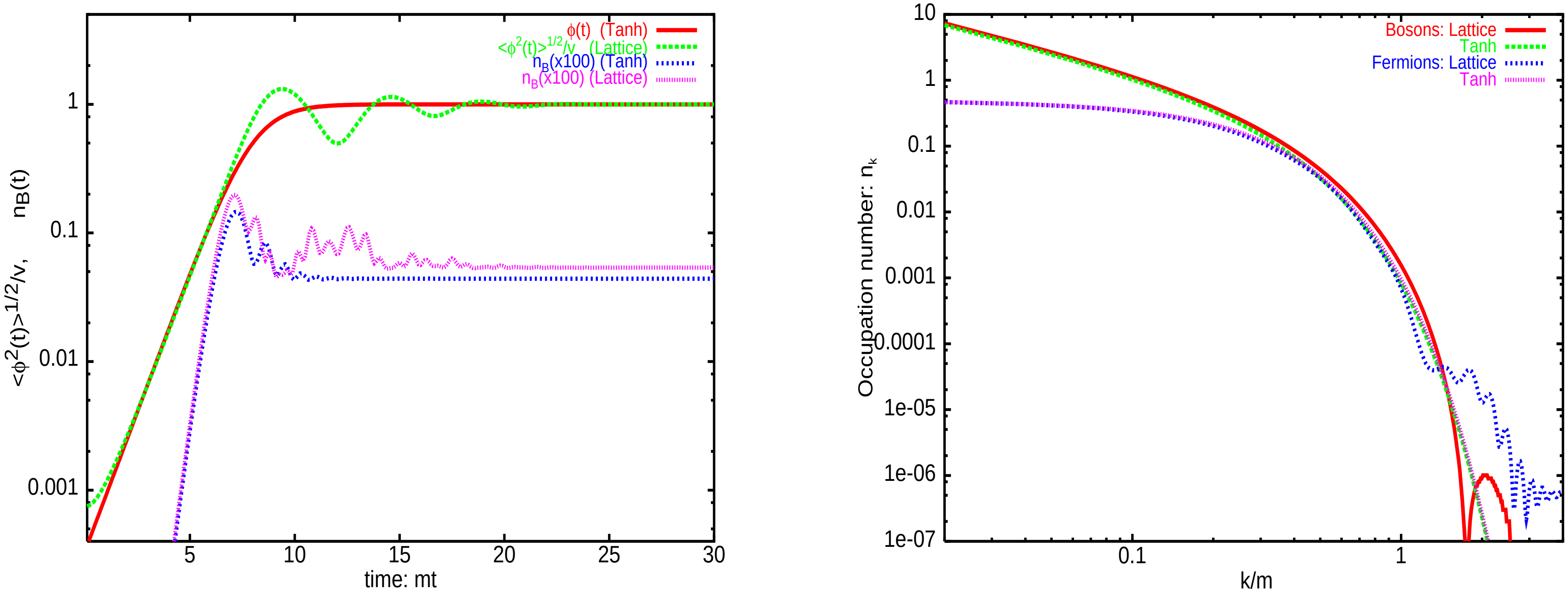
$$h_{ij} \propto \eta^{Q_i+Q_j} \ ,$$

with $\eta \simeq 1/\sqrt{300}$. Masses, decay widths and CP asymmetries:

$$2M_{2,3} > m_S > 2M_1 \ , \quad M_1 \simeq 10^{10}\ \mathrm{GeV}$$

$$\Gamma^0_{N_1} \simeq \frac{\eta^4}{8\pi}M_1 \ , \quad \Gamma^0_{N_{2,3}} \simeq \frac{\eta^2}{8\pi}M_{2,3} \ , \quad \Gamma^0_S \simeq \frac{\eta^4}{16\pi}m_S \ ,$$

$$\epsilon_1 \sim 0.1\ \eta^4 \ \sim 10^{-6}, \quad \epsilon_{2,3} \simeq 0.1\ \eta^2 \sim 3\times 10^{-4} \ .$$



