Large Primordial Non-Gaussianity from early Universe

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Primordial curvature perturbations

Komatsu et.al. 2008

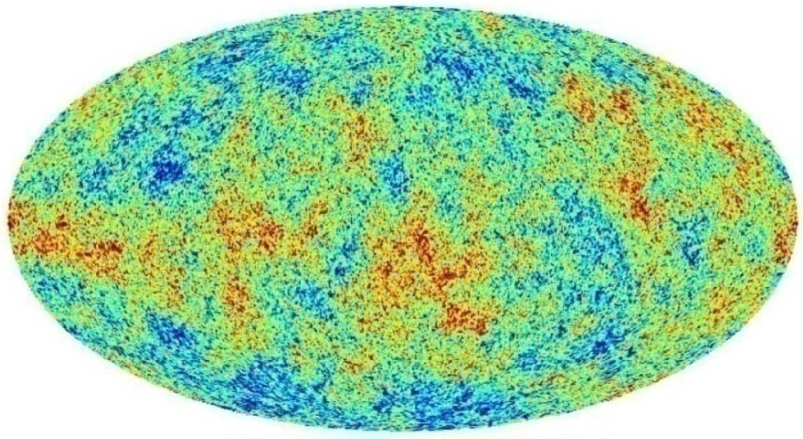
- Proved by CMB anisotropies

  nearly scale invariant $\quad n_s = 0.960 \pm 0.013$

  nearly adiabatic $\quad \alpha < 0.16, \quad S/\zeta \sim \sqrt{\alpha}$

  nearly Gaussian $\quad -9 < f_{\mathrm{NL}}^{\mathrm{local}} < 111$

$$\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{\mathrm{NL}}^{\mathrm{local}} \zeta_g(x)^2$$



- Generation mechanisms

  inflation

  curvaton

  collapsing universe (Ekpyrotic, cyclic)

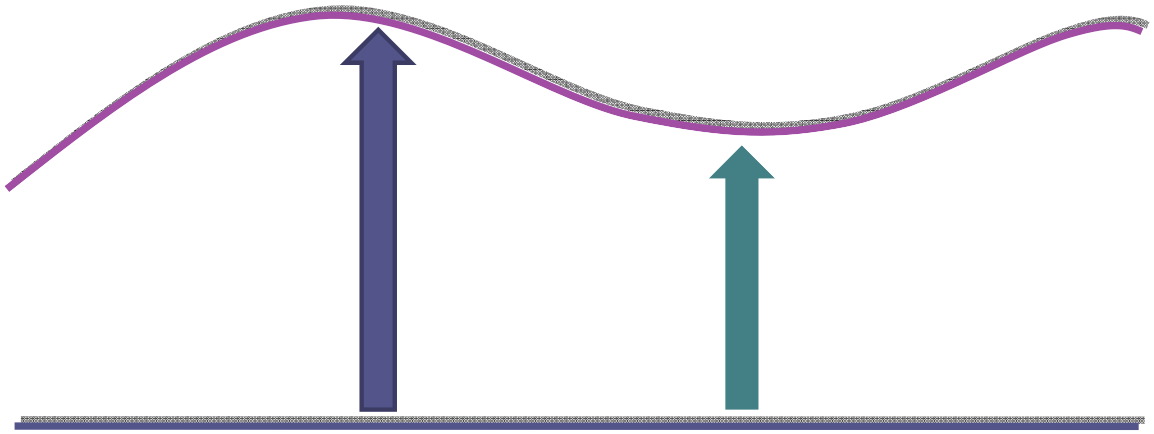
Generation of curvature perturbations

- Delta N formalism   Starobinsky ;85, Stewart&Sasaki '95

curvature perturbations on superhorizon scales

= fluctuations in local e-folding number

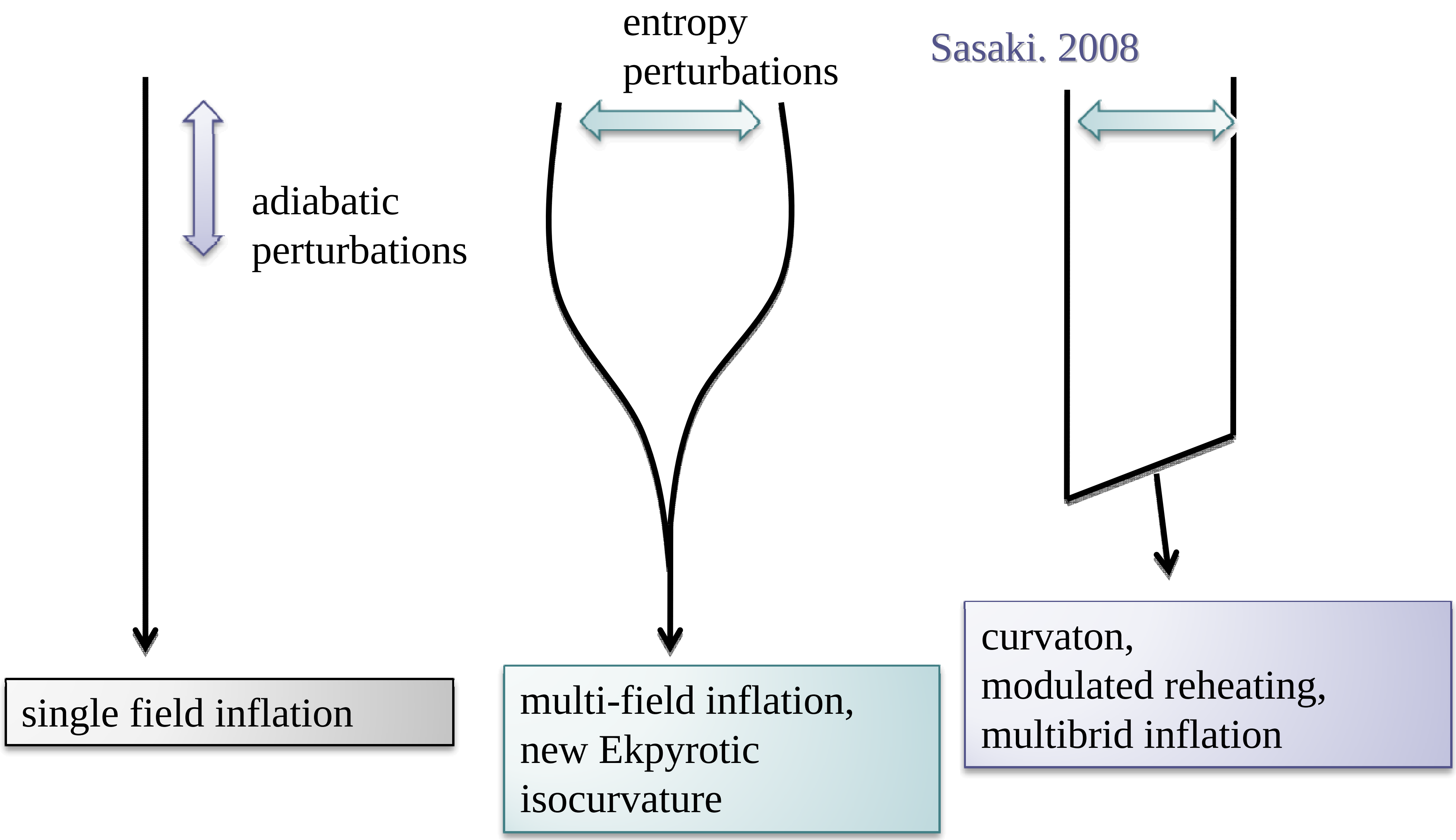
$$N(t,x^i) = \int_{t_i}^{t} dt' \, H(t',x^i) \qquad \zeta = N(t,x^i) - N_B(t)$$



