Thermodynamics in modified gravity

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I. Introduction

- Recent observations of Supernova (SN) Ia confirmed that the current expansion of the universe is accelerating.

  [Perlmutter *et al*. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999)]
  [Riess *et al*. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998)]
  [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)]

- There are two approaches to explain the current cosmic acceleration. **[Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)]**

< Gravitational field equation >

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Gravity **Matter**

$G_{\mu\nu}$: Einstein tensor

$T_{\mu\nu}$: Energy-momentum tensor

G : Gravitational constant

(1) **General relativistic approach** ⟶ **Dark Energy**

(2) **Extension of gravitational theory**

(1) General relativistic approach

- **Cosmological constant**
- **Scalar fields: X matter, Quintessence, Phantom, K-essence, Tachyon.**
- **Perfect fluid: Chaplygin gas**

(2) Extension of gravitational theory

- ***f*(*R*) gravity** $f(R)$: Arbitrary function of the Ricci scalar R

  [Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]

  [Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

  [Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]
- **Scalar-tensor theories**
- **Ghost condensates** **[Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405, 074 (2004)]**
- **Higher-order curvature term**
- **DGP braneworld scenario** **[Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]**

→ It is important to explore the theoretical features of such modified gravity theories.

⇨ We investigate **Thermodynamics** .

- The discovery of black hole (BH) entropy and the first law of BH thermodynamics with a Hawking temperature implies the fundamental connection between gravitation and thermodynamics. **[Bekenstein, Phys. Rev. D 7, 2333 (1973)]**

  [Bardeen, Carter and Hawking, Commun. Math. Phys. 31, 161 (1973)]

  [Hawking, Commun. Math. Phys. 43, 199 (1975)]

- It has been shown that the Einstein equation can be derived from the Clausius relation, $T\mathrm{d}S = \mathrm{d}Q$, in thermodynamics. **[Jacobson, Phys. Rev. Lett. 75, 1260 (1995)]**

  T : Temperature, S : Entropy, $\mathrm{d}Q$: Energy flux across the horizon

→ This approach has been applied to cosmological settings.

[Frolov and Kofman, JCAP 0305, 009 (2003)]

[Danielsson, Phys. Rev. D 71, 023516 (2005)], [Bousso, Phys. Rev. D 71, 064024 (2005)]

- It has been pointed out that in the theories where the Lagrangian density f is a non-linear function in terms of the Ricci scalar R , a non-equilibrium treatment is required such that the Clausius relation is modified as follows:

[Eling, Guedens and Jacobson, Phys. Rev. Lett. 96, 121301 (2006)]

$$\mathrm{d}\hat{S} = \mathrm{d}Q/T + \underline{\mathrm{d}_i\hat{S}}$$

Entropy production term

$$\hat{S} = \frac{FA}{4G}$$: Horizon entropy

$$F(R) = \frac{\mathrm{d}f}{\mathrm{d}R} \,,$$ A : Area of the horizon

＊ A hat denotes quantities in the non-equilibrium description of thermodynamics

⇨ **The variation of the quantity *F*(*R*) gives rise to the non-equilibrium term $\mathrm{d}_i\hat{S}$, which is absent in the Einstein gravity.**

→ In *f*(*R*) gravity and scalar-tensor theories, it has been discussed that a non-equilibrium description of thermodynamics is necessary provided that $\hat{S} = FA/(4G)$.

[Akbar and Cai, Phys. Lett. B 635, 7 (2006); Phys. Lett. B 648, 243 (2007)]

[Wu, Wang and Yang, Nucl. Phys. B 799, 330 (2008)]

[Wu, Wang, Yang and Zhang, Class. Quant. Grav. 25, 235018 (2008)]

[Cai and Cao, Phys. Rev. D 75, 064008 (2007)]

→ It is of interest to see whether an equilibrium description of thermodynamics is possible in such modified gravity theories.