Decay of false vacuum by growth of long wave-length modes, 'tachyonic preheating' (Garcia-Bellido, Morales '02); true vacuum reached at time $t_{\rm PH}$,

$$\langle S^\dagger S\rangle\Big|_{t=t_{\rm PH}} = v_{B-L}^2\ , \qquad t_{\rm PH} \simeq \frac{1}{2m_S}\ln\left(\frac{32\pi^2}{\lambda}\right)\ .$$

Initial state: nonrelativistic gas of S-bosons, $N_{2,3}$ and N_1 heavy neutrinos; energy fractions:

$$r_{N_1} \ll r_{N_{2,3}} \sim 10^{-3} \ll r_S \simeq 1 \ .$$

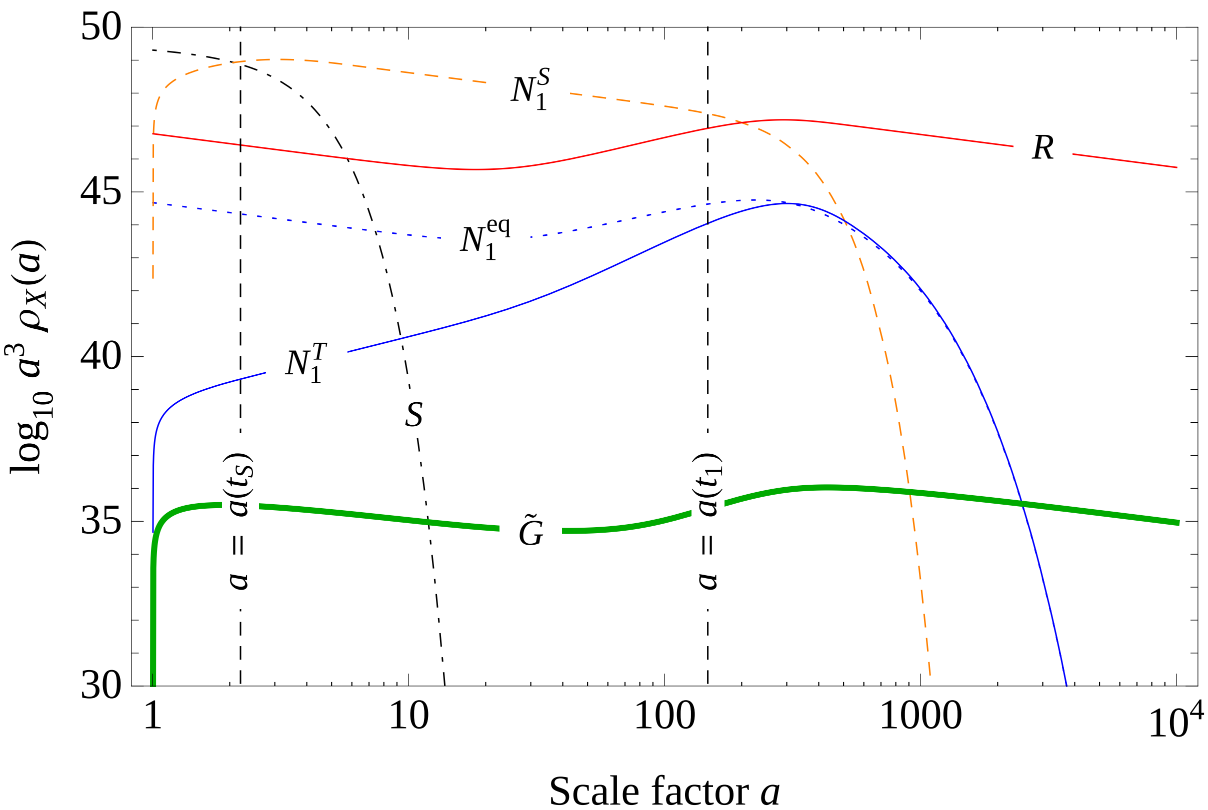
Time evolution: rapid $N_{2,3}$ decays, yields initial radiation, thermal N_1's and gravitinos; S decays produce nonthermal N_1's; N_1 decays produce most of radiation and baryon asymmetry; details of evolution described by Boltzmann equations.