$$\zeta = \Psi - \frac{\delta\rho}{3(\rho + P)}$$

$$\zeta_{in} = 0$$

How to generate delta N



- Suppose delta N is caused by some field fluctuations at horizon crossing

Lyth&Rodriguez '05

$$\zeta(t,x^i)=\sum_{I,J}\left(\frac{\partial N}{\partial\varphi_I}\delta\varphi_I+\frac{1}{2}\frac{\partial^2 N}{\partial\varphi_I\partial\varphi_J}\delta\varphi_I\delta\varphi_J+\ldots\right)$$

- Bispectrum $\left\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\right\rangle=(2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)B_\zeta(k_1,k_2,k_3)$

$$B_\zeta(k_1,k_2,k_3)$$

$$=\frac{N_{,I}N_{,J}N_{,IJ}}{N_{,I}N_{,I}}\left(P_\zeta(k_1)P_\zeta(k_2)+\text{perm.}\right)+N_{,I}N_{,J}N_{,K}B_{IJK}(k_1,k_2,k_3)$$

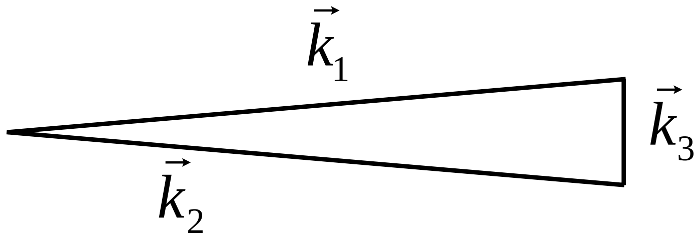
Local type ('classical')
(local in real space
=non-local in k-space)

Equilateral type ('quantum')
(local in k-space)

Observational constraints

- local type

  maximum signal for $k_3 \ll k_1, k_2$

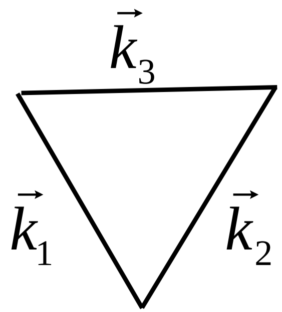


$$\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{\rm NL}^{\rm local} \zeta_g(x)^2$$

WMAP5 $-9 < f_{\rm NL}^{\rm local} < 111$

- Equilateral type

  maximum signal for $k_1 \approx k_2 \approx k_3$



WMAP5 $-151 < f_{\rm NL}^{\rm equil} < 253$

Theoretical predictions

	Standard inflation	Non-standard scenario
Single field	$f_{NL}^{\text{local, equil}} = O(\varepsilon,\eta) \square\ 1$	K-inflation, DBI inflation $f_{NL}^{\text{equil}} = 1/c_s^2 \square\ 1$ Features in potential Ghost inflation
Multi field	$f_{NL}^{\text{local}} = O(1)$ depending on the trajectory Rigopoulos, Shellard, van Tent ’06 Wands and Vernizzi ’06 Yokoyama, Suyama and Tanaka ‘07	DBI inflation $f_{NL}^{\text{equil}} = (1/c_s^2)/(1+T_{RS}^2) > 1$ curvaton $f_{\text{NL}}^{\text{local}} \square\ (5/4)\left(\rho/\rho_{curvaton}\right)_{decay}$ new Ekpyrotic (simplest model) $f_{NL}^{local} > (n_s - 1)^{-1}$ isocurvature perturbations (axion CDM) $f_{NL}^{local} \square\ 10^5 \alpha^3$

Three examples for non-standard scenarios

- (multi-field) K-inflation, DBI inflation

  Arroja, Mizuno, Koyama 0806.0619 JCAP

- (simplest) new ekpyrotic model

  Koyama, Mizuno, Vernizzi, Wands 0708.4321 JCAP

- (axion) CDM isocurvature model

  Hikage, Koyama, Matsubara, Takahashi, Yamaguchi
  (hopefully) to appear soon

K-inflation

- Non-canonical kinetic term

Amendariz-Picon et.al '99

$$S = \int d^4x\sqrt{-g}P(X,\phi),\ X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

- Field perturbations (leading order in slow-roll)

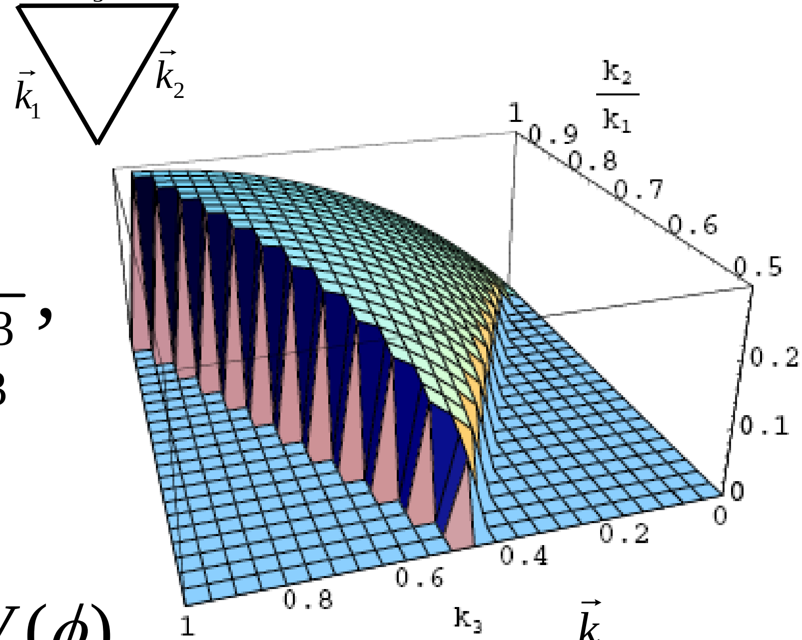
$$P_\zeta \simeq \frac{1}{c_s\varepsilon}\left(\frac{H}{M_{pl}}\right)^2, \quad r = \frac{P_T}{P_\zeta} = 16c_s\varepsilon$$