⇨ **We show that a picture of equilibrium thermodynamics does exist.**

II. Thermodynamics in modified gravity- non-equilibrium picture

$\mathcal{L}_{\mathrm{matter}}$: Matter Lagrangian

ϕ : Scalar field

∇_μ : Covariant derivative operator

f : Arbitrary function of R , ϕ and X

< Action >

$g = \det(g_{\mu\nu})$

$$I = \int d^4x \sqrt{-g} \left[\frac{f(R, \phi, X)}{16\pi G} + \mathcal{L}_{\mathrm{matter}} \right]$$

$X \equiv -(1/2)\, g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$: Kinetic term of ϕ

< Gravitational field equation >

$$F G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(\mathrm{matter})} + \frac{1}{2} g_{\mu\nu} (f - RF) + \nabla_\mu \nabla_\nu F - g_{\mu\nu} \Box F + \frac{1}{2} f_{,X} \nabla_\mu \phi \nabla_\nu \phi$$

$G_{\mu\nu} = R_{\mu\nu} - (1/2)\, g_{\mu\nu} R$

$R_{\mu\nu}$: Ricci tensor

< Equation of motion for ϕ >

$T_{\mu\nu}^{(\mathrm{matter})}$: Energy momentum tensor of ordinary matter (perfect fluids) with total energy density ρ_f and pressure P_f

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(f_{,X} \sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) + f_{,\phi} = 0$$

$$F \equiv \frac{\partial f}{\partial R}, \quad f_{,X} \equiv \frac{\partial f}{\partial X}, \quad f_{,\phi} \equiv \frac{\partial f}{\partial \phi}$$

$\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$: Covariant d'Alembertian

< Friedmann-Lemaître-Robertson-Walker (FLRW) space-time >

$$\mathrm{d}s^2 = h_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta + \tilde{r}^2 \mathrm{d}\Omega^2$$ $\tilde{r} = a(t)r\,,\ x^0 = t\,,\ x^1 = r$

$$h_{\alpha\beta} = \mathrm{diag}(-1, a^2(t)/[1-Kr^2])$$ $a(t)$: Scale factor

K : Cosmic curvature, $\mathrm{d}\Omega^2$: Metric of two-dimensional sphere with unit radius

→ Gravitational field equations in the FLRW background:

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3F}\left(\hat{\rho}_d + \rho_f\right)$$

$$\dot{H} - \frac{K}{a^2} = -\frac{4\pi G}{F}\left(\hat{\rho}_d + \hat{P}_d + \rho_f + P_f\right)$$

$\dot{} = \partial/\partial t$

$H = \dot{a}/a$

: Hubble parameter

→ Equation of motion for ϕ :

$$\frac{1}{a^3}\left(a^3\dot{\phi}f_{,X}\right)^{\cdot} = f_{,\phi}$$

- The Ricci scalar:

$$R = 6(2H^2 + \dot{H} + K/a^2)$$

< Continuity equation for perfect fluid >

$$\dot{\rho}_f + 3H(\rho_f + P_f) = 0$$

- The energy density $\hat{\rho}_d$ and pressure $\hat{P}_d$ of "dark" components:

$$\hat{\rho}_d \equiv \frac{1}{8\pi G}\left[f_{,X}X + \frac{1}{2}(FR - f) - 3H\dot{F}\right]$$

$$\hat{P}_d \equiv \frac{1}{8\pi G}\left[\ddot{F} + 2H\dot{F} - \frac{1}{2}(FR - f)\right]$$

$$\hat{T}^{(d)}_{\mu\nu} \equiv \frac{1}{8\pi G}\left[\frac{1}{2}g_{\mu\nu}(f - RF) + \nabla_\mu\nabla_\nu F - g_{\mu\nu}\Box F + \frac{1}{2}f_{,X}\nabla_\mu\phi\nabla_\nu\phi\right]$$

The Einstein equation: $G_{\mu\nu} = \frac{8\pi G}{F}\left(T^{(\mathrm{matter})}_{\mu\nu} + \hat{T}^{(d)}_{\mu\nu}\right)$

< Equation (corresponding to the continuity one) for $\hat{\rho}_d$ and $\hat{P}_d$ >

$$\dot{\hat{\rho}}_d + 3H(\hat{\rho}_d + \hat{P}_d) = \frac{3}{8\pi G}\left(H^2 + \frac{K}{a^2}\right)\dot{F}$$

$$F = \frac{\partial f}{\partial R}$$

For the theories with $\dot{F} \neq 0$ such as $f(R)$ gravity and scalar-tensor theories, the right-hand side does not vanish.

⇨ **The standard continuity equation does not hold.**

→ We proceed to the thermodynamic property of the theories explained above.

