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Non-equilibrium Condensation Process in a Holographic Superconductor

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Introduction

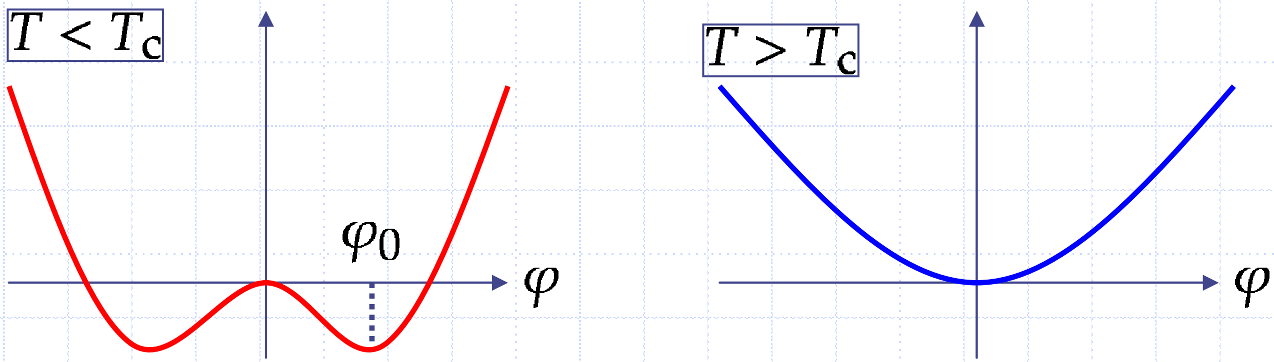
- AdS/CFT correspondence plays a central role in the study of the strongly coupled region of quantum field theory
  - In a limit, it is simply described by classical gravity theory on AdS
  - We should solve classical dynamics of gravitational systems
- Applications of AdS/CFT
  - We would like to get insights for strongly coupled systems from holographic approach
    - Quark-gluon plasma, superconductor, …

A phenomenological description of superconductor

- Landau-Ginzburg theory

$n_s = |\varphi|^2$: density of superconducting electron (order parameter)

Free energy : $$F = \alpha(T - T_c)|\varphi|^2 + \frac{\beta}{2}|\varphi|^4 + \cdots$$



The London equation is derived from this spontaneous symmetry breaking

Meissner effect

Holographic Superconductor

Hartnoll, Herzog and Horowitz (2008)

- This model was proposed as gravitational dual to superconductor
  - 4D Einstein-Maxwell-charged scalar
    - action:

$$S = \int d^4x \sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\nabla_\mu \Psi - iqA_\mu \Psi|^2 - m^2|\Psi|^2\right)$$

$(m^2 = -2/L^2)$

Temperature is introduced by adding a BH and a condensation is by a scalar field.

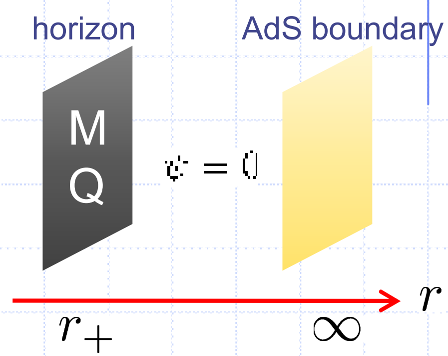
For low temperatures, there are two BH solutions with $\Psi = 0$ and $\Psi \neq 0$

This model causes the "superconducting" phase transition.