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$ sound speed

Garriga&Mukhanov '99

Bispectrum

LoVerde et.al. '07



$$B(k_1,k_2,k_3) = F(k_1,k_2,k_3)\frac{P_\zeta^{\ 2}}{k_1^3 k_2^3 k_3^3},$$

- DBI inflation

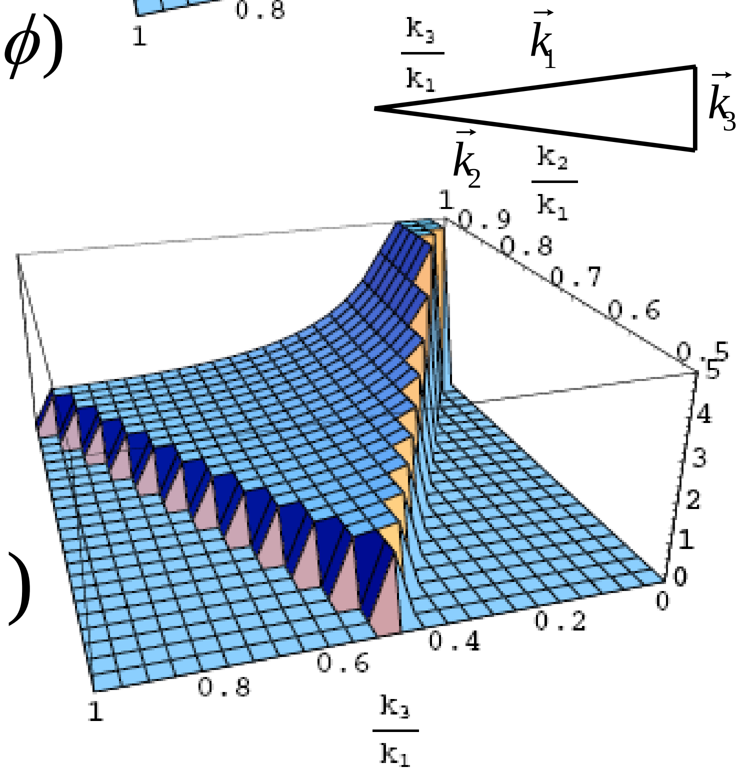
Aishahiha et.al. '04

$$P(X) = -\frac{1}{f(\phi)}\left(\sqrt{1-2f(\phi)X}-1\right)-V(\phi)$$

$$\Rightarrow \quad F(k_1,k_2,k_3) \propto \frac{1}{c_s^2}$$

cf local-type

$$F^{\text{local}}(k_1,k_2,k_3) = \frac{3}{10} f_{\text{NL}}^{\text{local}}(k_1^3+k_2^3+k_3^3)$$



Observational constraints

- (too) large non-Gaussianity

$$f_{NL}^{\text{eff}} = -\frac{35}{108}\frac{1}{c_s^2} \qquad \text{for equilateral configurations}$$

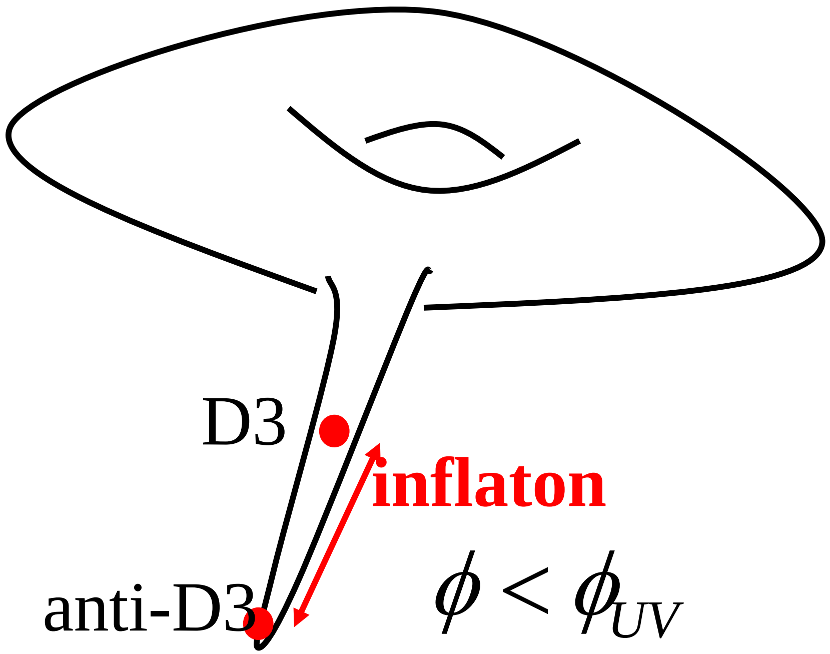
- Lyth bound

$$\frac{1}{M_p^2}\left(\frac{\Delta\phi}{\Delta N}\right)^2 = 6r$$

$$r = 16 c_s \varepsilon < 10^{-7}$$

$$1 - n_s \simeq 4\varepsilon \simeq 0.04 \pm 0.013$$

$$\Rightarrow \quad f_{NL}^{\text{eff}} > 300$$



Huston et.al '07, Kobayashi et.al '08

Multi-field model

Arroja, Mizuno, Koyama '08
Renaux-Petel, Steer, Langlois
Tanaka'08

$$S = \int d^4x \sqrt{-g} P(X^{IJ}, \phi), \ X^{IJ} = \frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J$$

$$P(X^{IJ}) = \tilde{P}(\tilde{X}), \quad \tilde{X} = X + \frac{b}{2}\left(X^2 - X_I^J X_J^I\right) \qquad \tilde{X}_0 = X_0$$

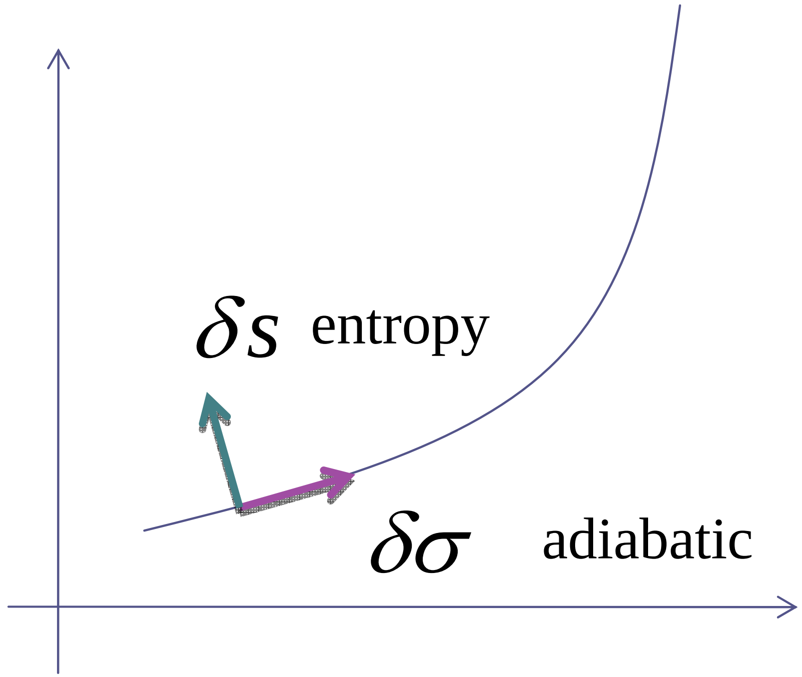
- Adiabatic and entropy decomposition

adiabatic sound speed

$$c_{ad}^2 = \frac{\tilde{P}_{,\tilde{X}}}{\tilde{P}_{,\tilde{X}} + 2X_0 \tilde{P}_{,\tilde{X}\tilde{X}}}$$