< Radius of the apparent horizon in the FLRW space-time >

$$\tilde{r}_A = \left(H^2 + \frac{K}{a^2} \right)^{-1/2} \longrightarrow \frac{F}{4\pi G} \mathrm{d}\tilde{r}_A = \tilde{r}_A^3 H \left(\hat{\rho}_d + \hat{P}_d + \rho_f + P_f \right) \mathrm{d}t$$

< Horizon entropy >

$$\hat{S} = \frac{FA}{4G}$$

Cf. For the Einstein gravity, $S = \dfrac{A}{4G}$

$A = 4\pi \tilde{r}_A^2$: Area of the apparent horizon

[Bekenstein, Phys. Rev. D 7, 2333 (1973)]

[Bardeen, Carter and Hawking, Commun. Math. Phys. 31, 161 (1973)]

[Hawking, Commun. Math. Phys. 43, 199 (1975)]

[Wald, Phys. Rev. D 48, 3427 (1993)], [Iyer and Wald, Phys. Rev. D 50, 846 (1994)]

[Brevik, Nojiri, Odintsov and Vanzo, Phys. Rev. D 70, 043520 (2004)]

[Eling, Guedens and Jacobson, Phys. Rev. Lett. 96, 121301 (2006)]

$$\frac{1}{2\pi \tilde{r}_A} \mathrm{d}\hat{S} = 4\pi \tilde{r}_A^3 H \left(\hat{\rho}_d + \hat{P}_d + \rho_f + P_f \right) \mathrm{d}t + \frac{\tilde{r}_A}{2G} \mathrm{d}F$$

→ The apparent horizon has the following Hawking temperature:

$$T = \frac{|\kappa|}{2\pi}$$

$$\kappa = -\frac{1}{\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right) = -\frac{\tilde{r}_A}{2}\left(2H^2 + \dot{H} + \frac{K}{a^2}\right)$$

$$= -\frac{2\pi G}{3F}\tilde{r}_A\left(\hat{\rho}_T - 3\hat{P}_T\right)$$: Surface gravity

$$\hat{\rho}_T \equiv \hat{\rho}_d + \rho_f$$: Total energy density

$$\hat{P}_T \equiv \hat{P}_d + P_f$$: Total pressure

< Misner-Sharp energy >

$$\hat{E} = \frac{\tilde{r}_A F}{2G}$$

Cf. For the Einstein gravity, $E = \frac{\tilde{r}_A}{2G}$

[Misner and Sharp, Phys. Rev. 136, B571 (1964)]

$$= V\frac{3F(H^2 + K/a^2)}{8\pi G} = V(\hat{\rho}_d + \rho_f)$$

$$V = 4\pi\tilde{r}_A^3/3$$

: Volume inside the apparent horizon

[Gong and Wang, Phys. Rev. Lett. 99, 211301 (2007)]

$$\longrightarrow \mathrm{d}\hat{E} = -4\pi\tilde{r}_A^3 H(\hat{\rho}_d + \hat{P}_d + \rho_f + P_f)\mathrm{d}t + 4\pi\tilde{r}_A^2(\hat{\rho}_d + \rho_f)\mathrm{d}\tilde{r}_A + \underline{\frac{\tilde{r}_A}{2G}\mathrm{d}F}$$

$$\Rightarrow T\mathrm{d}\hat{S} = -\mathrm{d}\hat{E} + 2\pi\tilde{r}_A^2(\hat{\rho}_d + \rho_f - \hat{P}_d - P_f)\mathrm{d}\tilde{r}_A + \underline{\frac{\tilde{r}_A}{2G}(1 + 2\pi\tilde{r}_A T)\,\mathrm{d}F}$$

< Work density >

[Hayward, Class. Quant. Grav. 15, 3147 (1998)]

[Hayward, Mukohyama and Ashworth, Phys. Lett. A 256, 347(1999)]

$$\boxed{\hat{W} = \frac{1}{2}(\hat{\rho}_d + \rho_f - \hat{P}_d - P_f)}$$

Cf. For $K = 0$, [Wu, Wang, Yang and Zhang, Class. Quant. Grav. 25, 235018 (2008)]

$$\Rightarrow T\mathrm{d}\hat{S} + \underline{T\mathrm{d}_i\hat{S}} = -\mathrm{d}\hat{E} + \hat{W}\mathrm{d}V$$

$$\mathrm{d}_i\hat{S} = -\frac{1}{T}\frac{\tilde{r}_A}{2G}(1 + 2\pi\tilde{r}_A T)\underline{\mathrm{d}F} = -\left(\frac{\hat{E}}{T} + \hat{S}\right)\underline{\frac{\mathrm{d}F}{F}}$$

$$= -\frac{\pi}{G}\frac{4H^2 + \dot{H} + 3K/a^2}{(H^2 + K/a^2)(2H^2 + \dot{H} + K/a^2)}\underline{\mathrm{d}F}$$

