Phenomenological Signature from Anisotropic Inflation

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AEG, Himmetoglu, Peloso [PRD81:063528]

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Outline

1. Introduction
   - Motivations
   - Anisotropic extensions of inflation
2. Signatures from anisotropic inflation
   - Background
   - Perturbations/Results

Statistical isotropy

- Inflation typically ends on a homogeneous/isotropic universe if it lasts long enough.
  $\Rightarrow$ Statistical distributions are isotropic.
- Assumption of statistical isotropy $\Leftrightarrow$ power spectrum $P(|\vec{k}|)$

$$\frac{\delta T}{T} = \sum a_{\ell m} Y_{\ell m} \qquad \langle a_{\ell m} a^{\star}_{\ell' m'} \rangle = C_\ell \, \delta_{\ell \ell'} \, \delta_{m m'}$$

- Nonstandard signatures from an early anisotropy may remain at observable scales, if duration of inflation is minimum. Signature:
  $P(\vec{k}) \Rightarrow \langle a_{\ell m} a^{\star}_{\ell' m'} \rangle \not\propto \delta_{\ell \ell'} \, \delta_{m m'}$

AEG, Contaldi, Peloso 2006

Statistical anisotropy in the data?

- $\ell = 2, 3$ are aligned and planar. `Tegmark et al. 2003` `de Oliviera-Costa et al. 2004`
- Explanation: Systematics? Astrophysical sources? Statistical fluke?
- Cosmological Source? An anisotropic stage of inflation could lead to anomalous alignment of low multipoles.
- ACW parametrization:
  $P(\vec{k}) = P_{\rm iso}(k)\left[1 + g_*(k)\,(\hat{k}\cdot\hat{n})^2\right]$ `Ackerman, Carroll, Wise 2007`
  Tested with WMAP5: $g_* = 0.15 \pm 0.039$ `Groeneboom, Eriksen 2008`
- Effect extends to $\ell \sim 400$.
- Missing factor. `Hanson, Lewis 2009`
  $\Rightarrow$ Refined analysis: $g_* = 0.29 \pm 0.031$. `Groeneboom et al. 2009`
- Axis of anisotropy aligned with ecliptic poles: Coincindence that favors astrophysical or systematic source.
- Increase of sensitivity in Planck. `Pullen, Kamionkowski 2007`
- g_* can still be used as a criterion for discriminating inflation models.

Anistropy + Inflation

Initial geometry?

- Scalar field in an initially homogeneous and anisotropic background. The formalism of linear perturbations constructed only recently. AEG, Contaldi, Peloso 2007 Perreira et al. 2007
- Cosmic no-hair conjecture: Homogeneous and anisotropic universe with cosmological constant
  $\Rightarrow$ Quick isotropization within a Hubble time. Wald 1983
- If isotropic inflation lasts the minimum amount, the largest scales may carry signatures from a previous anisotropic stage
  $\Rightarrow$ Duration needs to be tuned.

No sustainable anisotropy, tuned initial conditions
$\Rightarrow$ Way out: Include vector fields (with VEV $\neq 0$) to source the anisotropy.

Anistropy + Inflation

- Massless vector field with standard kinetic term $F_{\mu\nu}F^{\mu\nu} \rightarrow$ quick isotropization. Need to modify the action:

Vector fields?

1. Potential for the vector field $\Rightarrow V(A_\mu A^\mu)$ — Ford 1989
2. Fixed norm vector $\Rightarrow \lambda\,(A_\mu A^\mu - v^2)$ — Ackerman, Carroll, Wise 2007
3. Nonminimal coupling $\Rightarrow \xi\, R\, A_\mu A^\mu$ — Golovnev et al. 2008; Kanno et al. 2008; Chiba 2008; Dimopoulos, Karciauskas 2008
4. Kinetic coupling $\Rightarrow f(\phi)^2\, F_{\mu\nu}F^{\mu\nu}$ — Watanabe et al. 2009

- Cases 1–3 have broken U(1) symmetry $\Rightarrow$ Resulting longitudinal vector is a ghost leading to instabilities.
  Himmetoglu, Contaldi, Peloso 2008--2009
- Case 4 $\Rightarrow$ U(1) conserved $\Rightarrow$ no problematic longitudinal mode, no instability.
  Himmetoglu 2010

Anisotropic inflation: Background

- Action:

Watanabe et al. 2009

$$S = \int d^3x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \partial^\mu \phi - V(\phi) - \frac{1}{4} f(\phi)^2 F_{\mu\nu} F^{\mu\nu} \right]$$