entropy sound speed

$$c_{en}^2 = 1 + bX_0$$



- Multi-field k-inflation

Langlois&Renaux-Petel'08

$$P(X^{IJ}) = P(X) \qquad c_{en}^2 = 1$$

- Multi-field DBI inflation

Renaux-Petel, Steer, Langlois Tanaka'08

$$P(X^{IJ}) = -\frac{1}{f(\phi)}\left(\sqrt{-\det(g_{\mu\nu} - f(\phi)G_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J)} - 1\right) - V(\phi)$$

$$c_{en}^2 = c_{ad}^2$$

Final curvature perturbation

$$\zeta = \zeta_* + T_{RS} S_* \qquad \zeta = \frac{H}{\dot{\sigma}}\delta\sigma, \quad S = \frac{H}{\dot{s}}\delta s$$

Transfer from entropy mode T_{RS} Renaux-Petel, Steer, Langlois Tanaka'08

- Tensor to scalar ratio

$$r = 16\varepsilon c_s \frac{1}{1+T_{RS}^2}$$

- Bispectrum

k-dependence is the same as single field case!

$$f_{NL}^{\mathrm{eff}} = -\frac{35}{108}\frac{1}{c_s^2}\frac{1}{1+T_{RS}^2}$$

$$f_{NL}^{\mathrm{eff}} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2}, \quad \langle \zeta^2 \rangle \propto 1+T_{RS}^2, \quad \langle \zeta^3 \rangle \propto 1+T_{RS}^2$$

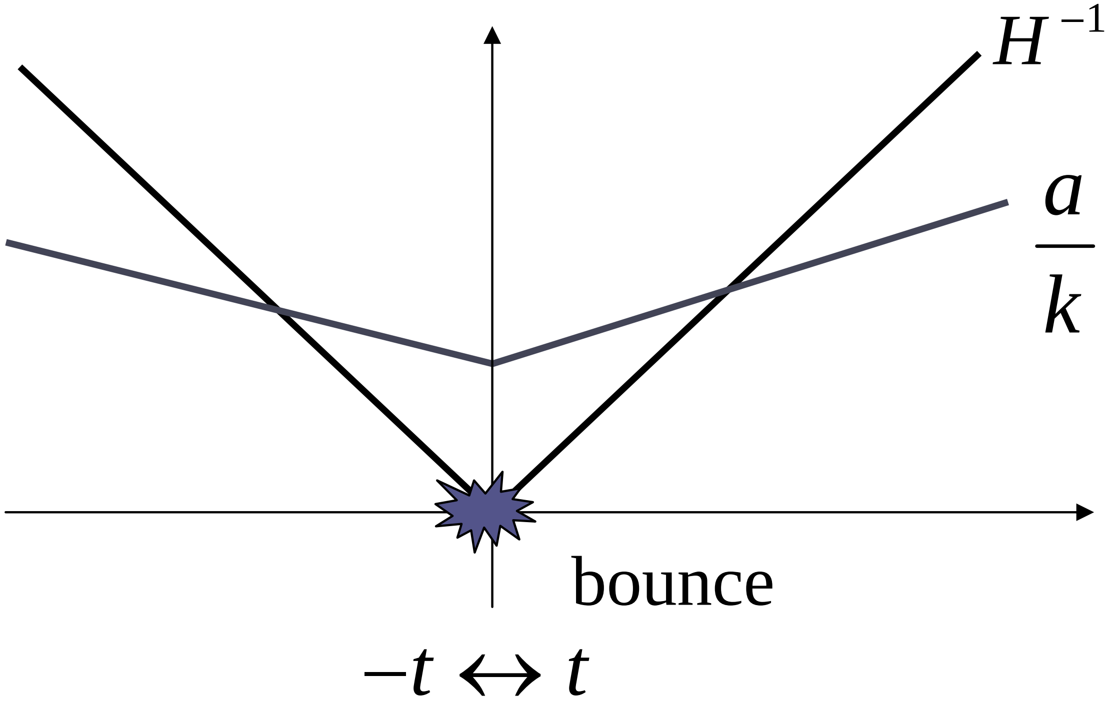
large transfer from entropy mode eases constraints

- Trispectrum Mizuno, Koyama, Arroja '09

different k-dependence from single field case?

New ekpyrotic models

- Collapsing universe



- Ekpyrotic collapse $a(t) = (-t)^n, \quad n$ □ 1

Khoury et.al. '01

- Old ekpyrotic model Khoury et.al. ‘01

$$V = -V_0 e^{-c\varphi} \qquad a(t) = (-t)^{2/c^2}$$

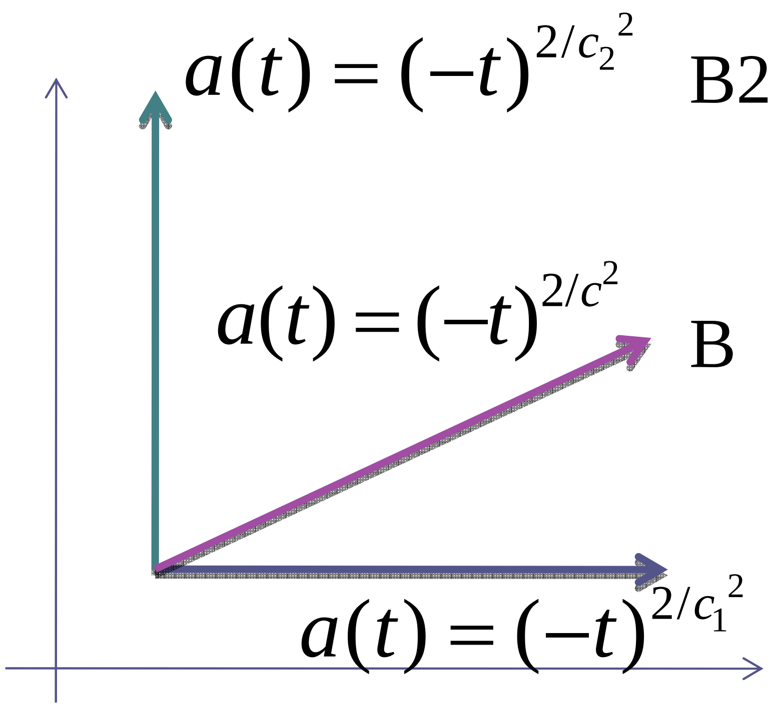
spectrum index for ζ is $n_s = 3$ Lyth ‘03

the bounce may be able to creat a scale invariant spectrum but it depends on physics at singularity