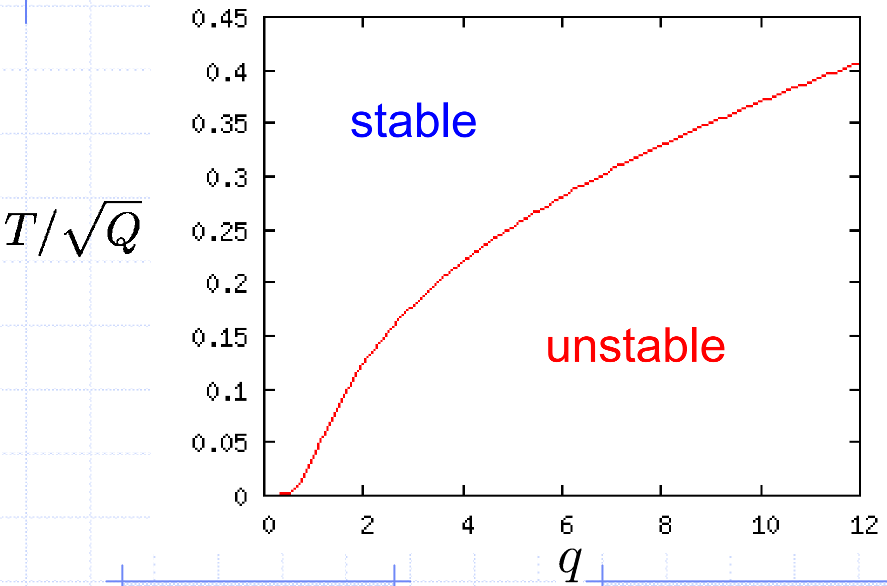
Reissner- Nordstrom-AdS with a **flat horizon**

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\vec{x}^2$$

$$\left[\quad f(r) = r^2 - \frac{2M}{r} + \frac{Q^2}{4r^2} \qquad A = Q\left(\frac{1}{r_+} - \frac{1}{r}\right)dt \qquad \psi = 0 \quad\right]$$



RN-AdS is unstable for low temperature.



The growing mode is in scalar field perturbation.

spontaneous U(1) symmetry breaking

This instability is identified with superconducting phase transition.

Two phases of black hole

- AdS Reissner-Nordström
  - Unbroken phase

Unstable at low temperature

$$g = r^2 - \frac{2M}{r} + \frac{Q^2}{4r^2}, \quad \alpha = \frac{Q}{r}, \quad \psi = \chi = 0$$

$$\begin{cases} ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2), \\ A = \alpha(r)dt, \quad \Psi = \psi(r), \end{cases}$$

Q : total charge, M : total mass

Zero temperature (extreme BH) : $Q = Q_e \equiv 2\sqrt{3} \qquad (L = r_+ = 1)$

- Hairy black hole
  - Broken phase $\psi \neq 0$ Numerical solution

For low temperature the scalar field can be a non-vanishing profile (condensation)

Interpretation of the (1+2)-dim. boundary

- We expand bulk fields at AdS boundary

$$\psi = \frac{\psi_1}{r} + \frac{\psi_2}{r^2} + \cdots \qquad \alpha = \mu + \frac{\rho}{r} + \cdots$$

Cf. $\eta_{\mu\nu} \Leftrightarrow \langle T_{\mu\nu} \rangle$, $\mu \Leftrightarrow \langle J \rangle = \rho$

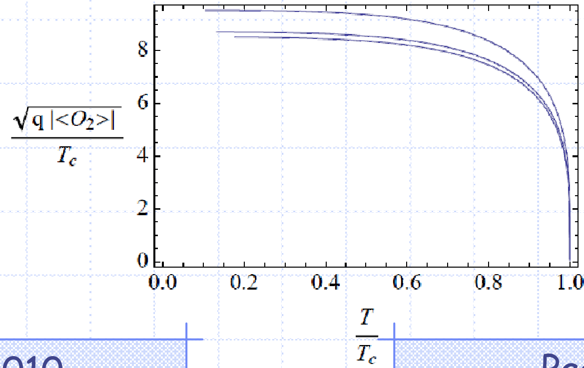
source ψ_i $\Leftrightarrow$ Operator in the CFT $\langle O_j \rangle = \psi_j \quad i, j = 1, 2 (i \neq j)$

