Hybrid inflation with a non-minimally coupled scalar field

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PART I: Non-minimally coupled χ field

Hybrid inflation with a nonminimally coupled χ field

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}(1-\kappa^2\xi\chi^2)R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - V(\phi,\chi)\right]$$

- Potential of Hybrid inflation

$$V(\phi,\chi) = \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

Effective Potential

$$V_{eff}(\phi,\chi) = V(\phi,\chi) + \frac{1}{2}\xi\chi^2 R = V(\phi,\chi) + 6\xi\chi^2 H^2$$

- effective mass of χ field during inflation

$$m^2_{eff}(\phi) = \left.\frac{d^2V_{eff}}{d\chi^2}\right|_{\chi=0} = -\lambda v^2(1-\kappa^2\xi v^2) + (g^2 + 2\xi\kappa^2 m^2)\phi^2$$

Non-minimally coupled χ field

critical value when the tachyonic stage begins

$$m_{eff}^2 = 0 \Longrightarrow \phi_c^2 = \frac{\lambda v^2 (1 - \kappa^2 \xi v^2)}{g^2 + 2\kappa^2 \xi m^2}$$

Comments

- For $\xi < -\frac{g^2}{2\kappa^2 m^2}$, the χ field is always tachyonic, irrespective of the value of ϕ, and inflation never happens.
- For $xi > \frac{1}{\kappa^2 v^2}$, inflation is never terminated by χ field.

non-minimal vs. minimal hybrid Inflation

- For $\xi > 0$, inflation is terminated later than standard hybrid inflatio ($\xi = 0$). ($\phi_c < \phi_{c,\xi=0}$)
- For $\xi < 0$, inflation is terminated earlier than standard hybrid inflatio ($\xi = 0$). ($\phi_c > \phi_{c,\xi=0}$)

ϕ *field dynamics in non-minimal hybrid inflation*

EoM for ϕ during inflation ($\chi = 0$)

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi \approx 0$$

background solution for ϕ with a slow-roll approximtions

$$\phi = \phi_c e^{-rH(t-t_c)}, \quad r = \frac{3}{2}\left(1 - \sqrt{1 - \frac{4m^2}{9H^2}}\right)$$

Conditions for Hybrid inflation in non-minimal hybrid inflation

$$\frac{|m_\chi|^2}{H^2} \gg 1, \quad (m_\chi^2 = -\lambda v^2(1 - \kappa^2\xi v^2)),$$

$$\lambda v^4 \gg m^2\phi_c^2 \Longrightarrow m^2 \ll \frac{g^2 v^2}{2(1 - 2\kappa^2\xi v^2)}$$

$$\kappa^2\xi\chi^2 \ll 1$$

χ field dynamics in non-minimal hybrid inflation

Equation of motion of χ field (Ignoring nonlinear interaction terms)

$$\chi_{,nn} + 3\chi_{,n} + (\beta - 12\xi)(e^{-2rn} - 1)\chi = 0, \quad n \equiv H(t - t_c)$$

Before $\phi = \phi_c$, χ field is massive

$$\chi(n) = e^{-\frac{3}{2}n}[c_1 J_\nu(z) + c_2 Y_\nu(z)],$$

$$\nu = \frac{1}{r}\sqrt{\frac{9}{4} + \beta - 12\xi}, \quad z = \frac{\sqrt{\beta - 12\xi}\, e^{-nr}}{r}$$

After $\phi = \phi_c$ with $rn \ll 1$

$$\chi(n) \approx e^{-\frac{3}{2}n}\left[d_1 \mathrm{Ai}\left(\frac{9}{4\delta} + \delta^{2/3} n\right) + d_2 \mathrm{Bi}\left(\frac{9}{4\delta} + \delta^{2/3} n\right)\right],$$

$$\delta = \sqrt{2(\beta + 12|\xi|)r}$$

Since $\mathrm{Ai}(x)$ is a decreasing function, we choose $d_1 = 0$

χ-field perturbations

- For a large scale perturbations ($k \gg aH$), since $\delta\chi \propto \chi$, before phase transition ($\phi > \phi_c$)

$$\delta\chi \approx e^{\frac{1}{2}(3-r)(N-N_*)} e^{i(\theta-\theta_*)} \delta\chi_*$$