- New ekpyrotic model

$$V = -V_1 e^{-c_1\varphi_1} - V_2 e^{-c_2\varphi_2}$$

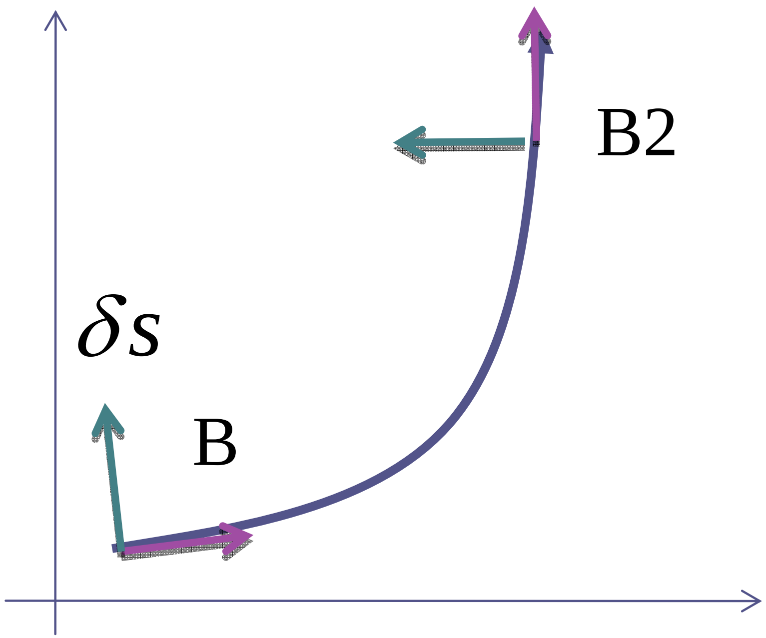
$$a(t) = (-t)^{2/c^2}, \frac{1}{c^2} = \frac{1}{c_1^2} + \frac{1}{c_2^2}$$

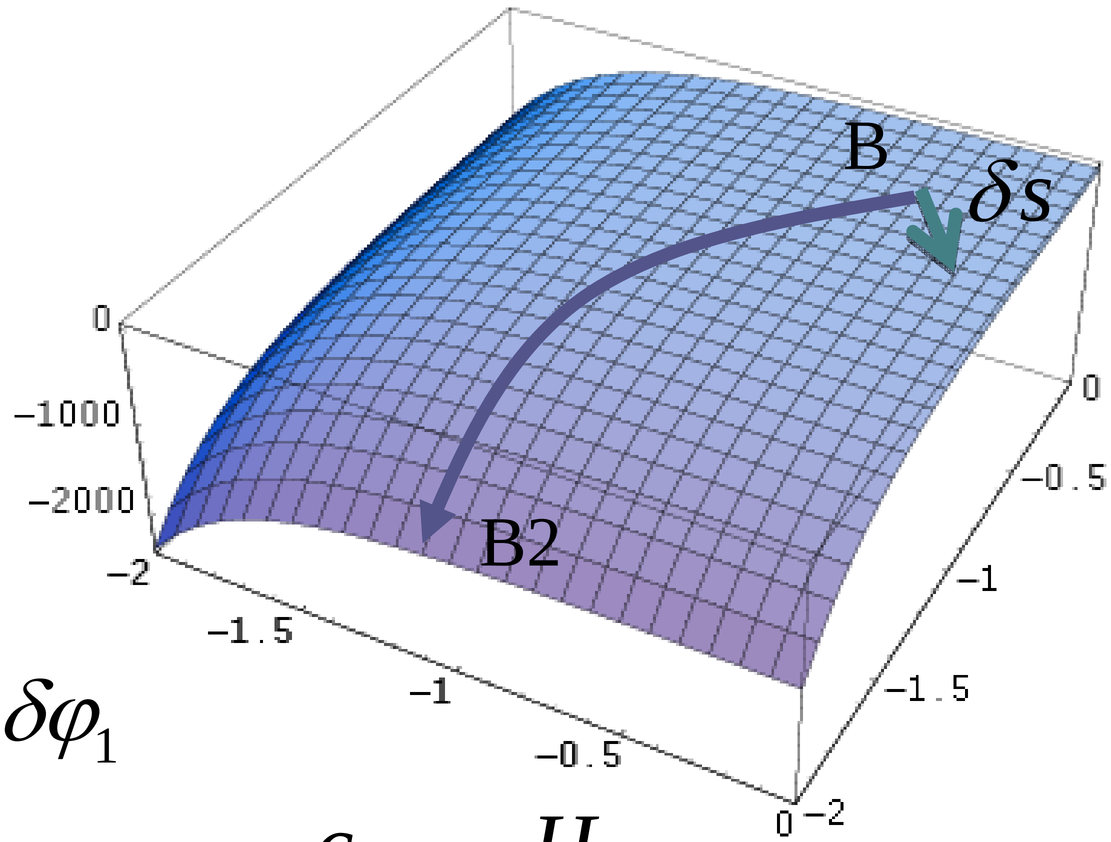


Lehners et.al , Buchbinder et.al. ‘07

- Multi-field scaling solution is unstable

entropy perturbation which has a scale invariant spectrum
is converted to adiabatic perturbations Koyama, Mizuno & Wands' 07





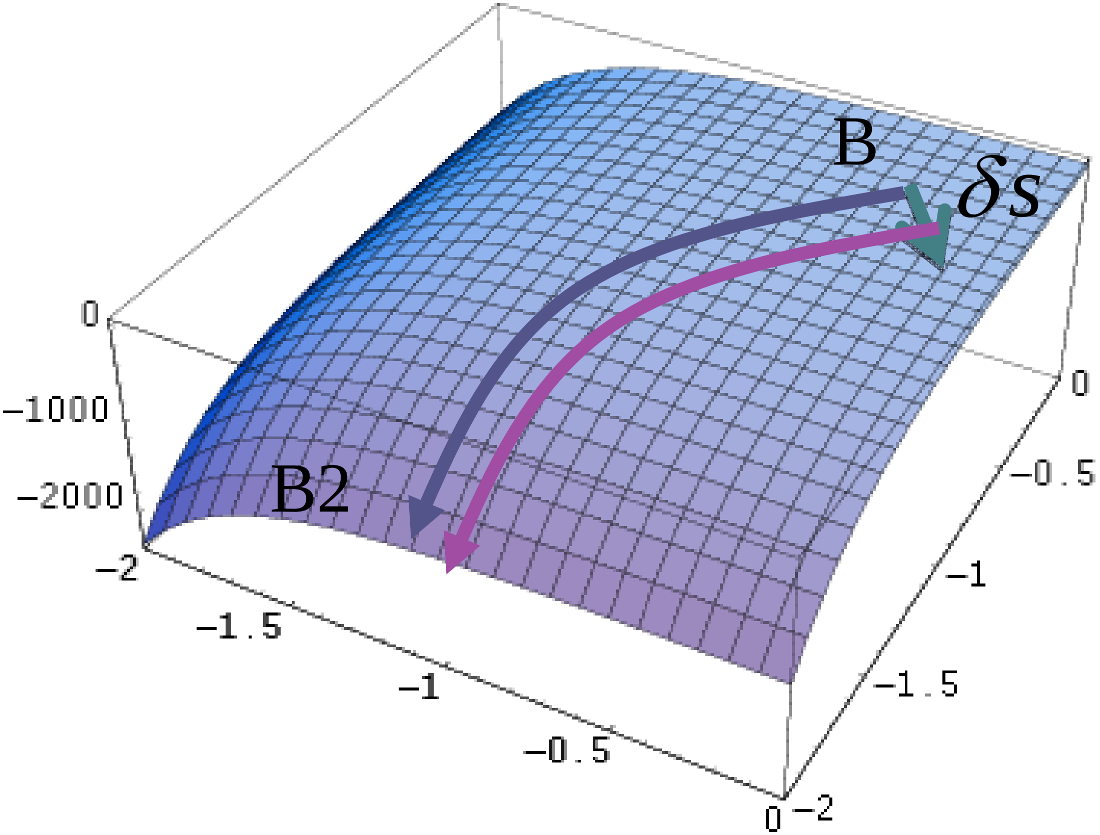
B: $$\delta s = \frac{c^2}{2}\frac{H}{2\pi}$$