: order parameter

•High T : $T > T_c \cdots \langle O \rangle = 0$

•Low T : $T < T_c \cdots \langle O \rangle \neq 0$

$\langle O \rangle \propto (T_c - T)^{1/2}$



Our study

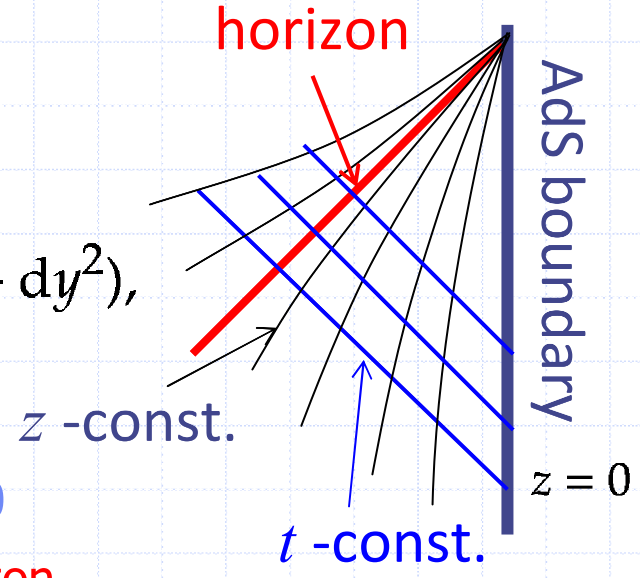
- We would like to investigate non-equilibrium condensation process
  - Is it true that the final state of the instability is the hairy BH?
  - How is the condensation process beyond the linear response theory?

- We will solve non-linear time evolutions of the Einstein-Maxwell-charged scalar system
  - Assuming plane symmetry
    - (1+1)-dimensional partial differential equations
  - The metric, Maxwell and complex scalar fields are coupled
    - We are taking account of backreaction for the spacetime
  - We consider perturbations of AdS-RN and study time evolutions
    - At low temperature an instability exists

Set up

- We assume that the spacetime has the plane symmetry
- The ingoing Eddington-Finkelstein coordinates
  - ansatz: $(z = 1/r)$

$$\begin{cases} ds^2 = -z^{-2}(F(t,z)dt^2 + dtdz) + \Phi^2(t,z)(dx^2 + dy^2), \\ A = \alpha(t,z)dt, \quad \psi(t,z), \quad \psi^*(t,z) \end{cases}$$



Constant-t surfaces are null
AdS boundary is located at z=0

We can extend time slices inside the horizon

Equations of motion

$$\Phi \equiv e^{-\phi}, \quad f \equiv z^{-2}F, \quad r = 1/z$$

- Evolution equations:

$$\begin{cases} -4(\dot{\phi})' + 8\phi'\dot{\phi} + \frac{1}{2}\alpha'^2 + m^2|\psi|^2 - \frac{6}{L^2} = 0, \\ -2(\dot{\alpha})' + f'\alpha' + 4\dot{\phi}\alpha' - 2iq(\psi\dot{\psi}^* - \psi^*\dot{\psi}) = 0, \\ 2(\dot{\psi})' + iq\alpha'\psi - 2\dot{\phi}\psi' - 2\phi'\dot{\psi} - m^2\psi = 0, \\ 2(\dot{\psi}^*)' - iq\alpha'\psi^* - 2\dot{\phi}\psi^{*\prime} - 2\phi'\dot{\psi}^* - m^2\psi^* = 0, \\ 2\alpha'^2 + 8\dot{\phi}\phi' - 2f'' - 2(\dot{\psi}\psi^{*\prime} + \dot{\psi}^*\psi') = 0, \end{cases}$$

$$\begin{cases} ' = \partial_r, \\ \dot{} = \partial_t + f/2\partial_r \end{cases}$$

etc.

- Constraint equations:

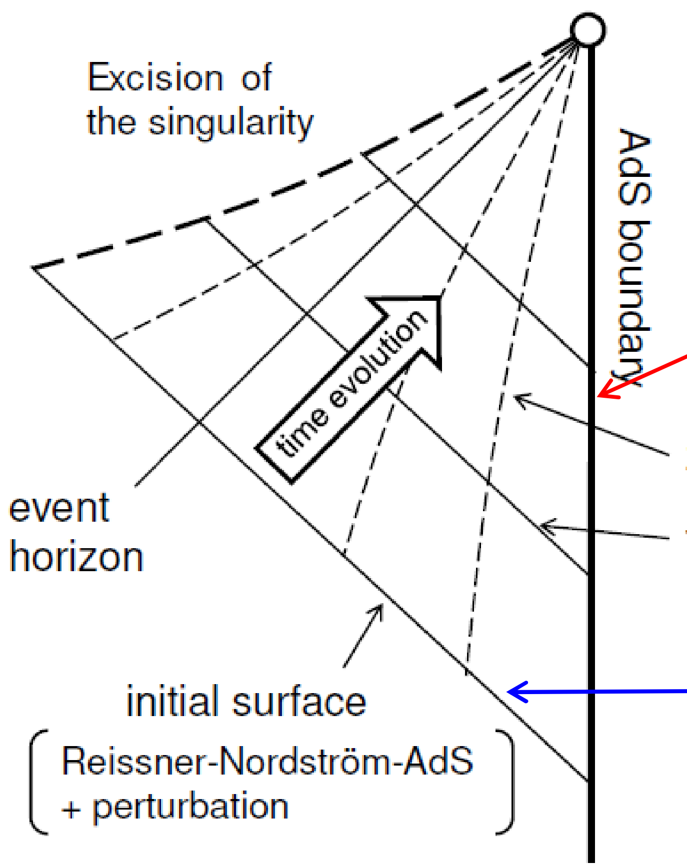
$$2\ddot{\phi} - 2\dot{\phi}^2 - f'\dot{\phi} - |\dot{\psi}|^2 = 0,$$

$$2\phi'' - 2\phi'^2 - |\psi'|^2 = 0,$$

$$-\alpha'' + 2\phi'\alpha' + iq(\psi\psi^{*\prime} - \psi^*\psi') = 0,$$

Initial conditions and boundary conditions must satisfy these constraint equations.

Schematic picture of our calculation