After phase transition ($\phi_e < \phi < \phi_c$)

$$\delta\chi \approx e^{\frac{1}{2}(3-r)N_*} e^{-\frac{3}{2}N_c} \left(\frac{\phi_i}{\phi_c}\right)^{\frac{1}{2}} \delta\chi_* e^{i(\theta-\theta_c)} e^{-\frac{3}{2}n} \frac{\mathrm{Bi}(\frac{9}{4\delta^{4/3}} + \delta^{3/2} n)}{\mathrm{Bi}(\frac{9}{4\delta^{4/3}})}$$

- In the subhorizon scales ($k \ll aH$), WKB approximation provides assuming the adiabatic initial vaccum state at $\eta \to -\infty$

$$\delta\chi \approx \frac{1}{a}\left[4\left(k^2 + \frac{\beta - 12\xi}{\eta^2}\left(\frac{\phi}{\phi_c}\right)^2\right)\right]^{-1/4} e^{-i\int \omega d\eta}$$

At the horizon crossing time, $\delta\chi_*$ becomes

$$\delta\chi_* = \frac{H}{(2k^3)^{1/2}(\beta + 12|\xi|)^{1/4}} \left(\frac{\phi_*}{\phi_c}\right)^{-1/2}$$

χ-field perturbations

During superhorizon evolution,

$$\frac{\delta\chi}{\chi} = -\zeta_0 \frac{r\phi_i}{(\beta - 12\xi)^{1/4}\chi_i} \left(\frac{\phi_c}{\phi_*}\right)^{1/2} \left(\frac{k}{H}\right)^{3(1-r)/2} e^{-i(\psi_* - \psi_i)}$$

where ζ_0 is the amplitude of the "adiabatic" curvature perturbation

$$\zeta_0 = -\frac{H}{(2k^3)^{1/2} r\phi_i e^{-rN_*}}$$

Non-adiabatic curvature perturbations

Evolution of the curvature perturbation in Einstein frame

$$\zeta' = \frac{H}{H'}\frac{k^2}{a^2}\Phi + H\left(\frac{\delta\phi}{\phi'} - \frac{\delta\chi}{\chi'}\right)\Upsilon, \quad ' \equiv \frac{d}{d\hat{t}}, \cdot \equiv \frac{d}{dt}$$

- $\Upsilon = \frac{1}{2}\left(\frac{\Omega^{-2}\dot{\phi}^2 - \dot{\chi}^2}{\Omega^{-2}\dot{\phi}^2 + \dot{\chi}^2}\right)' - (\ln\Omega)_{,\chi}\chi'\left(\frac{\Omega^{-2}\dot{\phi}^2}{\dot{\chi}^2 + \Omega^{-2}\dot{\phi}^2}\right)^2,$
- conformal transformation:$\hat{g}_{\mu\nu} = \Omega^2(\chi)g_{\mu\nu}, \Omega^2 = 1 - \kappa^2\xi\chi^2$
- t : cosmic time in Jordan frame, $\hat{t}$: cosmic time in Einstein frame
- ζ is invariant under the conformal transformation

curvature perturbation for the superhorzion scales

$$\zeta = \zeta_0\left[1 + \frac{r\phi_i}{(\beta - 12\xi)^{\frac{1}{4}}\chi_i}\left(\frac{\phi_c}{\phi_*}\right)^{\frac{1}{2}}\left(\frac{k}{H}\right)^{\frac{3}{2}(1-r)} e^{-i(\psi_* - \psi_i)}\int_0^{n_f} dn\frac{\Omega\chi\nu}{\dot{\chi}}\right]$$

Classical Backreaction I

- Nonlinear terms in χ field equation becomes important at $\chi = \chi_{(1)}$

$$\chi_{(1)}^2 = \frac{v^2}{1 - \kappa^2 \xi v^2}$$

- Nonlinear terms in ϕ field equation becomes important at $\chi = \chi_{(2)}$

$$\chi_{(2)} = \frac{m^2}{g^2}$$

- Since $\chi_{(2)}^2 \ll \chi_{(1)}^2$, the nonlinear effects in ϕ field direction appear earlier than those in χ field direction.

$$\frac{\chi_{(2)}^2}{\chi_{(1)}^2} = \frac{(1 - \kappa^2 \xi v^2) m^2}{g^2 v^2} \ll 1$$