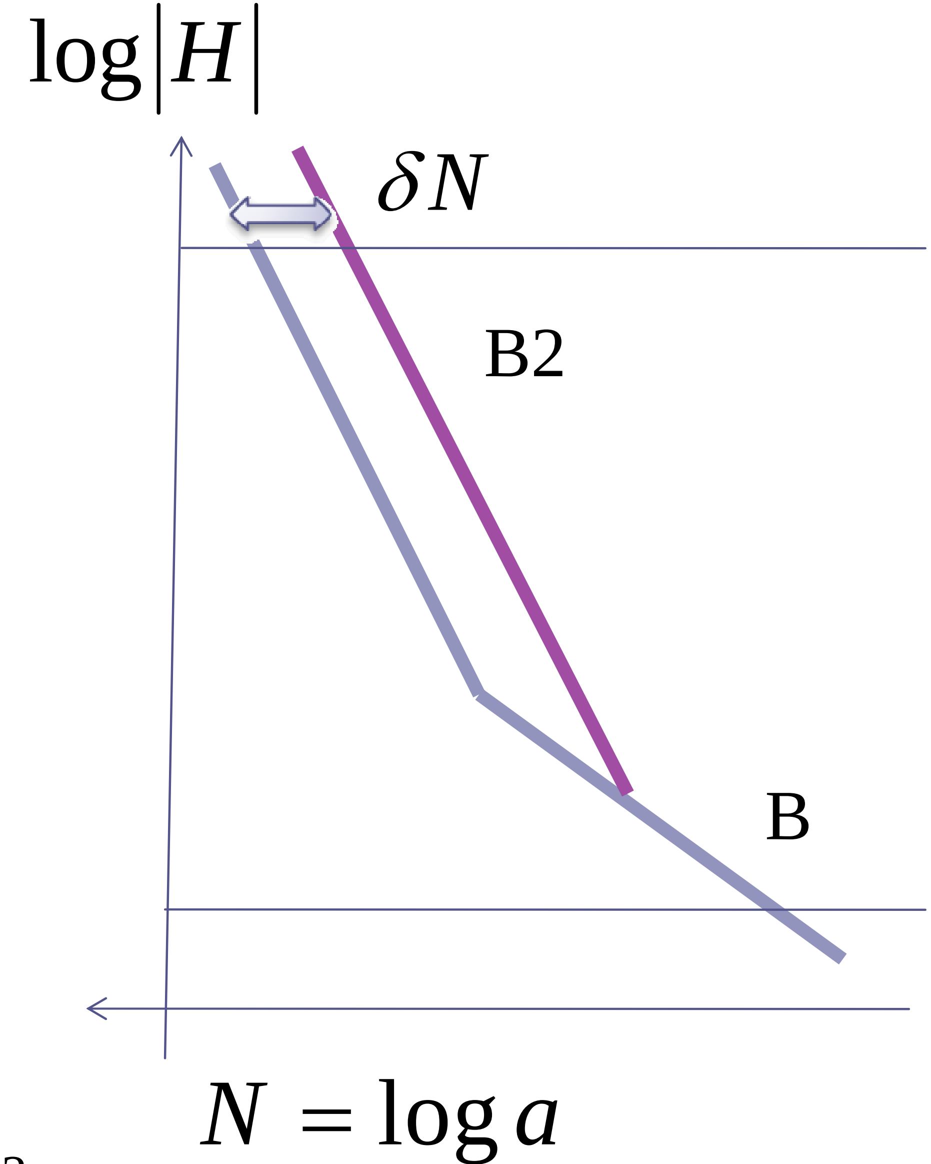
$$\zeta \simeq 0$$

B2: $$\delta s \propto \delta\varphi_1$$

$$\zeta \propto \delta\varphi_2 = \frac{c_2}{\sqrt{c_1^2 + c_2^2}}\frac{H_T}{2\pi}$$

- Delta-N formalism





$$\delta N = \frac{dN}{ds}\delta s + \frac{1}{2}\frac{d^2 N}{ds^2}\delta s^2$$

- Predictions   Koyama, Mizuno, Vernizzi & Wands' 07

$$n_s - 1 = 4\left(\frac{1}{c_1^2} + \frac{1}{c_2^2}\right) > 0$$

$$r = 0$$

$$f_{\rm NL}^{\rm local} = \frac{5}{12} c_1^2 > \frac{5}{3}(n_s - 1)^{-1}$$

- Generalizations

  changing potentials   Buchbinder et.al 07

  conversion to adiabatic perturbations in kinetic domination   Lehners &Steinhardt '08