→ $\mathrm{d}_i\hat{S}$ can be interpreted as a term of entropy production in the non-equilibrium thermodynamics.

$$\boxed{\mathrm{d}_i\hat{S} \propto \mathrm{d}F}$$

- The theories with $F = \mathrm{constant}$ lead to $\mathrm{d}_i\hat{S} = 0$.

⟶ The first-law of equilibrium thermodynamics holds.

- The theories with $\mathrm{d}F \neq 0$, including *f*(*R*) gravity and scalar-tensor theories, give rise to the additional term $\mathrm{d}_i\hat{S}$.

→ The existence of $\mathrm{d}_i\hat{S}$ $(\neq 0)$ is related to the fact that for $\dot{F} \neq 0$ the standard continuity equation does not hold as follows:

$$\dot{\hat{\rho}}_d + 3H(\hat{\rho}_d + \hat{P}_d) = \underline{\frac{3}{8\pi G}\left(H^2 + \frac{K}{a^2}\right)\dot{F}}$$

⇨ **If it is possible to define energy density and pressure of dark components satisfying the standard continuity equation, the non-equilibrium description of thermodynamics may not be necessary.**

III. Equilibrium description of thermodynamics in modified gravity

< Rewriting the gravitational field equations >

$$3F_0\left(H^2+\frac{K}{a^2}\right) = 8\pi G\left(\rho_d+\rho_f\right)$$

$$-2F_0\left(\dot{H}-\frac{K}{a^2}\right) = 8\pi G\left(\rho_d+P_d+\rho_f+P_f\right)$$

< Redefinition of the energy density and pressure of dark components >

$$\rho_d \equiv \frac{1}{8\pi G}\left[f_{,X}X+\frac{1}{2}(FR-f)-3H\dot{F}+3(F_0-F)\left(H^2+\frac{K}{a^2}\right)\right]$$

$$P_d \equiv \frac{1}{8\pi G}\left[\ddot{F}+2H\dot{F}-\frac{1}{2}(FR-f)-(F_0-F)\left(2\dot{H}+3H^2+\frac{K}{a^2}\right)\right]$$

⇨ $\dot{\rho}_d+3H\left(\rho_d+P_d\right)=0$: Standard continuity equation

Differences between $\hat{\rho}_d$, $\hat{P}_d$ and ρ_d, P_d

→ The constant F_0 can be chosen arbitrarily as long as $F_0 > 0$.

- The natural choice is $F_0 = 1$, in which the entropy and the Misner-Sharp energy reduce to the standard forms in the Einstein gravity as $S = \frac{A}{4G}$ and $E = \frac{\tilde{r}_A}{2G}$, respectively.

∗ In what follows, we take $F_0 = 1$.

$$T\mathrm{d}S = 4\pi\tilde{r}_A^3 H(\rho_d + P_d + \rho_f + P_f)\mathrm{d}t - 2\pi\tilde{r}_A^2(\rho_d + P_d + \rho_f + P_f)\mathrm{d}\tilde{r}_A$$

$$E = \frac{\tilde{r}_A}{2G} = V(\rho_d + \rho_f)$$

$$\mathrm{d}E = -4\pi\tilde{r}_A^3 H(\rho_d + P_d + \rho_f + P_f)\mathrm{d}t + 4\pi\tilde{r}_A^2(\rho_d + \rho_f)\mathrm{d}\tilde{r}_A$$

⇒ $$T\mathrm{d}S = -\mathrm{d}E + W\mathrm{d}V$$

: **First law of equilibrium thermodynamics**

$$W = \frac{1}{2}(\rho_d + \rho_f - P_d - P_f)$$

$$\dot{S} = 6\pi H V \tilde{r}_A \underline{(\rho_d + \rho_f + P_d + P_f)} = -\frac{2\pi}{G}\frac{H(\dot{H} - K/a^2)}{(H^2 + K/a^2)^2}$$