- Vector field VEV aligned with x–axis.

$$\langle A_\mu \rangle = (0,\ A_1(t),\ 0,\ 0)$$

- Background geometry: Axisymmetric Bianchi-I

$$ds^2 = -dt^2 + a(t)^2 dx^2 + b(t)^2 \left[dy^2 + dz^2 \right]$$

- $V(\phi) = \frac{m^2}{2} \phi^2$
- Slow roll + Small anisotropy $\Rightarrow \rho_A \sim \text{constant if}$

$$f(\phi) = \exp\left[2\, c\, \frac{\phi^2}{M_{\rm pl}^2} \right]$$

compatible with isotropic attractor

- With $c \gtrsim 1 \Rightarrow$ growing anisotropy.

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c=1
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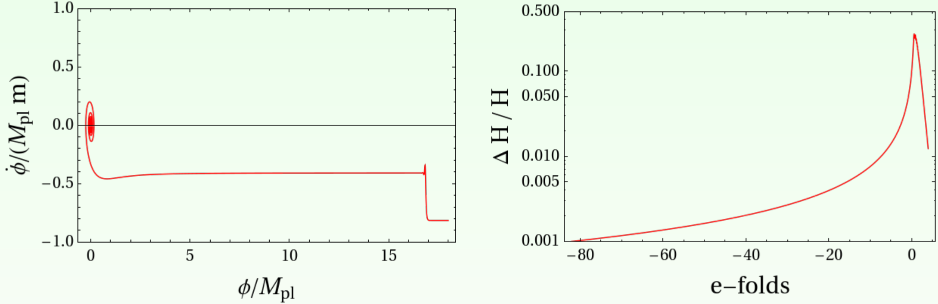
Anisotropic inflation: Background

Example: $V = \frac{1}{2}\, m^2\, \phi^2$, $\quad f(\phi) = \exp\left[\frac{c\,\phi^2}{2\,M_{\rm pl}^2}\right]$, $\quad \left.\frac{\Delta H}{H}\right|_{\rm in} = 10^{-20}$, $\quad \phi_{\rm in} = 18\, M_{\rm pl}$, $\quad c = 2$



Two attractor solutions

1. Isotropic slow roll $\Rightarrow 3\, H\, \dot\phi = -m^2\, \phi$
2. Anisotropic slow roll $\Rightarrow 3\, H\, \dot\phi = -\frac{m^2\,\phi}{c}$

- In attractor–2, anisotropy increases $\Rightarrow \frac{\Delta H}{H} = \frac{2}{3}\, \frac{c-1}{c^2}\, \frac{M_{\rm pl}^2}{\phi^2}$

Anisotropic inflation: Perturbations

Physical degrees of freedom

- DOF
  $$\delta g_{\mu\nu} \Longrightarrow 10$$
  $$\delta\phi \Longrightarrow 1$$
  $$\delta A_\mu \Longrightarrow 4$$
  15
- Gauge
  $$x^\mu \to x^\mu + \xi^\mu \Longrightarrow -4$$
  $$\delta A_\mu \to \delta A_\mu + \partial_\mu \alpha \Longrightarrow -1$$
  -5
- Nondynamical
  $$\delta g_{0\mu} \Longrightarrow -4$$
  $$\delta A_0 \Longrightarrow -1$$
  -5

5 physical degrees

- Isotropic case: Spherical symmetric background
  $\Longrightarrow$ All three degrees decouple
- Symmetry under rotations around x–axis
  $\Longrightarrow$ Two decoupled subsets $5 = 2 + 3$

Anisotropic inflation: Perturbations

Decomposition of perturbations: AEG, Himmetoglu, Peloso 2010

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\,\Phi & a\,\partial_x\chi & b\,(\partial_i B + B_i) \\ & -2\,a^2\,\Psi & a\,b\,\partial_x\left(\partial_i\tilde{B} + \tilde{B}_i\right) \\ & & b^2\left[-2\,\Sigma\,\delta_{ij} + 2\,E_{,ij} + E_{(i,j)}\right] \end{pmatrix}$$

$$\delta A_\mu = (\delta A_0,\, \delta A_1,\, \partial_i \delta A + \delta A_i)\ , \qquad \delta\phi$$

- Gauge $\Rightarrow$ Keep nondynamical modes, set $\tilde{B} = \Sigma = E = E_i = \delta A = 0$.
- Integrate these out using the constraint equations.

