



Prof. K. Sato's group as of 1986 (6 years after proposing inflation)

(Part of) UTAP/RESCEU as of 2008

Sarujima (Monkey Island) in Tokyo Bay

On this slot of the symposium, originally Virginia Trimble was supposed to give a talk on the history of the concept **Multiverse** with the title "APERIO KOSMOI: Multiple Universes from the Ancients to 1981"

but she could not come here in the end, because she could not pass through the security check at Los Angels Airport....???



1981?

Inflation based on the first-order phase transition K. Sato MNRAS 195(1981)467; PLB99(1981)66, A. Guth PRD23(1981)347

cf New inflation A. Linde PLB108(1982)389, Albrechet & Steinhardt PRL 48(1982)1220
 R² theory A. Starobinskiy PLB91(1980)99
 Chaotic inflation A. Linde PLB129(1983)177

1981

MULTI-PRODUCTION OF UNIVERSES BY FIRST-ORDER PHASE TRANSITION OF A VACUUM

Katsuhiko SATO, Hideo KODAMA, Misao SASAKI^a and Kei-ichi MAEDA Department of Physics, Kyoto University, Kyoto 606, Japan

^a Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

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Gauge theories with spontaneously broken symmetries give rise to a cosmological phase transition of a vacuum. We show that if the phase transition is strongly of first order, such gauge theories combined with general relativity yield a surprising prediction; although the Creator might have made a unitary universe, many mini-universes are produced sequentially afterward as a result of the phase transition.





eternal inflation of Vilenkin and Linde

Reporting that Professor Sato won Nishina Memorial Prize

The paper of the multiproduction of the Universes was epoch-making in the sense that the conventional cosmology dealing with "the one and only Universe" was replaced by the new cosmology pushing "our Universe among many possible universes."



triggered a transition of the vision of the Universe され, 我々の宇宙が実現する確率まで議論されるように なっているが, こうした研究の背景には, 佐藤氏を嚆矢 とする上述のような宇宙観の変遷があることを忘れては んならない.

天文月報1991年3月号

Astronomical Herald March, 1991 (by Astronomical Society of Japan)

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Fig. 2.

eternal inflation of Vilenkin and Linde

So much for the Multiverse

In the current paradigm of Inflationary Cosmology, in which the seeds of large scale structures and the anisotropy of CMB are explained in terms of quantum fluctuations of scalar fields,

We must deal with the quantum ensemble of the universes whether there are many universes or only one.

We are observing one realization of the ensemble from a single point.



What can we learn about cosmophysics by observing only one Universe?









Jun'ichi Yokoyama (RESCEU, U. Tokyo) 横山順一 Theories predict an ensemble average, and at best statistical distribution around it.

We can observe only one realization of the ensemble.

Comparison between theories and observations is done incorporating the cosmic variance.

We would be happy if our theory agrees with observation within the cosmic variance.

What should we do if it doesn't?

It depends on the degree of deviation, of course.

5-year WMAP data. TT angular power spectrum



Theoretical curve of the best-fit \land CDM model with a power-law initial spectrum

Even the binned data have some deviations from the power-law model.

Errors are dominated by the cosmic variance up to l=407.

Becoimple power flaim or interdiate flootexation spectrum sufficient?



From the viewpoint of observational cosmology, the spectral shape of primordial curvature perturbation should be determined purely from observational data without any theoretical prejudice.

Plan Inverse Analysis

Shown at Poster #C07 by Ryo Nagata

Maximum Likelihood Matrix Method Forward Analysis Markov-Chain Monte-Carlo Method

Conclusion

Maximum Likelihood Matrix Method

As confirmed by WMAP observation, temperature fluctuation $\frac{\delta T}{T} \partial_{\sigma} \varphi \mathbf{G} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm} \partial_{\sigma} \varphi \mathbf{\zeta}$ is Gaussian distributed.

$$\langle a_{\ell m} \rangle = 0, \quad \langle a_{\ell m} a^*_{\ell' m'} \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{m m'},$$

with
$$C_{\ell} = \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{(2\ell+1)^2} |X_{\ell}(k)|^2 P(k)$$

