Particle production induced by Vacuum decay in Real time formalism

arXiv:2006.10986 [hep-th]

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Motivation

L. Parker, *Phys. Rev.* **183** (1969) 1057.

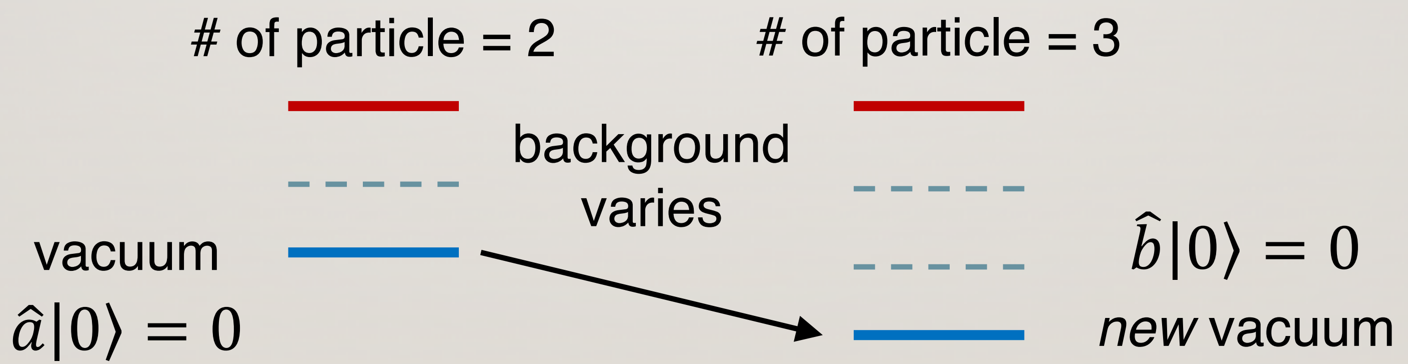
DYNAMICAL BACKGROUND & PARTICLE PRODUCTION

Background field varies

⇒ definition of “vacuum” changes

⇒ particle production

This **inevitably** occurs!



L. H. Ford, *Phys. Rev. D* **35** (1987) 2955. *etc.*

DYNAMICAL BACKGROUND & PARTICLE PRODUCTION

The **more sharply** background field varies,

the **more** particles are produced

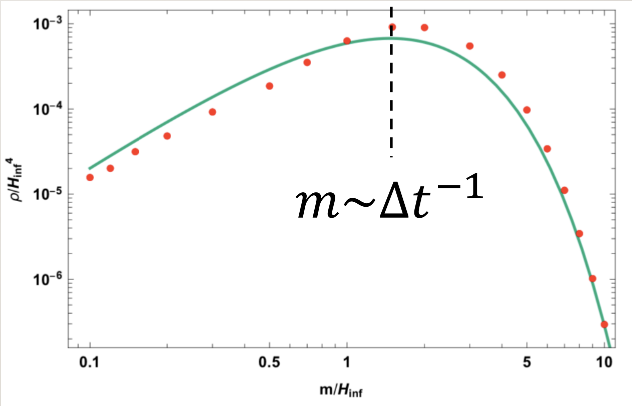
e.g. fermion production after inflation

$$\boldsymbol{\rho \cong 2\times 10^{-3} e^{-4m\Delta t} m^2 H_{inf}^2}$$

m : fermion mass

$\boldsymbol{\Delta t}$: transition time scale from inflation

H_{inf} : Hubble parameter during inflation



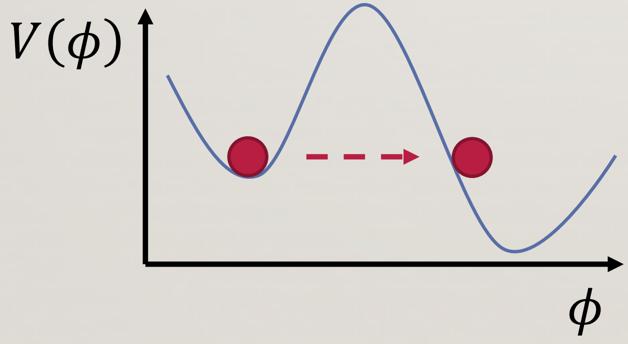
SH and J. Yokoyama, *Phys. Lett. B* **798** (2019) 135024.