The asymptotic form of the scalar field

$$\psi(t,z) = z\psi_1(t) + z^2\psi_2(t) + \cdots$$

At the AdS boundary (z=0),
we give the boundary condition

$$\psi_1(t) = 0$$

At the initial surface,
we give the initial condition,

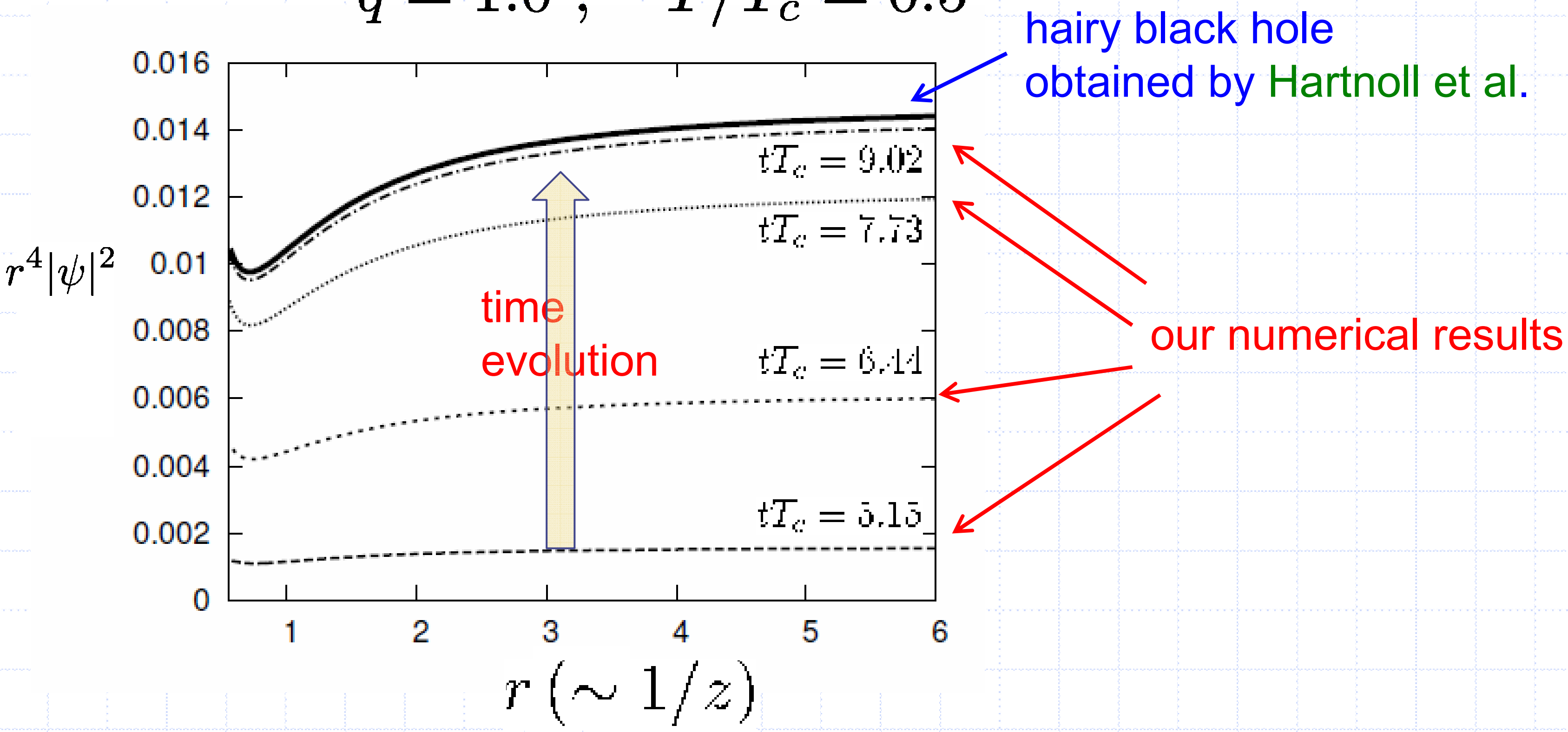
$$\psi/z^2 = \frac{A}{\sqrt{2\pi}\,\delta}\exp\left[-\frac{(z-z_m)^2}{2\delta^2}\right]$$