Classical Backreaction II

- $|\xi| \ll 1$:

$$\zeta \simeq \zeta_0\left[1-\frac{\sqrt{2}\beta m^2(g^2+2m^2\kappa^2\xi)}{r^{1/2}\lambda v^2 g^2(\beta-12\xi)^{3/4}}e^{i(-\psi_*+\psi_i)}\left(\frac{\phi_*}{\chi_*}\right)\left(\frac{k}{H}\right)^{\frac{3}{2}-r}\right]$$

(Can re-produce the minimal case ($\xi = 0$) results due to classical back-reaction (Abolhasani & Firouzjahi, 1005.2934))

- $|\xi| \gg 1$:

$$\zeta \simeq \zeta_0\left[1-\frac{\pi r^{11/6}e^{-i(\psi_*-3\psi_i+2\psi_e)}}{3\cdot 2^{5/6}\kappa^2\xi^2(\beta-12\xi)^{13/12}}\left(\frac{\beta(g^2+2\kappa^2\xi m^2)}{\lambda v^2(\beta-12\xi)}\right)^{3/2r}\times\frac{\phi_i^{3/r}}{\chi_i^3}\left(\frac{\phi_c}{\phi_*}\right)^{1/2}\left(\frac{k}{H}\right)^{3(1-r)/2}\right]$$

It is necessary to consider the quantum backreaction effect to check the effct of the entropy perturbations on the adiabatic perturbations during tachyonic stage. (In Progress!)

PART II: Hybrid Inflation with a Non-minimally Coupled Inflaton

Hybrid inflation with a nonminimally coupled inflaton

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}(1-\kappa^2\xi\phi^2)R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - V(\phi,\chi)\right]$$

Potential of Hybrid inflation

$$V(\phi,\chi) = \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\mu\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

- global minima: $(\phi,\chi) = (0,\pm v)$
- During inflation, χ field stays at the false vacuum ($\chi = 0$)
- effective mass of χ: $m_{eff}^2 = -\lambda v^2 + g^2\phi^2$; tachyonic instability occurs at $\phi^2 = \phi_c^2 = \lambda v^2/g^2$ and inflation ends

Classification of the potential in Einstein frame I

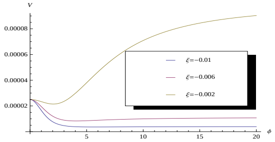
Class I

$$\frac{m^4}{\mu\lambda v^4} < 1$$

i) $\frac{m^2}{\kappa^2\lambda v^4} < |\xi| < \frac{\mu}{\kappa^2 m^2}$: one local minimum at $\phi = \phi_e$

ii) $\frac{m^2}{\kappa^2\lambda v^4} > |\xi| > 0$: monotonically increasing

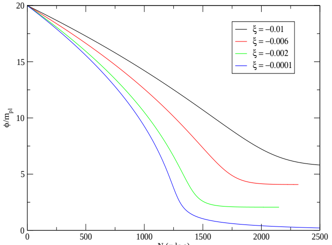
iii) $|\xi| > \frac{\mu}{\kappa^2 m^2}$: monotonically decreasing



- ϕ_e: local minimum

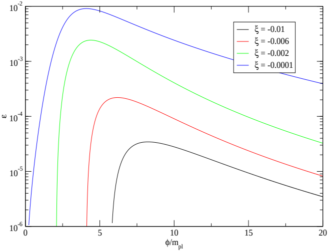
$$\phi_e^2 = -\frac{m^2 + \kappa^2\lambda v^4 \xi}{\mu + \kappa^2 m^2 \xi}$$

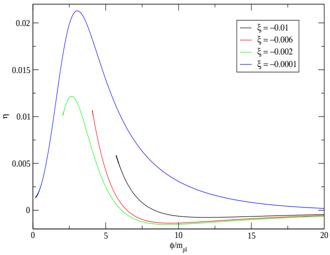
- At $\phi = \phi_c \left(\equiv \frac{\lambda^{1/2} v}{g} \right)$, tachyonic instability occurs and inflation ends.
- $\phi_e \leqslant \phi_c$