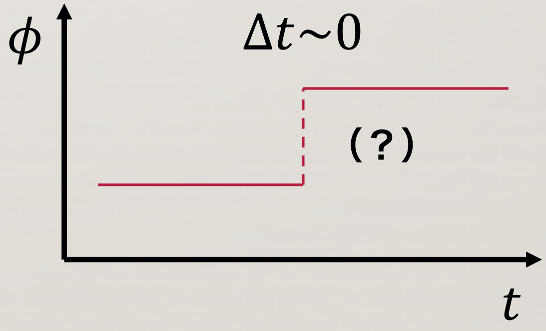
QUANTUM TUNNELING & PARTICLE PRODUCTION

Quantum tunneling may seem “instantaneous” in real time

⇒ Does this mean $\boldsymbol{\Delta t \sim 0}$ and **effective** particle production?

We may have to take particle production into account when discussing tunneling! *e.g.* **vacuum decay**





Analysis

OUR MODEL

Background field: ϕ

Coupled field: χ ← to be produced

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\underline{(M_0^2 + g\phi^2)}\chi^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \underline{V(\phi)}$$

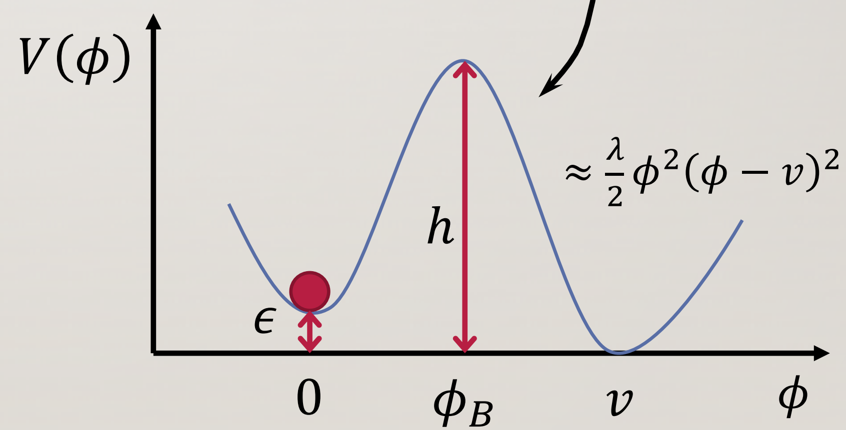
effective mass of χ

$\equiv M_\chi^2$

⇓

effective frequency of χ

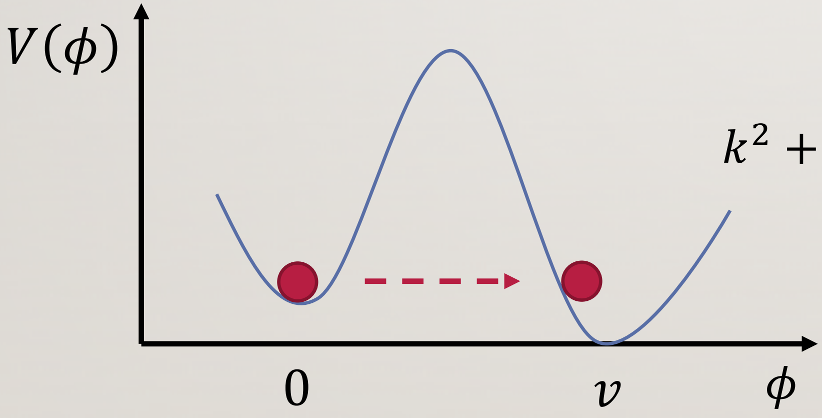
$\omega_k^2(t) \equiv k^2 + M_\chi^2(t)$

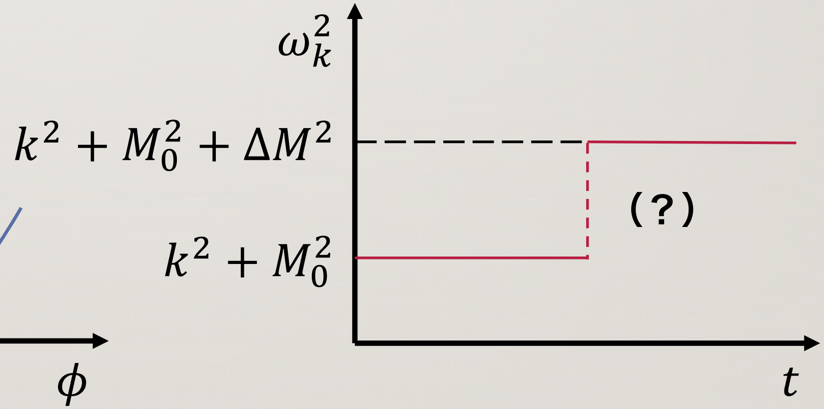


OUR MODEL

When ϕ tunnels from $\phi = 0$ to $\phi = v$,

M_χ^2 changes from M_0^2 to $M_0^2 + gv^2 \equiv M_0^2 + \Delta M^2$





Tunneling of background field $\Rightarrow$ Change of effective mass

EVOLUTION OF MODE FUNCTION

χ's mode function χ_k obeys the following EoM:

$$\frac{d^2\chi_k(t)}{dt^2} + \omega_k^2(t)\chi_k(t) = 0$$

The time-varying ω_k^2 **displaces** χ_k from the vacuum state

$$\chi_k(t_f) = \alpha_k \chi_{k,\mathrm{vac}}(t_f) + \beta_k \chi_{k,\mathrm{vac}}^*(t_f)$$

$|\beta_k|^2$ = produced particle number density n_k

The solution of EoM $\boldsymbol{\chi_k(t)}$ tells us amount of produced χ

EVOLUTION OF MODE FUNCTION

Therefore, essential problems are:

- How to describe $\omega_k(t)$ during tunneling of ϕ?
- How to solve the EoM of χ_k under $\omega_k(t)$?

How to describe $\omega_k(t)$ during tunneling of ϕ?

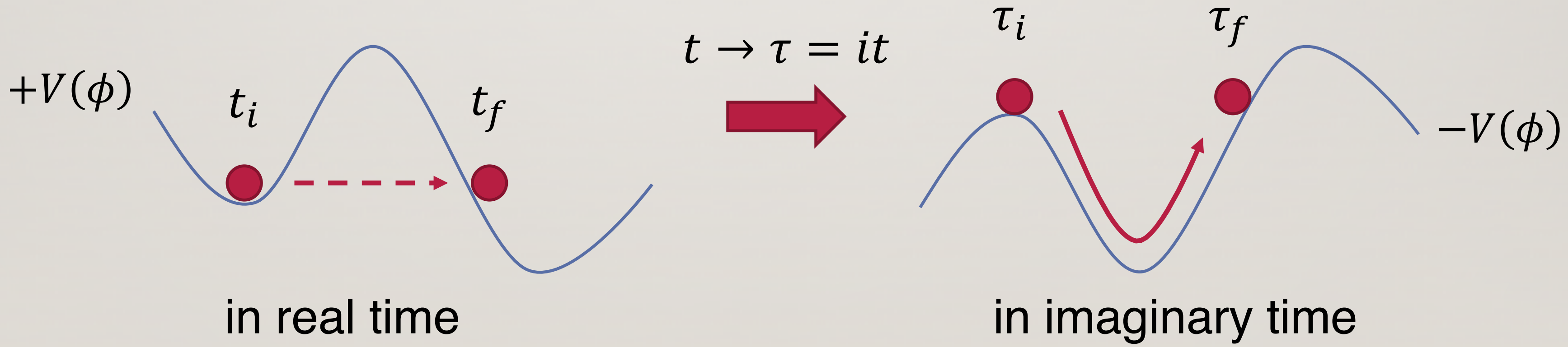
S. Coleman, *Phys. Rev. D* **15** (1977) 2929.

IMAGINARY TIME FORMALISM

Quantum tunneling

$\Downarrow$ Wick rotation $t \to \tau = it$

Semi-classical motion **with upside-down potential $-V(\phi)$ in imaginary time τ** (bounce solution)



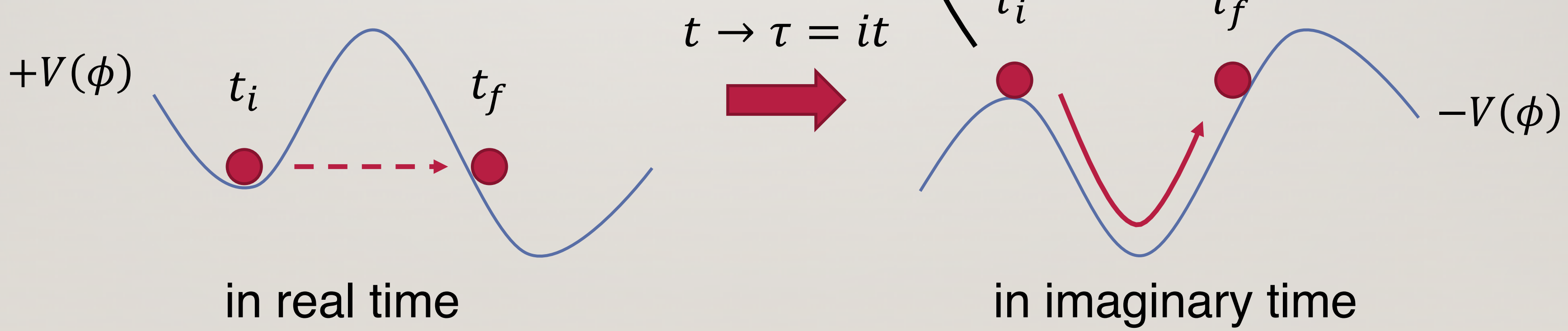
V. A. Rubakov, *Nucl. Phys. B* **245** (1984) 481. *etc.*