Primordial Power spectrum

The probability distribution function (PDF) for each multipole

is given by

$$\mathcal{P}[a_{\ell m}|P(k)] = \frac{1}{\pi C_{\ell}} \exp\left(-\frac{|a_{\ell m}|^2}{C_{\ell}}\right) \quad (m \neq 0),$$

$$\mathcal{P}[a_{\ell 0}|P(k)] = \frac{1}{\sqrt{2\pi C_{\ell}}} \exp\left(-\frac{|a_{\ell 0}|^2}{2C_{\ell}}\right), \qquad P(k)$$

The likelihood function is their products.

r products.
$$\mathcal{L}[\{a_{\ell m}\}|P(k)] = \prod_{\ell, m \ge 0} \mathcal{P}[a_{\ell m}|P(k)].$$

Maximum Likelihood analysis

We insert the observed values

$$C_{\ell}^{obs} \equiv \frac{1}{2\ell + 1} \sum_{m = -\ell}^{\ell} |a_{\ell m}|^2 - N_{\ell}$$

to the above PDF and

regard it as a PDF for the power spectrum P(k).

(N_{ℓ} : dispersion of observational noises)

$$\mathcal{S} \equiv -2\ln\mathcal{L}[\{C_{\ell}^{obs}\}|P(k)]$$
$$= \sum_{\ell=\ell_{\min}}^{\ell_{\max}} (2\ell+1) \left[\frac{C_{\ell}^{obs}+N_{\ell}}{C_{\ell}}+\ln\left(\frac{C_{\ell}+N_{\ell}}{C_{\ell}^{obs}+N_{\ell}}\right)\right] + (const.)$$

Likelihood function for P(k)

$$S \equiv -2 \ln \mathcal{L}[\{C_{\ell}^{obs}\}|P(k)]$$

$$= \sum_{\ell=\ell_{\text{min}}}^{\ell_{\text{max}}} (2\ell+1) \left[\frac{C_{\ell}^{obs} + N_{\ell}}{C_{\ell}} + \ln \left(\frac{C_{\ell}}{C_{\ell}^{obs}} + N_{\ell} \right) \right] + (const.)$$
should be multiplied by the sky coverage factor f_{sky} .
$$with \qquad C_{\ell}^{obs} \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 - N_{\ell}$$
 χ^2 distribution with degree $2\ell + 1$

We assume the values of global cosmological parameters are fixed (to the WMAP best-fit values), and maximize the likelihood function with regard to the power spectrum P(k).

$$\begin{split} \text{We solve} \quad \frac{\delta S}{\delta P(k)} &= \sum_{\ell} f_{sky} \frac{k^2}{\pi} \frac{|X_{\ell}(k)|^2}{2\ell + 1} \frac{C_{\ell} - C_{\ell}^{obs}}{(C_{\ell} + N_{\ell})^2} = 0, \\ cf \quad \frac{\delta C_{\ell}}{\delta P(k)} &= \frac{2k^2}{\pi} \frac{|X_{\ell}(k)|^2}{(2\ell + 1)^2} \\ \frac{\delta C_{\ell}}{\delta P(k)} &= \frac{2k^2}{\pi} \frac{|X_{\ell}(k)|^2}{(2\ell + 1)^2} \\ \frac{\delta C_{\ell}}{(2\ell + 1)^2} \frac{|X_{\ell}(k)|^2}{2\ell + 1} \int dk' \frac{2k'^2}{\pi} \frac{|X_{\ell}(k')|^2}{(2\ell + 1)^2} \\ P(k') &= \sum_{\ell = \ell_{\rm min}}^{\ell_{\rm max}} \frac{k^2 f_{sky}}{\pi (C_{\ell} + N_{\ell})^2} \frac{|X_{\ell}(k)|^2}{2\ell + 1} \\ C_{\ell}^{obs} \\ C_{\ell}^{obs} \end{split}$$

We obtain a matrix equation.

$$\sum_{\ell, k'} D_{k\ell} G_{\ell k'} P_{k'} = \sum_{\ell} D_{k\ell} C_{\ell}^{obs}.$$

$$\sum_{\ell, k'} D_{k\ell} G_{\ell k'} P_{k'} = \sum_{\ell} D_{k\ell} C_{\ell}^{obs}.$$

$$\begin{bmatrix} \ell \\ D_{k\ell} \end{bmatrix} \begin{bmatrix} k \\ G_{\ell k'} \end{bmatrix} \begin{bmatrix} k \\ P_{k'} \end{bmatrix} = \begin{bmatrix} \ell \\ D_{k\ell} \end{bmatrix} \begin{bmatrix} c_{\ell}^{obs} - N_{\ell} \end{bmatrix} C_{\ell}^{obs} - N_{\ell} \\ C$$

#k dimensional square matrix

but we cannot invert it as it is, because the transfer function contained there act as a smoothing function.