Non-Gaussianity from isocurvature perturbations

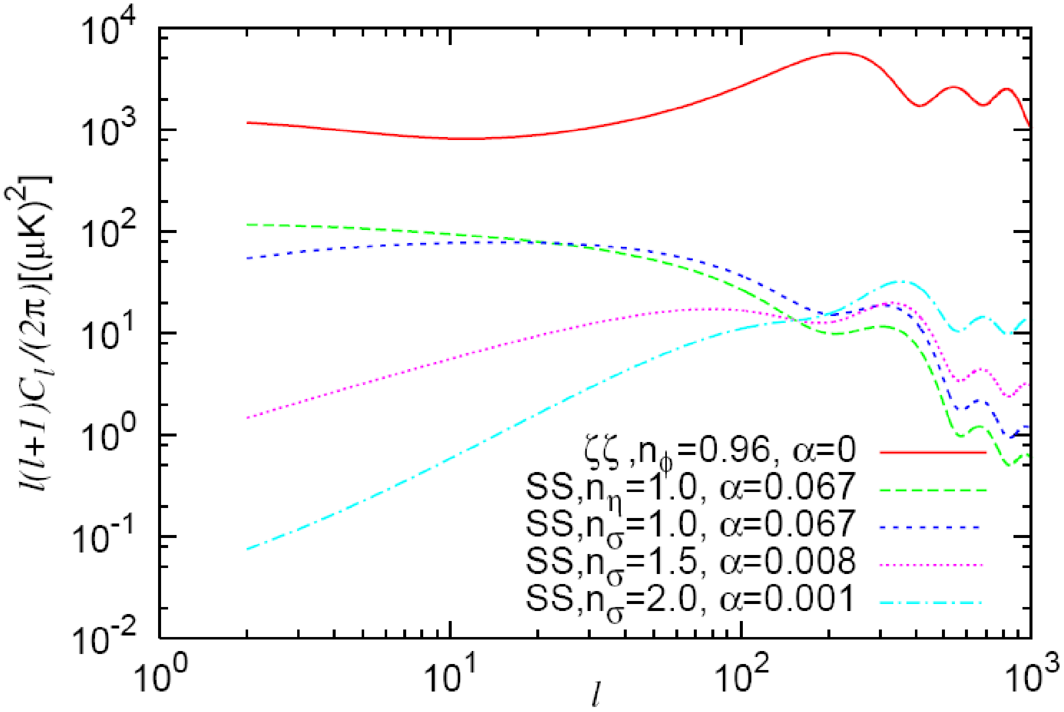
- (CDM) Isocurvature perturbations are subdominant

$$\alpha = \frac{P_S}{P_S + P_\zeta} < 0.067 \quad (n_S = 1)$$

$$< 0.008 \quad (n_S = 1.5)$$

$$< 0.001 \quad (n_S = 2)$$

- Non-Gaussianity can be large



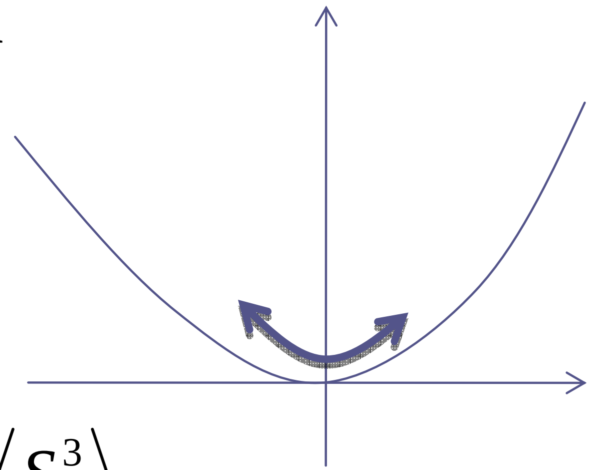
Boubekeur& Lyth '06, Kawasaki et.al '08, Langlois et.al '08

Axion CDM

- Massive scalar fields without mean

$$\delta\rho_{CDM} \sim m^2\sigma^2$$

Linde&Mukhanov ‘97, Peebles ’98, Kawasaki et.al‘08



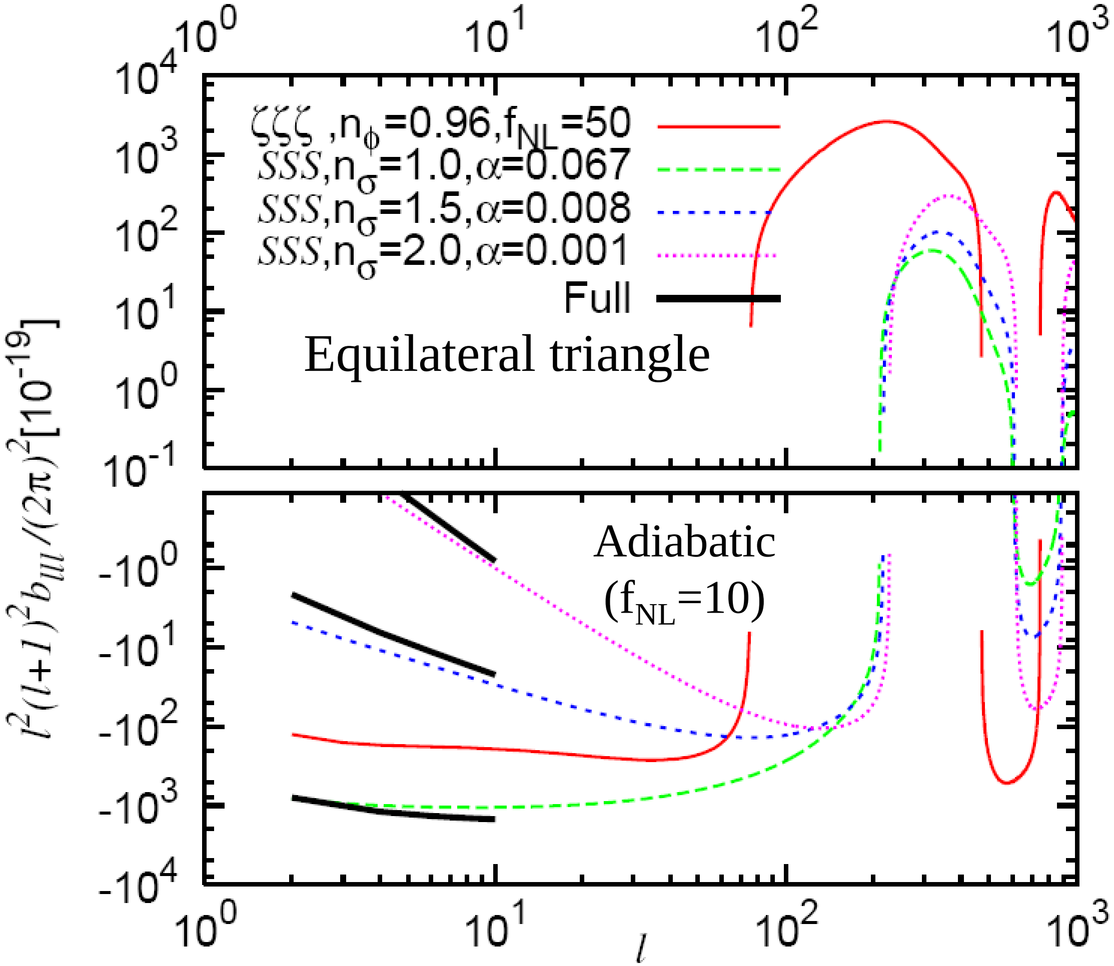
- Entropy perturbations

$$S = \frac{\delta\rho_{CDM}}{\rho_{CDM}} - \frac{3\delta\rho_r}{4\rho_r} \equiv \eta_g{}^2 - \left\langle \eta_g{}^2 \right\rangle \qquad \frac{\left\langle S^3 \right\rangle}{\left\langle S^2 \right\rangle^{3/2}} \sim O(1)$$