Quadratic action, formally

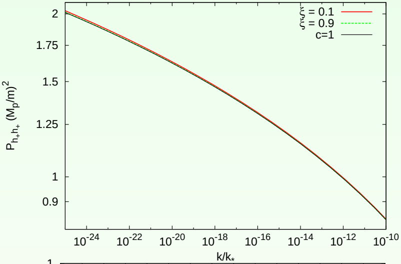
$$S_s^{(2)} = \frac{1}{2}\int dt\, d^3k \left[\dot{Y_s}^\dagger\, \dot{Y_s} - Y^\dagger\, \Omega_s^2\, Y_s\right]\ , \quad Y_s \equiv \begin{pmatrix} V_+ \\ H_+ \\ \Delta_+ \end{pmatrix}$$

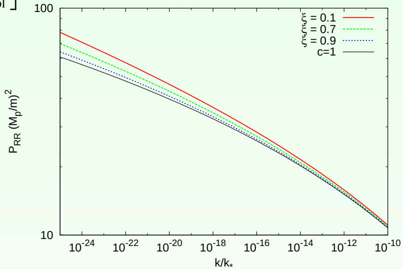
$$S_v^{(2)} = \frac{1}{2}\int dt\, d^3k \left[\dot{Y_v}^\dagger\, \dot{Y_v} - Y_v^\dagger\, \Omega_v^2\, Y_v\right]\ , \quad Y_v \equiv \begin{pmatrix} H_\times \\ \Delta_\times \end{pmatrix}$$

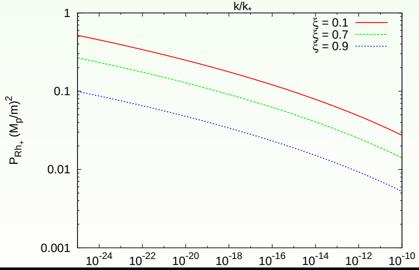
- After isotropization: $V_+ \to v$, $H_+ \to h_+$, $H_\times \to h_\times$; $\Omega^2 \to$ diagonal .
- $\Omega^2 \Rightarrow$ Nondiagonal, time dependent $\Rightarrow$ Scalar-Tensor correlation.
- Deep inside horizon $(H \ll p) \Rightarrow \Omega_s^2,\ \Omega_v^2 \sim p^2\mathbb{1} + \mathcal{O}(H)$.
  $\Rightarrow$ Eigenvalues/vectors evolve adiabatically. Well defined vacuum.
  Quantization of coupled bosons $\Leftarrow$ Nilles et al. 2001

Anisotropic inflation: Results

Spectra for $V = \frac{1}{2}\, m^2\, \phi^2\,, \quad f(\phi) = \exp\left[\frac{c\,\phi^2}{2\,M_{\rm pl}^2}\right]\,, \quad c - 1 = 10^{-5}. \quad \xi \equiv \hat{k}\cdot\hat{n}$







- $\langle h_+ h_+ \rangle$ and $\langle h_\times h_\times \rangle$ are nearly identical. Angular dependence mild.
- Even very small $c - 1$ gives a noticeable angular dependence to $\langle \mathcal{R}\mathcal{R} \rangle$. Slightly larger power than standard ($c = 1$) case.
- Characteristic $\langle \mathcal{R} h_+ \rangle$ signal is smaller than diagonal correlators.

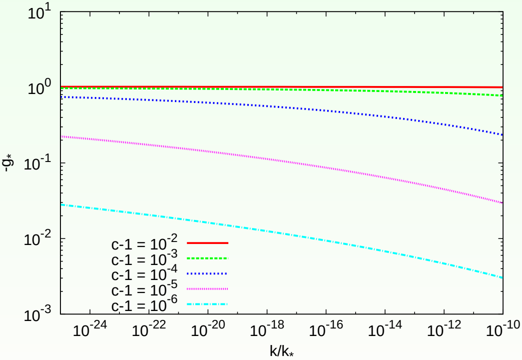
Results in agreement with

`Dulaney, Gresham 2010 -- Watanabe, Kanno, Soda 2010`

Anisotropic inflation: Results

We fit the $\mathcal{R}$ spectrum to ACW parametrization

$$P(\vec{k}) = P_{\rm iso}(k)\left[1 + g_*(k)\,\xi^2\right]$$



- ACW parametrization is accurate $\Rightarrow g_*$ independent of direction.
- $\Delta H/H \sim \mathcal{O}(10^{-6})$ $\Rightarrow |g_*| \sim \mathcal{O}(10^{-1})$. Very different magnitudes.
- For all c values $\Rightarrow g_* < 0$.