If we introduce some appropriate prior to the power spectrum, we can reconstruct it.

Bayes theorem

$$\mathcal{P}\left[P(k)|\{C_{\ell}^{obs}\}\right] = \frac{\mathcal{P}\left[\{C_{\ell}^{obs}\}|P(k)\right]\mathcal{P}\left[P(k)\right]}{\mathcal{P}\left[\{C_{\ell}^{obs}\}\right]} \xrightarrow{\text{Prior}}$$

Prior for P(k): "smoothness condition" cf (Tocchini-Valentini, Hoffman & Silk 05)

$$\mathcal{P}[P(k)] \propto \exp\left[-\epsilon \int dk \left(\frac{dk^3 P(k)}{dk}\right)^2\right] \equiv e^{-\epsilon \mathcal{J}[P(k)]}$$

With this prior, the maximum likelihood equation

$$\frac{\delta S}{\delta P(k)} = \sum_{\ell} f_{sky} \frac{k^2}{\pi} \frac{|X_{\ell}(k)|^2}{2\ell + 1} \frac{C_{\ell} - C_{\ell}^{obs}}{(C_{\ell} + N_{\ell})^2} = 0,$$

is modified to

$$\frac{\delta}{\delta P(k)} \Big(\mathcal{S}[P(k)] + \epsilon \mathcal{J}[P(k)] \Big) = 0$$

The value of ε is chosen so that the reconstructed power spectrum does not oscillate too much (in particular, to negative values) and that recalculated C_{ℓ} 's agree with observation well.

Test Calculations

start with a power spectrum with oscillatory modulation $A(k) \equiv k^3 P(k) = \underline{A(k/k_0)^{n-1}} + B \operatorname{si}$ d: distance to LSS (13.4 Gpc)

reconstruct

calculate



Test Calculations

$$A(k) \equiv k^{3} P(k) = A(k/k_{0})^{n-1} + B \sin\left(\frac{2\pi}{T}kd\right) \implies C_{\ell} \implies A(k)$$



Test Calculations

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Test Calculations: Summary

- * Resolution depends on ε .
- * Locations of peaks/dips are reproduced quite accurately.
- * Always returns equal or smaller amplitudes = smoothed spectrum.
- * Gives a conservative bound on any deviation from the power law
- ★ If we find some deviation, actual power spectrum should have even larger deviation.



Application to WMAP5 data

* We fix cosmological parameters to the best fit values of the power-law Λ CDM model based on WMAP5.

$$h = 0.723, \ \mathfrak{M}_m = 249, \ \Omega_b = 0.0432,$$

 $\Omega_\Lambda = 0.751, \ \mathfrak{O} = 0.089$
 \square distance to the last scattering surface $d = 13.4$ Gpc

* We make 50000 samples of C_{ℓ} based on observed mean values and scatters around them based on the proper likelihood function of WMAP and perform inversion for each sample.

Application to WMAP5 data: Results



 $2.1 \times 10^{-3} \,\mathrm{Mpc}^{-1} \le k \le 2.7 \times 10^{-2} \,\mathrm{Mpc}^{-1}$

d = 13.4Gpc

 $kd \approx \ell$

Recalculate the angular power spectrum



fits the observational data with binning $\Delta \ell = 4$ well.