As long as the null energy condition is satisfied:

$$\rho_T + P_T \equiv \rho_d + \rho_f + P_d + P_f \geq 0\,,$$

$\dot{S} \geq 0$ $\longrightarrow$ **The horizon entropy increases.**

< Reasons of realization of the equilibrium picture >

(i) **There is $T^{(d)}_{\mu\nu}$ satisfying the local conservation law:** $\underline{\nabla^{\mu} T^{(d)}_{\mu\nu} = 0}$

(ii) $S = CA,$ C **: Constant**

→ The Einstein equation

$$G_{\mu\nu} = 8\pi G \left(T^{(\mathrm{matter})}_{\mu\nu} + T^{(d)}_{\mu\nu} \right)$$

$$T^{(d)}_{\mu\nu} \equiv \frac{1}{8\pi G}\left[\frac{1}{2} g_{\mu\nu}(f - R) + \nabla_{\mu}\nabla_{\nu}F - g_{\mu\nu}\Box F + \frac{1}{2} f_{,X}\nabla_{\mu}\phi\nabla_{\nu}\phi + (1 - F)R_{\mu\nu} \right]$$

$$\nabla^{\mu} T^{(\mathrm{matter})}_{\mu\nu} = 0$$

$$\nabla^{\mu} G_{\mu\nu} \doteq 0$$

< Relation between S and $\hat{S}$ >

$$\mathrm{d}S = \frac{1}{F}\mathrm{d}\hat{S} + \frac{1}{F}\frac{2H^2 + \dot{H} + K/a^2}{4H^2 + \dot{H} + 3K/a^2}\mathrm{d}_i\hat{S}$$

$$\mathrm{d}_i\hat{S} = -\frac{6\pi}{G}\frac{4H^2 + \dot{H} + 3K/a^2}{H^2 + K/a^2}\frac{\mathrm{d}F}{R}$$

The difference between S and $\hat{S}$ appears in modified gravity theories with $\mathrm{d}F \neq 0$, while $S = \hat{S}$ in the Einstein gravity.

⇨ **The change of the horizon entropy S in the equilibrium framework involves the information of both $\mathrm{d}\hat{S}$ and $\mathrm{d}_i\hat{S}$ in the non-equilibrium framework.**

IV. Summary

- We have shown that **it is possible to obtain a picture of equilibrium thermodynamics** on the apparent horizon in the expanding cosmological background for a wide class of modified gravity theories with the Lagrangian density $f(R, \phi, X)$, such as $f(R)$ gravity and scalar-tensor theories.
  - **This comes from a suitable definition of an energy momentum tensor of the "dark" component that respects to a local energy conservation.**
  - **The horizon entropy S in equilibrium thermodynamics is proportional to the horizon area A with a constant coefficient.**
- **The equilibrium description in terms of S is convenient because it takes into account the contribution of both the horizon entropy $\hat{S}$ in non-equilibrium thermodynamics and an entropy production term $\mathrm{d}_i\hat{S}$.**

< Further results >

- We have applied our formalism to

(a) *f*(*R*) inflation models with the Lagrangian density:

$$f(R) = R + \alpha R^n$$

$$\alpha > 0, \quad (1+\sqrt{3})/2 < n \le 2$$

Cf. For $n = 2$,

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

(b) viable *f*(*R*) dark energy models:

λ, R_c, n

: Positive constants

$$f(R) = R - \lambda R_c \left[1 - \left(1 + \frac{R^2}{R_c^2} \right)^{-n} \right]$$

[Starobinsky, JETP Lett. 86, 157 (2007)]

For $\lambda = \mathcal{O}(1)$, R_c is of the order of the current cosmological Ricci scalar R_0.

$$f(R) = R - \lambda R_c \tanh\left(\frac{R}{R_c} \right)$$

[Tsujikawa, Phys. Rev. D 77, 023507 (2008)]

⇨ **We have found that for a flat cosmological background with a decreasing Hubble parameter, S globally increases with time.**

Backup Slides

Abstract

- We show that it is possible to obtain a picture of equilibrium thermodynamics on the apparent horizon in the expanding cosmological background for a wide class of modified gravity theories with the Lagrangian density $f(R, \phi, X)$, where R is the Ricci scalar and X is the kinetic energy of a scalar field ϕ.