Anisotropic inflation: Conclusion

- Carried out a complete phenomenological study of a stable model with sustainable anisotropy.
- Resulting anisotropy signature has an opposite sign than the observed value.
- Consequence of broken 3D rotational symmetry
  $\Rightarrow$ Time evolving, nondiagonal mass matrix for the modes.
  Characteristic signature $\Rightarrow$ Nonzero scalar-tensor correlator.
  For this model, too small to be interesting.
- Nevertheless, provides the tools for future studies which can reproduce the WMAP feature or lead to new predictions.

EXTRA SLIDE: Background evolution

- Equation for A_i solved analytically. Remaining equations

$$3\,H^2 - 3\,h^2 = \frac{1}{M_{\rm pl}^2}\left(\frac{\dot\phi^2}{2} + V(\phi) + \rho_A\right)$$

$$2\,\dot H + 3\,H^2 + 3\,h^2 = \frac{1}{M_{\rm pl}^2}\left(\frac{\dot\phi^2}{2} + V(\phi) - \frac{\rho_A}{3}\right)$$

$$\ddot\phi + 3\,H\,\dot\phi + V'(\phi) = 2\,\rho_A\,\frac{f'(\phi)}{f(\phi)}$$

$$\dot h + 3\,h\,H = \frac{2}{3\,M_{\rm pl}^2}\,\rho_A$$

$$H \equiv \frac{1}{3}\left(\frac{\dot a}{a} + 2\,\frac{\dot b}{b}\right)$$

$$h \equiv \frac{1}{3}\left(\frac{\dot b}{b} - \frac{\dot a}{a}\right)$$

$$\rho_A \equiv \frac{p_A^2}{2\,b^4\,f(\phi)^2}$$

- Isotropic limit $\rho_A \to 0$, $h \to 0$
- Slow roll + Small anisotropy $\Rightarrow \rho_A \sim \text{constant if}$

$$f(\phi) = \exp\left[\frac{2\,c}{M_{\rm pl}^2}\int^{\phi}\frac{V(\bar\phi)}{V'(\bar\phi)}\,d\bar\phi\right] \longleftarrow$$

c=1
compatible with
isotropic attractor

- With $c \gtrsim 1 \Rightarrow$ growing anisotropy.

EXTRA SLIDE: Definition of spectra

$$\frac{1}{2}\langle A\left(t,\,\vec{x}\right)\;B\left(t,\,\vec{y}\right)+B\left(t,\,\vec{y}\right)\;A\left(t,\,\vec{x}\right)\rangle\equiv$$

$$\int\frac{dk}{k}\int_0^1 d\xi\,\cos\left(k\,\xi\,r_L\right)J_0\left(k\,\sqrt{1-\xi^2}\,r_T\right)\;P_{AB}$$

$$\longrightarrow\int\frac{dk}{k}\,\frac{\sin\left(k\,r\right)}{k\,r}\,P_{AB}\qquad\text{(isotropic limit)}$$

where

$$\begin{aligned} r_L &\equiv \left|\hat{n}\cdot\left(\vec{x}-\vec{y}\right)\right| \\ r_T &\equiv \sqrt{\left(\vec{x}-\vec{y}\right)^2-\left[\hat{n}\cdot\left(\vec{x}-\vec{y}\right)\right]^2} \\ \xi &\equiv \hat{k}\cdot\hat{n} \end{aligned}$$

and

$$P_{AB}\equiv\frac{k^3}{2\,\pi^2}\,\mathrm{Re}\left(A_k\,B_k^*\right)$$

EXTRA SLIDE: Coupled systems

Examples

- For cosmological perturbations,
  3D rotational symmetry $\Rightarrow$ Decoupled fields
  Relaxing the symmetry $\Rightarrow$ Coupling

  AEG, Contaldi, Peloso 2007
- Preheating mechanism can play a key role in decay of SUSY flat directions.
  D–term potential $\Rightarrow$ Coupling between scalar fields.