* Theoretically, different k modes are uncorrelated.

$$\left\langle \Phi_{k}(t)\Phi_{k'}^{*}(t)\right\rangle = P(k,t)\delta^{3} \partial k'$$

* Observationally reconstructed spectrum is correlated with nearby

k-modes.

$$C_{\ell} = \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{(2\ell+1)^2} |X_{\ell}(k)|^2 P(k)$$



limited by the transfer function

Error estimation

* Calculate the covariance matrix from $\mathcal{N}=50000$ samples of the reconstructed power spectra.

$$K_{ij} \equiv \frac{1}{\mathcal{N}} \sum_{\alpha=1}^{\mathcal{N}} A_{\alpha}(k_i) A_{\alpha}(k_j) - \frac{1}{\mathcal{N}} \sum_{\alpha=1}^{\mathcal{N}} A_{\alpha}(k_i) \frac{1}{\mathcal{N}} \sum_{\beta=1}^{\mathcal{N}} A_{\beta}(k_j)$$
$$\equiv \langle \langle A_{\alpha}(k_i) A_{\alpha}(k_j) \rangle \rangle_{\alpha} - \langle \langle A_{\alpha}(k_i) \rangle \rangle_{\alpha} \langle \langle A_{\beta}(k_j) \rangle \rangle_{\beta},$$

 Diagonalize the covariance matrix to constitute mutually independent band powers. The number of band powers is chosen so that their widths do not overlap with each other.



Result of band power decomposition





Deviation around $kd \approx \ell \approx 40$ can be seen even in the binned C_{ℓ} but those at 125 can not be seen there.

(Nagata & JY 08)





Statistical distribution according to WMAP likelihood function.

Statistical analysis of 50000 samples generated according to WMAP's likelihood function shows that the probability to find a deviation above 3.3σ is 10^{-3} . This is small.

But we have observed one such an event out of 40 band powers. $10^{-3} \times 40=0.04$. This is large. I would be happy to live in a Universe which is realized in a "standard" theory with the probability of 4%.



If we try to interpret the deviation from the band-power analysis only, we may well conclude that it is just a realization of a rare event among many random realizations of quantum ensemble.



But if we look at the original unbinned angular power spectrum we find some nontrivial oscillatory structures that may have originated in features in the primordial power spectrum.

NB Correlation between different multipoles is less than 1%.



In fact, if we change the wavenumber domain of decomposition slightly, we obtain a dip rather than an excess even for the band power analysis.

Forward Analysis

Assume various shapes of modified power spectrum P(k) with three additional parameters in addition to the standard power-law.

Perform Markov-Chain Monte Carlo analysis with CosmoMC with these three additional parameters in addition to the standard 6 parameter Λ CDM model.

Transfer function shows that C_{ℓ} depends on P(k) with $kd \ge \ell$.

$$C_{\ell} = \int \frac{d^3k}{(2\pi)^3} \frac{4\pi}{(2\ell+1)^2} |X_{\ell}(k)|^2 P(k)$$



If we add some extra power on P(k)at $kd \approx 125$, it would modify all C_{ℓ} 's with $\ell \leq kd \approx 125$.



Simply adding an extra power around $kd \approx 125$ does not much improve the likelihood, because it modifies the successful fit of power-law model at smaller ℓ 's. Consider power spectra which change C_{ℓ} 's only locally.



kd

χ^2_{eff} improves as much as 21 by introducing 3 additional parameters.

	power law	v^{Λ} type	W type	S type	Δ_{\max}	WMAP5	Planck
Ω_b	0.0438	0.0443	0.0435	0.0440	0.0005	0.0030	
Ω_m	0.256	0.260	0.257	0.256	0.004	0.027	
Ω_{Λ}	0.744	0.740	0.743	0.744	0.004	0.015	
H_0 $^{(a)}$	72.1	71.8	72.0	72.1	0.3	2.7	
$\ln(10^{10}A_{0.002})^{(b)}$	3.173	3.155	3.187	3.146	0.027	0.047	
n_s	0.964	0.969	0.954	0.970	0.010	0.015	0.0045
$10^2\Omega_b h^2$	2.274	2.280	2.260	2.285	0.011	0.062	0.017
$\Omega_c h^2$	0.1094	0.1100	0.1096	0.1094	0.0006	0.0063	0.0016
au	0.0864	0.0831	0.0786	0.0812	0.0078	0.017	0.005
$z_{ m re}$	10.9	10.6	10.3	10.4	0.6	1.4	
$\Delta\chi^2_{ m eff}$	0	-18	-16	-21			

(Ichiki, Nagata, JY, 08)

If χ^2 improves by 2 or more, it is worth introducing a new parameter, according to Akaike's information criteria (AIC).