⟶ This comes from a suitable definition of an energy momentum tensor of the "dark" component that respects to a local energy conservation in the Jordan frame.

⟶ In this framework, the horizon entropy S corresponding to equilibrium thermodynamics is equal to a quarter of the horizon area A in units of gravitational constant G, as in Einstein gravity. For a flat cosmological background with a decreasing Hubble parameter, S globally increases with time, as it happens for viable $f(R)$ inflation and dark energy models.

- We also demonstrate that the equilibrium description in terms of the horizon entropy S is convenient because it takes into account the contribution of both the horizon entropy $\hat{S}$ in non-equilibrium thermodynamics and an entropy production term.

→ In $f(R)$ gravity and scalar-tensor theories, it has been discussed that a non-equilibrium description of thermodynamics is necessary provided that $\hat{S} = FA/(4G)$.

[Akbar and Cai, Phys. Lett. B 635, 7 (2006); Phys. Lett. B 648, 243 (2007)]

[Wu, Wang and Yang, Nucl. Phys. B 799, 330 (2008)]

[Wu, Wang, Yang and Zhang, Class. Quant. Grav. 25, 235018 (2008)]

[Cai and Cao, Phys. Rev. D 75, 064008 (2007)]

- The appearance of a non-equilibrium entropy production term $\mathrm{d}_i\hat{S}$ is intimately related to the theories in which the derivative of the Lagrangian density f with respect to R is not constant.

→ It is of interest to see whether an equilibrium description of thermodynamics is possible in such modified gravity theories.

⇨ **We show that a picture of equilibrium thermodynamics does exist.**

< Connections between thermodynamics and modified gravity > (No. 7)

Cf. [Brustein and Hadad, Phys. Rev. Lett. 103, 101301 (2009)]

[KB, Geng, Nojiri and Odintsov, Europhys. Lett. 89, 50003 (2010)]

- **$f(R)$ gravity** [Akbar and Cai, Phys. Lett. B 635, 7 (2006); Phys. Lett. B 648, 243 (2007)]

  [Gong and Wang, Phys. Rev. Lett. 99, 211301 (2007)]

  [Wu, Wang and Yang, Nucl. Phys. B 799, 330 (2008)]

  [Wu, Wang, Yang and Zhang, Class. Quant. Grav. 25, 235018 (2008)]

  [Elizalde and Silva, Phys. Rev. D 78, 061501 (2008)]

  [KB and Geng, Phys. Lett. B 679, 282 (2009)]

- **Scalar-tensor theories**

  [Cai and Cao, Phys. Rev. D 75, 064008 (2007)]

- **Gauss-Bonnet and Lovelock gravity**

  [Paranjape, Sarkar and Padmanabhan, Phys. Rev. D 74, 104015 (2006)]

  [Cai, Cao, Hu and Kim, Phys. Rev. D 78, 124012 (2008)]

- **Braneworld models**

  [Sheykhi, Wang and Cai, Nucl. Phys. B 779, 1 (2007); Phys. Rev. D 76, 023515 (2007)]

  [Ge, Phys. Lett. B 651, 49 (2007)]

  [Wu, Yang and Zhang, arXiv:0710.5394 [hep-th]]

Appendix

< Equilibrium description of thermodynamics in the Einstein frame >

$$I = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{F(\phi)}{2\kappa^2}R + \omega(\phi)X - V(\phi)\right] - \int \mathrm{d}^4x\,\mathcal{L}_M(g_{\mu\nu},\Psi_M)$$

$$\tilde{g}_{\mu\nu} = F g_{\mu\nu}$$ **: Conformal transformation**

$$\varphi \equiv \int \mathrm{d}\phi\sqrt{\frac{3}{2}\left(\frac{F_{,\phi}}{\kappa F}\right)^2 + \frac{\omega}{F}}\,, \qquad U = \frac{V}{F^2}$$

$$I_E = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\varphi)^2 - U(\varphi)\right] - \int \mathrm{d}^4x\,\mathcal{L}_M(F(\phi)^{-1}\tilde{g}_{\mu\nu},\Psi_M)$$

∗ A tilde represents quantities in the Einstein frame.