- Dominant nG may come from isocurvature perturbations

$$f_{NL}^{\mathrm{eff}} \sim \frac{\left\langle S^3 \right\rangle}{\left\langle \zeta^2 \right\rangle^2} \sim \alpha^{\frac{3}{2}} \frac{1}{\left\langle \zeta^2 \right\rangle^{1/2}} \sim \alpha^{\frac{3}{2}} 10^5$$

Bispectrum of CMB from the isocurvature perturbation



$$\left(\frac{S}{N}\right)^2 = \sum_{2\le l_1\le l_2\le l_3} I^2_{l_1 l_2 l_3} \frac{b_{l_1 l_2 l_3}{}^2}{C_{l_1} C_{l_2} C_{l_3} \Delta_{l_1 l_2 l_3}}$$

Noise: WMAP 5-year

$$\left(\frac{S}{N}\right)_{adi} = \left(\frac{S}{N}\right)_{iso}$$

$$f_{NL}^{\text{eff}} = 41\left(\frac{\alpha}{0.067}\right)^{3/2}$$

The isocurvature perturbations can generate large non-Gaussianity (f_{NL}~40)

WMAP5 constraints -Minkowski functional

Hikage, Koyama, Matsubara, Takahashi, Yamaguchi '08

- Minkowski functional measures the topology of CMB map (=weighted some of bispectrum)

no detection of non-G from isocurvature perturbations and get constraints

$$\alpha < 0.070 \quad (n_\eta = 1)$$

$$\alpha < 0.042 \quad (n_\eta = 1.5)$$

$$\alpha < 0.0064 \quad (n_\eta = 2)$$

comparable to constraints from power spectrum

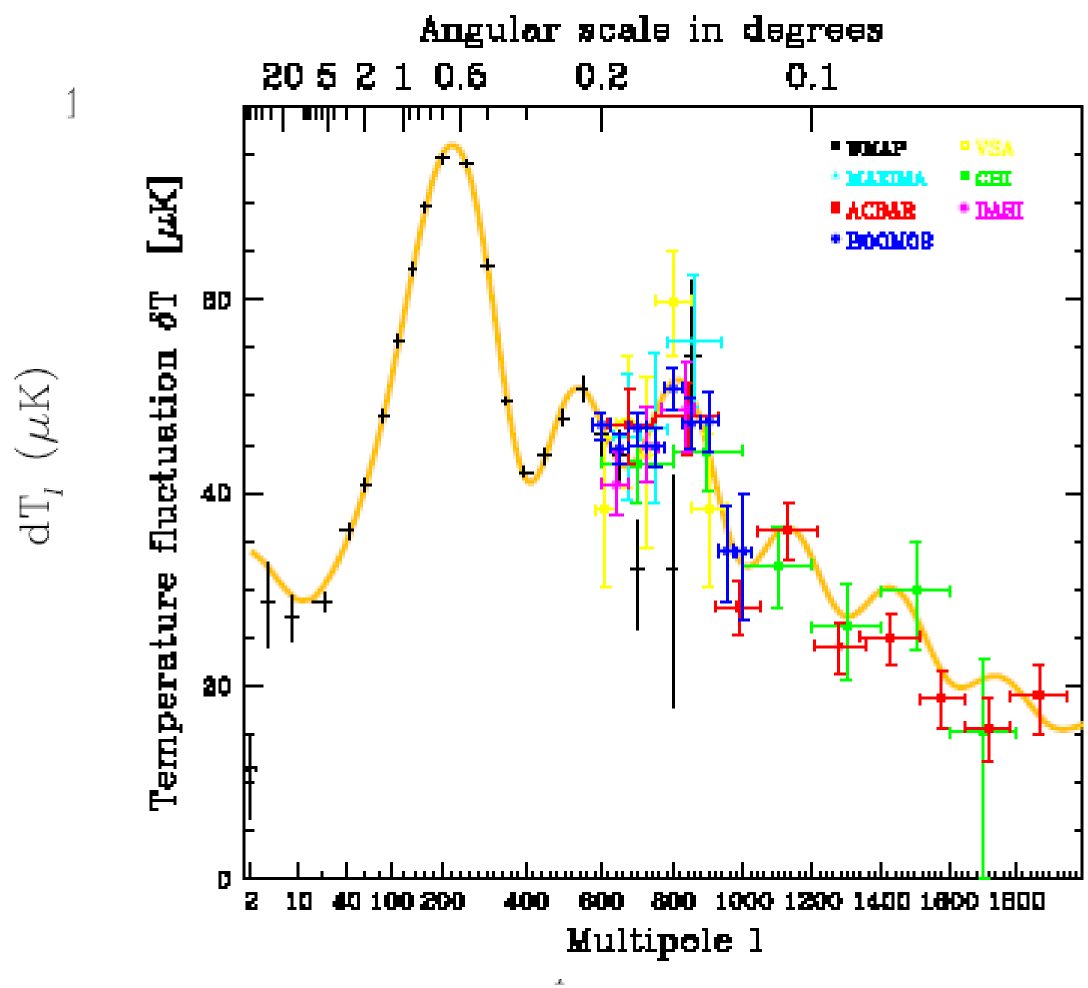
Theoretical predictions

	Standard inflation	Non-standard scenario
Single field	$f_{NL}^{\text{local, equil}} = O(\varepsilon, \eta) \ll 1$	K-inflation, DBI inflation $f_{NL}^{\text{equil}} = 1/c_s^2 \gg 1$ Features in potential Ghost inflation
Multi field	$f_{NL}^{\text{local}} = O(1)$ depending on the trajectory	DBI inflation $f_{NL}^{\text{equil}} = (1/c_s^2)/(1+T_{RS}^2) > 1$ curvaton $f_{\text{NL}}^{\text{local}} \sim (5/4)\left(\rho/\rho_{curvaton}\right)_{decay}$ new Ekpyrotic (simplest model) $f_{NL}^{local} > (n_s - 1)^{-1}$ isocurvature perturbations (axion CDM) $f_{NL}^{local} \sim 10^5 \alpha^3$

Conclusions

- Power spectrum from pre-WMAP to post-WMAP

- bispectrum
  WMAP 8year
  Planck



Do everything you can now!

Axion CDM

Classical mean of axion $a = f_a \theta_a$

quantum fluctuations $\left\langle \delta a^2 \right\rangle = H_{\rm inf} / 2\pi$

if $f_a \theta_a < H_{\rm inf} / 2\pi$

$$\frac{\delta \rho_a}{\rho_a} = \left(\frac{\delta a}{a_*} \right)^2 , \quad a_* = \frac{H_{\rm inf}}{2\pi}$$

$$S = r \frac{\delta \rho_a}{\rho_a} , \quad r = \frac{\Omega_a}{\Omega_{\rm cdm}}$$

$$r = 1.8 \times 10^{-2} \left(\frac{F_a}{10^{12}\,\mathrm{GeV}} \right)^{-0.82} \left(\frac{a_*}{10^{11}\,\mathrm{GeV}} \right)^2 \left(\frac{0.11}{\Omega_{\rm cdm} h^2} \right)$$