For typical parameter choices, e.g.,

$$\widetilde{m_1} = 10^{-3} \text{ eV} \ , \quad M_1 = 10^{10} \text{ GeV} \ , \quad \epsilon_1 = -\eta^2 \epsilon_{2,3} = 10^{-6} \ ,$$
$$m_{\widetilde{G}} = 100 \text{ GeV} \ , \quad m_{\widetilde{g}} = 800 \text{ GeV} \ ,$$

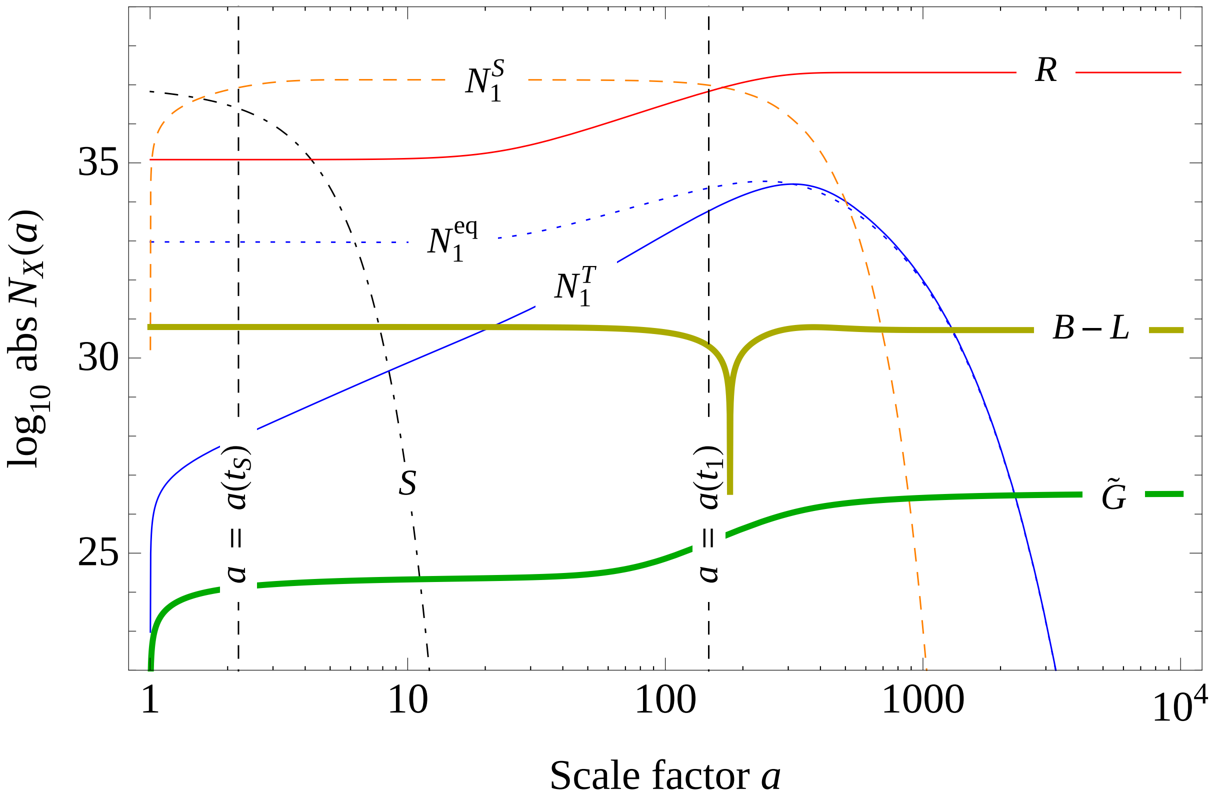
one obtains consistent values of matter and dark matter abundances, $\eta_B = 1.6 \times 10^{-7} > \eta_B^{\text{CMB}} \simeq 6.2 \times 10^{-10}$ and $\Omega_{\tilde{G}} h^2 = 0.11$; in general correlation between absolute neutrino mass scale and the gravitino mass !!