Comparison with other non power-law, non standard models (based on 3 year WMAP data)

Goodness of Fit, $\Delta \chi^2_{\text{eff}} \equiv -2 \ln \mathcal{L}$, for *WMAP* Data only Relative to a Power-Law ACDM Model

Model Number	Model	$\Delta \chi^2_{ m eff} \equiv -\Delta (2 \ln \mathcal{L})$	N _{par}
M1	Scale-invariant fluctuations $(n_s = 1)$	6	5
M2	No reionization ($\tau = 0$)	7.4	5
M3	No dark matter ($\Omega_c = 0, \Omega_\Lambda \neq 0$)	248	6
M4	No cosmological constant ($\Omega_c \neq 0, \Omega_{\Lambda} =$	0) 0	6
M5	Power law ACDM	0	6
M6	Quintessence ($w \neq -1$)	0	7
M7	Massive neutrino ($m_{\nu} > 0$)	-1	7
M8	Tensor modes $(r > 0)$	0	7
M9	Running spectral index $(dn_s/d \ln k \neq 0)$	-4	7
M10	Nonflat universe ($\Omega_k \neq 0$)	-2	7
M11	Running spectral index and tensor modes	-4	8
M12	Sharp cutoff	-1	7
M13	Binned $\Delta^2_{\mathcal{R}}(k)$	-22	20

Note.—A worse fit to the data is $\Delta \chi^2_{\rm eff} > 0$.

(Spergel et al 07)

Running spectral index improves χ_{eff}^2 by 4. AIC OK Running + tensor improve χ_{eff}^2 by 4. AIC marginal (Our analysis of 5 year WMAP data shows that Running improves χ_{eff}^2 only by 1.8. AIC No)



Note.—A worse fit to the data is $\Delta \chi^2_{\text{eff}} > 0$.

Binned power spectrum does not improve χ^2_{eff} sufficiently, if binning is done with no reference to the observational data.

It is very difficult to improve the fit. Inverse analysis is very important! Unlike our reconstruction methods, MCMC calculations use not only TT data but also TE data.

$$\Delta \chi^2_{eff}$$
 due to improvement of TT fit = -12.5
 $\Delta \chi^2_{eff}$ due to improvement of TE fit = -8.5

It is intriguing that our modified spectra improve TE fit significantly even if we only used TT data in the beginning.

TT(temp-temp) data and model

TE(temp-Epol) data and model



Posterior distribution in MCMC calculation with

$$k^{3}P(k) = A\left(\frac{k}{k_{0}}\right)^{n-1} + B\left(\frac{k}{k_{0}}\right)^{n-1} \exp\left(-\frac{(k-k_{*})^{2}}{\kappa^{2}}\right) \cos\left(\pi\frac{k-k_{*}}{\kappa}\right)$$



Probability to find $B < 1.43 \times 10^{-10}$ is only 2.2×10^{-5} . (for $k_*d = 100 \square 150$) (tentative) The tentative probability that the primordial power spectrum $P(k,t_i) = \langle |\Phi_k(t_i)|^2 \rangle$ has a nonvanishing modulation (at some wave number) is estimated to be ~99.98%.

Is it due to some nontrivial physics during inflation?? or just a rare event ($\sim 0.02\%$) in the standard theory?

The presence of such a fine structure changes the estimate of other cosmological parameters at an appreciable level.



- * If we wish to evaluate the values of the cosmological parameters of our current Universe with high accuracy, we should take possible nontrivial, non-power-law features into account.
- * Whether they have any physical origin or are just a particular realization of random fluctuations, they are properties of our own Universe.
- * We should investigate their characteristic features (and impact on other parameters), even if this may not be an physics issue.

Astronomer's Universe and Physicist's universe





To find something interesting

Abstract information by Fourier decomposition.

CONCLUSION

With the next generation (or perhaps next-to-next generation) of higher precision observations, Cosmology will inevitably turn to Astronomy from Physics. This could be regarded as a triumph of physics.





A brief history of Katsubilza Sata

born on Augus PhD from Kyo Hayashi) Kyoto Univers University of T Dean of Facult Director of RE President of IA 1988-1991), pr 1998, 2005-200 the 5th Inoue Fo the 36th Nishin



A brief history of Katsuhiko Sato



List of the graduate students supervised by Professor Sato



Thank you, Professor Sato. We wish you a happy life after retirement from Physics Dept & RESCEU.