  Olive, Peloso 2006

In these examples, frequency matrix is nondiagonal and time evolving $\Rightarrow$ coupled mode functions

$$\phi_i(\eta, \vec{x}) = \int \frac{d^3k}{(2\,\pi)^{3/2}}\, \mathrm{e}^{i\,\vec{k}\cdot\vec{x}} \left[v_{ij}(\eta, k)\, \hat{a}_j(\vec{k}) + v_{ij}^{\star}(\eta, k)\, \hat{a}_j^{\dagger}(-\vec{k}) \right]$$

We use the generalized Bogolyubov formalism for quantization of such systems. Nilles, Peloso, Sorbo 2001

EXTRA SLIDE: Quantization of coupled bosons

Nilles, Peloso, Sorbo 2001

$$S = \frac{1}{2}\int d^3k\, d\eta\, \left(\phi'^{\,\dagger}\,\phi' - \phi^\dagger\,\Omega^2\,\phi\right)$$

- Nondiagonal, time dependent frequency matrix:
$\phi^\dagger\Omega^2\phi = \underbrace{(\phi^\dagger\, C)}_{\tilde\phi^\dagger}\underbrace{(C^T\,\Omega^2\,C)}_{\omega^2_{\rm diag}}\underbrace{(C^T\,\phi)}_{\tilde\phi}$

- Kinetic Mixing: $\phi'^{\,\dagger}\phi' = \tilde\phi'^{\,\dagger}\,\tilde\phi' + \tilde\phi'^{\,T}\,\Gamma\,\tilde\phi + \tilde\phi\,\Gamma^T\,\tilde\phi'^{\,\dagger} + \tilde\phi^\dagger\, C'^{\,T}\, C'\,\tilde\phi$
$(\Gamma \equiv C^T\, C')$

- $\tilde\phi_i = \left[\frac{1}{\sqrt{2\,\omega}}\left(\underbrace{{\rm e}^{-i\int^t \omega\,dt} A}_{\alpha} + \underbrace{{\rm e}^{i\int^t \omega\,dt} B}_{\beta}\right)\right]_{ij}\hat a_j(\vec k) + [\cdots]^\star_{ij}\;\hat a^\dagger_j(-\vec k)$

(Bogolyubov Matrices)

$$\alpha\,\alpha^\dagger - \beta^\star\,\beta^T = \mathbb{1}\,, \qquad \alpha\,\beta^\dagger - \beta^\star\,\alpha^T = 0$$

EXTRA SLIDE: Quantization of coupled bosons

- $$\mathcal{H} = \frac{1}{2}\,(\hat{a}^\dagger\,,\,\hat{a})\begin{pmatrix}\alpha^\dagger & \beta^\dagger \\ \beta^T & \alpha^T\end{pmatrix}\begin{pmatrix}\omega & 0 \\ 0 & \omega\end{pmatrix}\underbrace{\begin{pmatrix}\alpha & \beta^\star \\ \beta & \alpha^\star\end{pmatrix}\begin{pmatrix}\hat{a} \\ \hat{a}^\dagger\end{pmatrix}}_{\begin{pmatrix}\hat{b} \\ \hat{b}^\dagger\end{pmatrix}}$$
- Time dependent annihilation/creation operators: $\begin{pmatrix}\hat{b} \\ \hat{b}^\dagger\end{pmatrix}$

  : $\mathcal{H} := \omega_i\,\hat{b}_i^\dagger\,\hat{b}_i$.
- Occupation numbers: $N_i(t) = \langle \hat{b}_i^\dagger\,\hat{b}_i\rangle = \left(\beta^\star\,\beta^T\right)_{ii}$
- Equations of motion:

$$\begin{aligned} \alpha' &= (-i\,\omega - I)\alpha + \left(\frac{\omega'}{2\,\omega} - J\right)\beta \qquad & I &\equiv \frac{1}{2}\left(\sqrt{\omega}\,\Gamma\,\frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}}\,\Gamma\,\sqrt{\omega}\right) \\ \beta' &= (i\,\omega - I)\beta + \left(\frac{\omega'}{2\,\omega} - J\right)\alpha \qquad & J &\equiv \frac{1}{2}\left(\sqrt{\omega}\,\Gamma\,\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,\Gamma\,\sqrt{\omega}\right) \end{aligned}$$

Anti-Hermitian: Rotate produced states. Preserves total $N(t)$

Hermitian: Particle production.

- Adiabaticity condition

$$\left[\frac{\omega'}{\omega^2} - 2\,\frac{1}{\sqrt{\omega}}\,J\,\frac{1}{\sqrt{\omega}}\right]_{ij} = \left[\frac{\omega'}{\omega^2} - \left(\Gamma\frac{1}{\omega} - \frac{1}{\omega}\Gamma\right)\right]_{ij} \ll 1$$