$$3(\tilde{H}^2 + K/\tilde{a}^2) = \kappa^2\,(\tilde{\rho}_\varphi + \tilde{\rho}_f)$$

$$2(\dot{\tilde{H}} - K/\tilde{a}^2) = -\kappa^2(\tilde{\rho}_\varphi + \tilde{P}_\varphi + \tilde{\rho}_f + \tilde{P}_f)$$

$$\tilde{T} = \frac{|\tilde{\kappa}_s|}{2\pi}, \quad \tilde{S} = \frac{\tilde{A}}{4G}, \quad \tilde{E} = \frac{\tilde{\bar{r}}_A}{2G}$$

$$\tilde{\bar{r}}_A = (\tilde{H}^2 + K/\tilde{a}^2)^{-1/2}$$

$$\tilde{A} = 4\pi\tilde{\bar{r}}_A^2$$

$$\tilde{\kappa}_s = -[1 - (\mathrm{d}\tilde{\bar{r}}_A/\mathrm{d}\tilde{t})/(2\tilde{H}\tilde{\bar{r}}_A)]/\tilde{\bar{r}}_A$$

$$\mathrm{d}\tilde{t} = \sqrt{F}\,\mathrm{d}t\,, \qquad \tilde{a} = \sqrt{F}a$$

$$H = \sqrt{F}\left(\tilde{H} - \frac{1}{2F}\frac{\mathrm{d}F}{\mathrm{d}\tilde{t}}\right)$$

$\Rightarrow$ $$\tilde{T}\mathrm{d}\tilde{S} = -\mathrm{d}\tilde{E} + \tilde{W}\mathrm{d}\tilde{V}$$

$$\tilde{S} = \frac{\pi}{G}\frac{1}{\tilde{H}^2}\,, \qquad \tilde{E} = \frac{1}{2G}\frac{1}{\tilde{H}}$$

Cf. Expressions in the Einstein frame

$$S = \pi/(GH^2), \quad E = 1/(2GH)$$

→ **In general, the horizon entropy in the Einstein frame $\tilde{S}$ is different from that in the equilibrium picture in the Jordan frame S .**

No. A4

$$-\partial \mathcal{L}_M/\partial\varphi = \sqrt{-\tilde{g}}\kappa Q(\varphi)\tilde{T}_M$$

$$Q(\varphi) \equiv -F_{,\varphi}/(2\kappa F), \qquad \tilde{T}_M \equiv -\tilde{\rho}_f + 3\tilde{P}_f$$

$$H = \sqrt{F}\left(\tilde{H} - \frac{1}{2F}\frac{\mathrm{d}F}{\mathrm{d}\tilde{t}}\right) = \sqrt{F}\left(\tilde{H} + \kappa Q\frac{\mathrm{d}\varphi}{\mathrm{d}\tilde{t}}\right)$$

$$\mathrm{d}E = (\mu/\sqrt{F})\mathrm{d}\tilde{E}$$

$$\mu \equiv \frac{1 + \kappa Q[\ddot{\varphi} + (Q_{,\varphi}/Q - \kappa Q)\dot{\varphi}^2 - \tilde{H}\dot{\varphi}]/\dot{\tilde{H}}}{(1 + \kappa Q\dot{\varphi}/\tilde{H})^2}$$

$$W\mathrm{d}V = \frac{\mu(3 + \mu\dot{\tilde{H}}/\tilde{H}^2)}{\sqrt{F}(3 + \dot{\tilde{H}}/\tilde{H}^2)}\tilde{W}\mathrm{d}\tilde{V}$$

$$T\mathrm{d}S = \frac{\mu(2 + \mu\dot{\tilde{H}}/\tilde{H}^2)}{\sqrt{F}(2 + \dot{\tilde{H}}/\tilde{H}^2)}\tilde{T}\mathrm{d}\tilde{S}$$

- **In scalar-tensor theories in which F and μ dynamically change in time, the equilibrium description of thermodynamics in the Jordan frame is not identical to that in the Einstein frame.**

- **We regard that the frame in which the baryons obey the standard continuity equation $\rho_{\mathrm{matter}} \propto a^{-3}$, i.e., the Jordan frame, is the "physical" frame where physical quantities are compared with observations and experiments.**

⇨ **The direct construction of the equilibrium thermodynamics in the Jordan frame is not only versatile but is physically well motivated.**