Thermal and nonthermal energy densities



Comoving energy densities of thermal and nonthermal N_1's, gravitinos and radiation as functions of scale factor a.

Thermal and nonthermal number densities

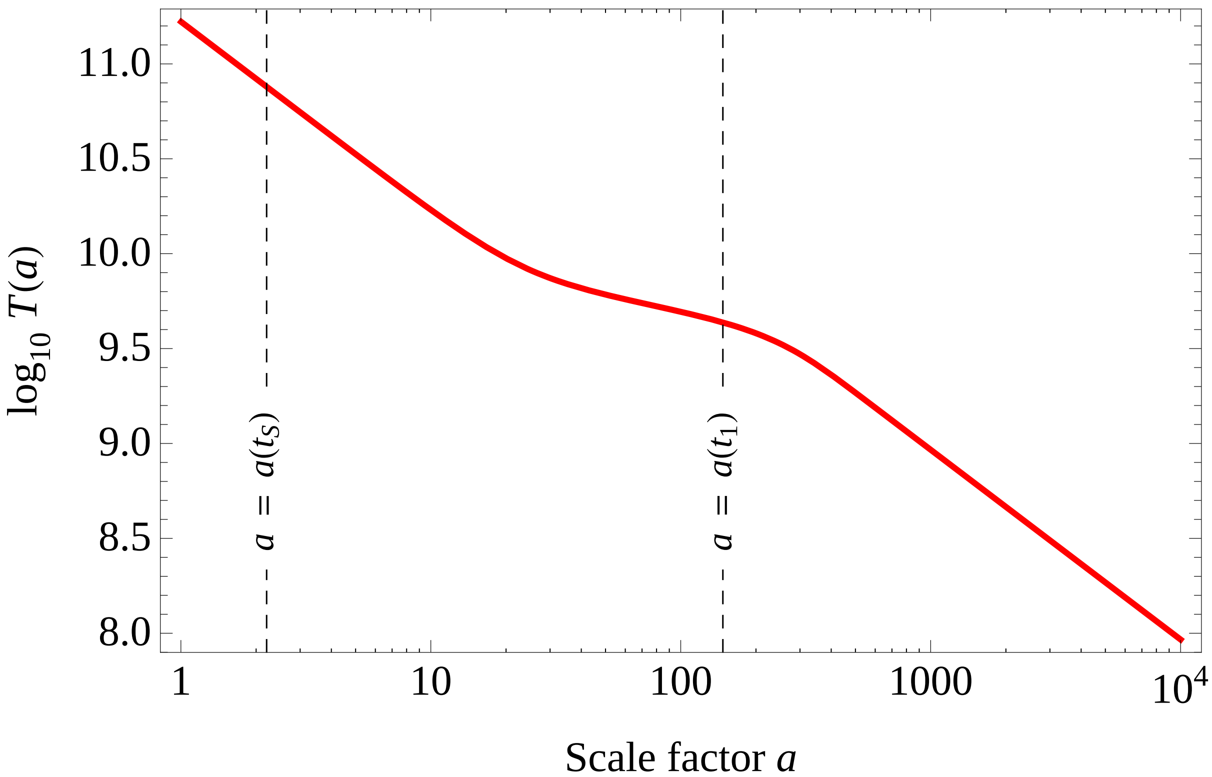


Comoving number densites of thermal and nonthermla $N_1's$, $B-L$, gravitinos and radiation as functions of scale factor a.

SUMMARY

- Leptogenesis is very active field of research!
- Considerable progress towards full quantum treatment of leptogenesis; first results: temperature dependent CP asymmetries, kinetic equations for flavoured leptogenesis
- Close connection between leptogenesis and dark matter; gravitino dark matter viable possibility!
- Leptogenesis has strong impact on possible collider signatures of supersymmetry
- Remaining challenge: complete picture of leptogenesis, dark matter and inflation

“Reheating temperature”



Temperature of thermal part of energy density as function of scale factor.