Class I: $\frac{m^4}{\mu\lambda v^4} < 1$

$(\lambda = 1, v^2 = 10^{-2} m_{pl}^2, m^2 = 10^{-6} m_{pl}^2, \mu = 10^{-6})$





$$n_s - 1 = 2\eta - 6\epsilon \approx 2\eta \begin{cases} > 0 & \text{blue titled spectrum} \\ < 0 & \text{red tilted spectrum} \end{cases}$$

Coupling parameter	Region	Spectrum
$\frac{m^2}{\kappa^2\lambda v^4} < \lvert\xi\rvert < \frac{\mu}{\kappa^2 m^2}$	Vacuum-dominated	Red
	Local minimum	Blue
	Large field	Red
$0 < \lvert\xi\rvert < \frac{m^2}{\kappa^2\lambda v^4}$	Vacuum-dominated	Blue
	Large field	Red
$\lvert\xi\rvert > \frac{\mu}{\kappa^2 m^2}$	Vacuum-dominated	Red

Classification of the potential in Einstein frame II

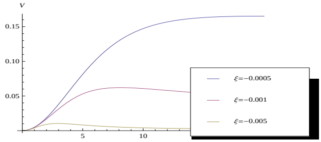
Class II

$$\frac{m^4}{\mu\lambda v^4} > 1$$

i) $\frac{\mu}{\kappa^2 m^2} < |\xi| < \frac{m^2}{\kappa^2\lambda v^4}$: one local maximum

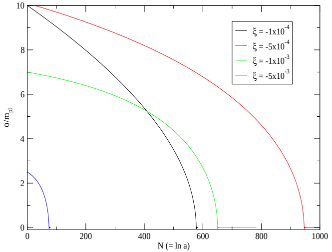
ii) $\frac{\mu}{\kappa^2 m^2} > |\xi| > 0$: monotonically increasing

iii) $|\xi| > \frac{m^2}{\kappa^2\lambda v^4}$: monotonically decreasing



- ϕ_e: local maximum

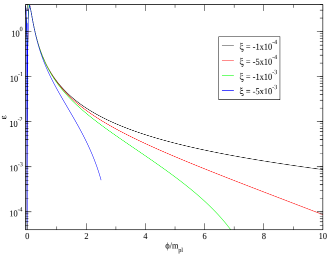
$$\phi_e^2 = -\frac{m^2 + \kappa^2\lambda v^4 \xi}{\mu + \kappa^2 m^2 \xi}$$

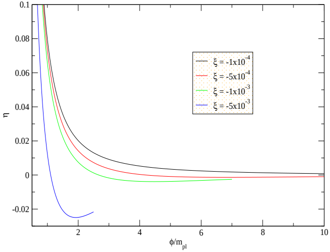


Class II: Spectrum

$$(\lambda = 1, v^2 = 10^{-2} m_{pl}^2, m^2 = 10^{-2} m_{pl}^2, \mu = 10^{-4})$$

slow-roll parameter





Coupling parameter	Region	Spectrum
$\frac{\mu}{\kappa^2 m^2} < \lvert\xi\rvert < \frac{m^2}{\kappa^2 \lambda v^4}$	Vacuum-dominated	Blue
	Local maximum	Red
$0 < \lvert\xi\rvert < \frac{\mu}{\kappa^2 m^2}$	Vacuum-dominated	Blue
	Large field	Red
$\lvert\xi\rvert > \frac{m^2}{\kappa^2 \lambda v^4}$	Vacuum-dominated	Red

Summary

- We consider hybrid inflation when an inflaton or a waterfall field couples to gravity nonminimally.
- For a waterfall field coupled to gravity ($|\xi| \gg 1$):
  1. We investigate the effect of the entropy perturbations due to the waterfall field on the adiabatic perturbations.
  2. We are investigating the effect of the classical and quantum backreaction effect on the curvature perturbations which are induced by the entropy perturbations.
- For an inflaton coupled to gravity:
  1. In Einstein frame the potential has a local minimum or is monotonically increasing as ϕ increases for $\frac{m^4}{\mu\lambda v^4} < 1$ depending on $|\xi|$. For $\frac{m^4}{\mu\lambda v^4} > 1$, the potential has a local maximum or is monotonically increasing which can generate inflation.
  2. The scalar spectral index shows blue-tilted or red-tilted spectrum depending on the coupling parameter $|\xi|$.