with $A = 0.01$, $\delta = 0.05$ and $z_m = 0.3$.

Other functions are determined by solving constraint equations.

Snapshots of time evolution

$$q = 1.0\ , \quad T/T_c = 0.5$$



The dynamical solution approaches the hairy black hole.

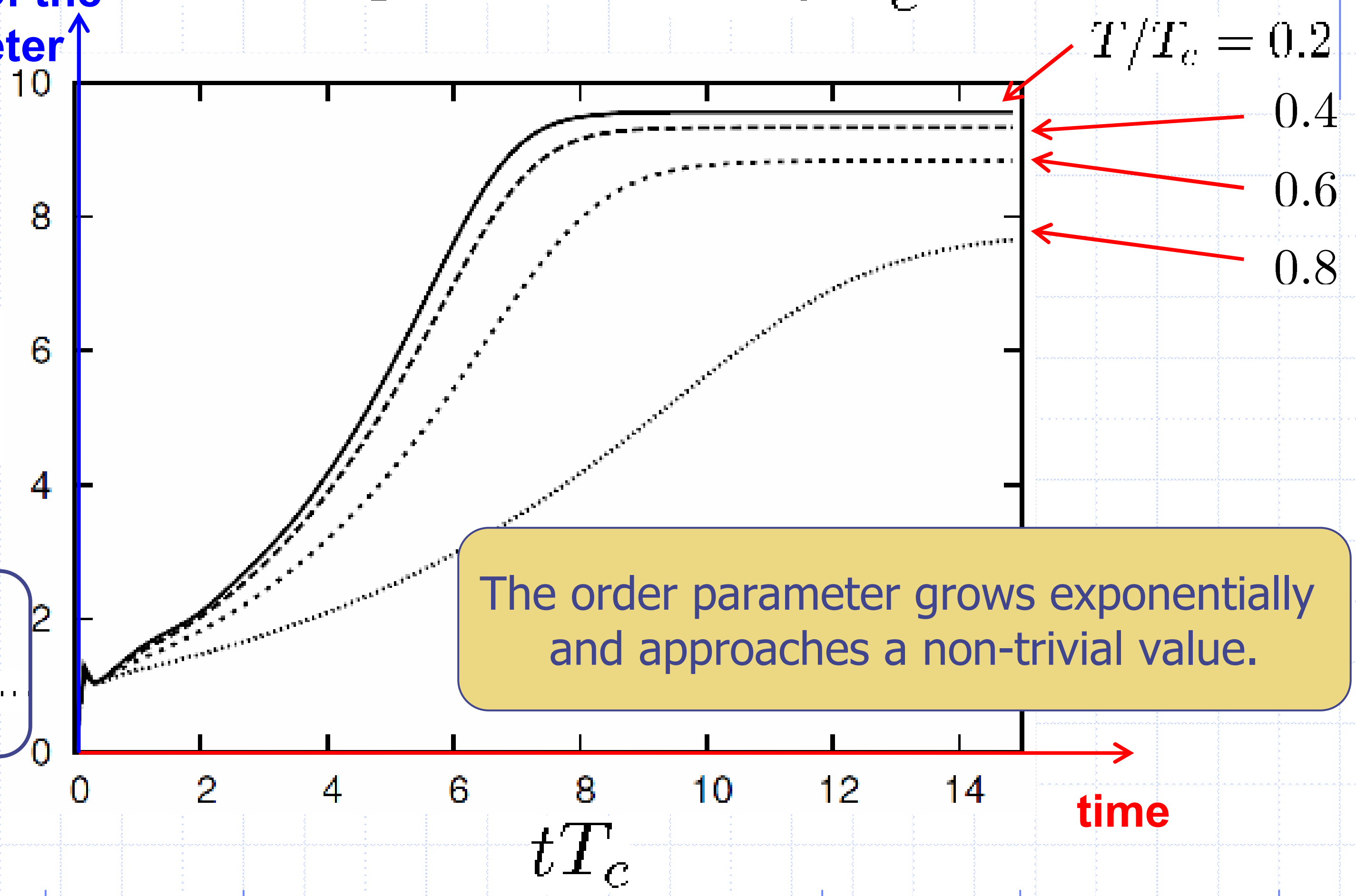
⇨ For the most symmetric perturbation, the hairy BH is stable.

The dynamics of the superconducting order parameter

$$q = 1.5 \quad T < T_c$$

square root of the order parameter

$$\frac{|q\langle \mathcal{O}_2 \rangle|^{1/2}}{T_c}$$

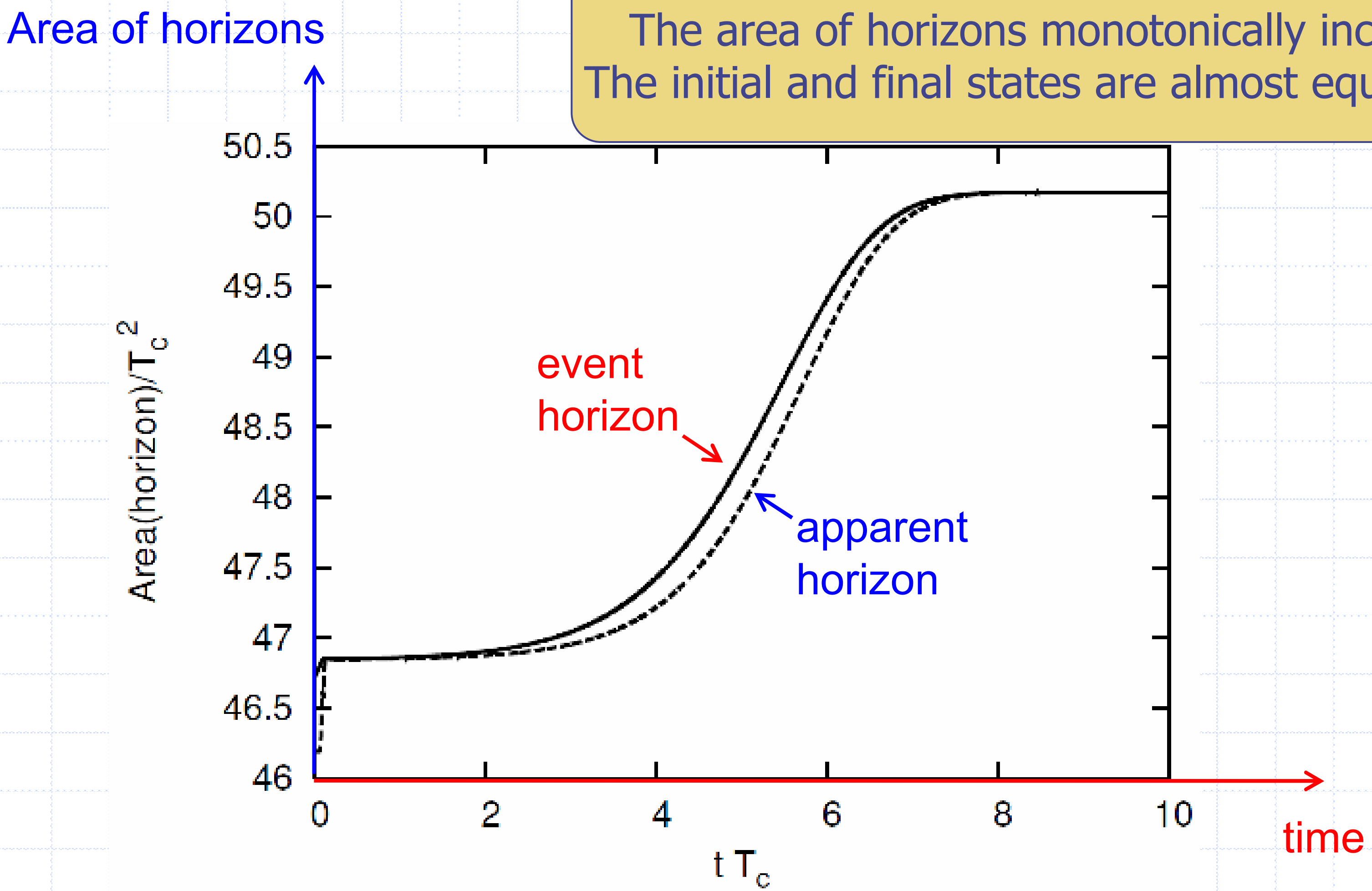


$$\langle \mathcal{O}_2 \rangle \equiv \psi_2$$

$$\phi(t,z) = z\phi_1(t) + z^2\psi_2(t) + \cdots$$

Evolution of horizons

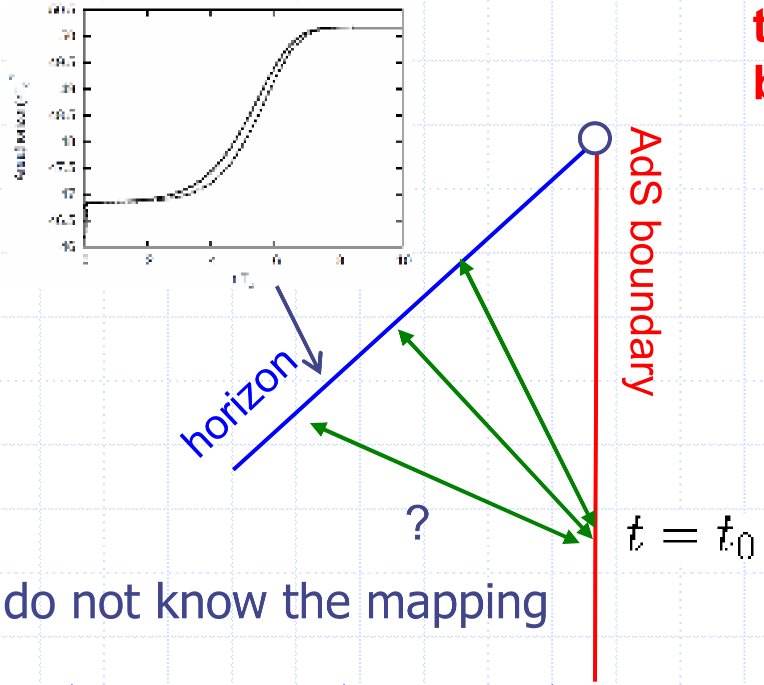
The area of horizons monotonically increase.
The initial and final states are almost equilibrium.



Does the area of horizon correspond to the entropy?

For stationary spacetimes,
the entropy and temperature of CFT on the boundary are given by
the area and Hawking temerature of black hole horizon

For dynamical spacetimes,
there is ambiguity of the mapping
between the AdS boundary and horizons.

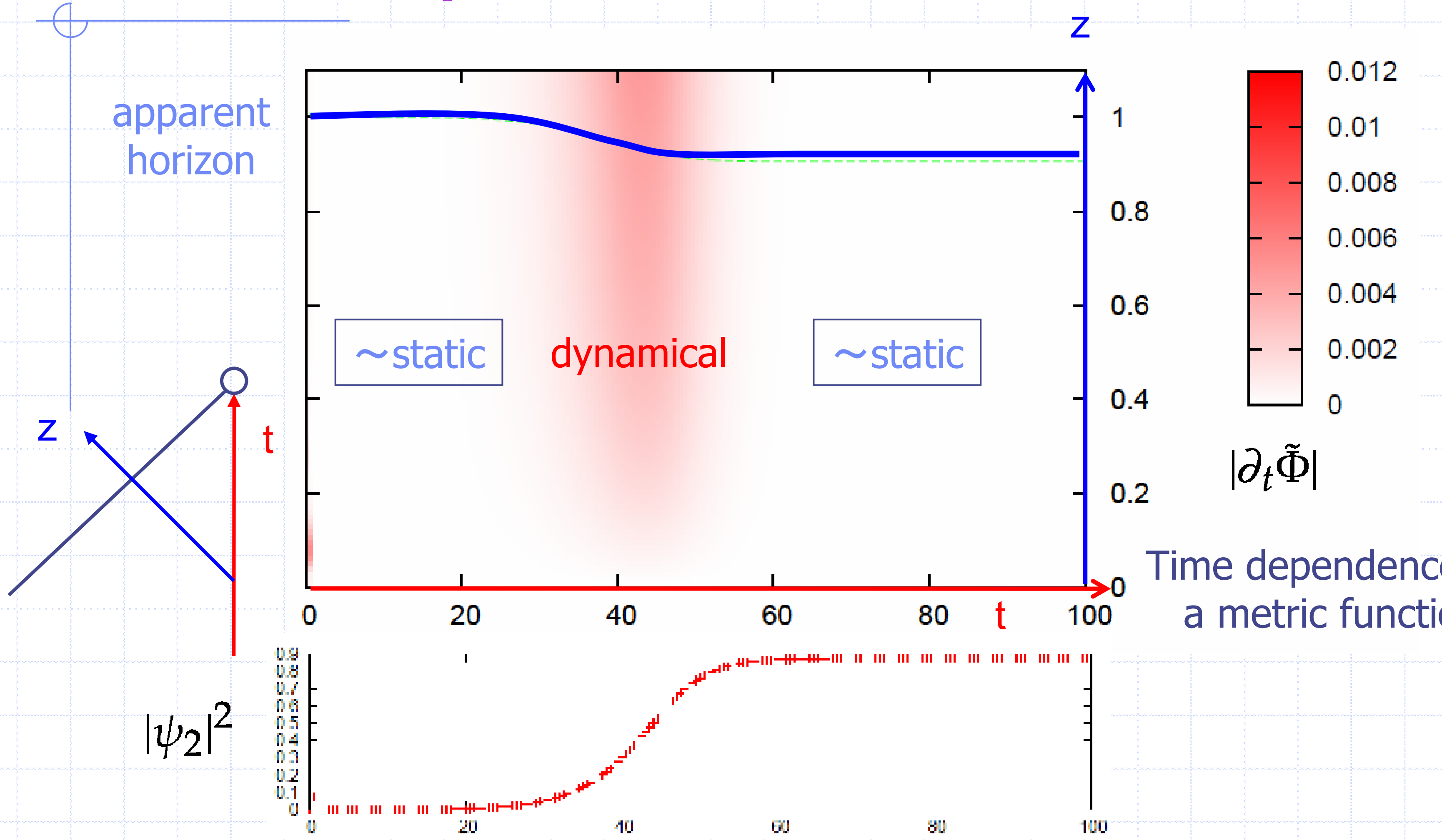


We do not know the mapping

We are using
ingoing Eddington-Finkelstein coordinates

Our choice is the null mapping.
But there are many other choices

Time dependence of the bulk



Time dependence of a metric function

Summary

- We have studied a non-equilibrium condensation process in a holographic superconductor model
  - We have solved non-linear time evolutions in the Einstein-Maxwell-complex scalar system
- The final state of the instability of RN is the hairy BH obtained by Hartnoll et al.
  - The hairy BH is stable against plane symmetric perturbations
- We have studied the time evolution of horizons in the bulk
  - It seems to be reasonable that the mapping between the horizon and the boundary is null