IMAGINARY TIME FORMALISM

⇒ ω_k can also be described by τ

$$\frac{d^2\chi_k(\tau)}{d\tau^2} - \left(k^2 + M_0^2 + g\phi^2(\tau)\right)\chi_k(\tau) = 0$$

substitute the bounce solution $\phi(\tau)$



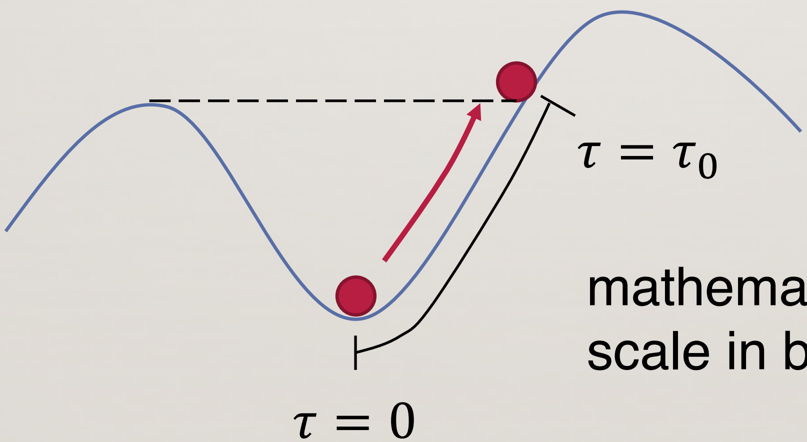
V. A. Rubakov, *Nucl. Phys. B* **245** (1984) 481. *etc.*

IMAGINARY TIME FORMALISM

"*Imaginary* transition time scale τ_0" gives a exponential suppression $e^{-4\omega_k\tau_0}$ on particle production

(Rubakov 1984, Yamamoto+ 1995)

What is a *physical* meaning of this τ_0?



mathematically, τ_0 is a time scale in bounce solution

J. Blanco-Pillado+, *JCAP* **12** (2019) 001. *etc.*

REAL TIME FORMALISM

Quantum tunneling **in real time**

- Wigner function method (Hertzberg & Yamada 2019)
- Flyover vacuum decay (J. Blanco-Pillado+ 2019)

We use this

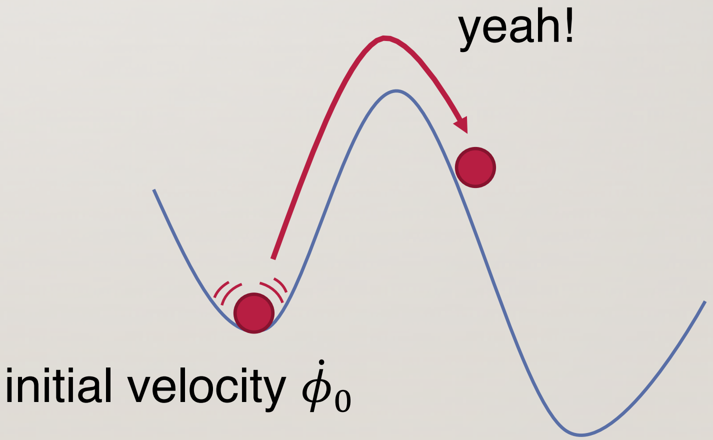
J. Blanco-Pillado+, *JCAP* **12** (2019) 001. *etc.*

FLYOVER VACUUM DECAY

Concept

ϕ obtains **initial velocity $\dot{\phi}_0$** from **quantum fluctuation**

⇒ If $\dot{\phi}_0 > \dot{\phi}_{th}$, ϕ *semi-classically* climbs over the barrier



J. Blanco-Pillado+, *JCAP* **12** (2019) 001. *etc.*

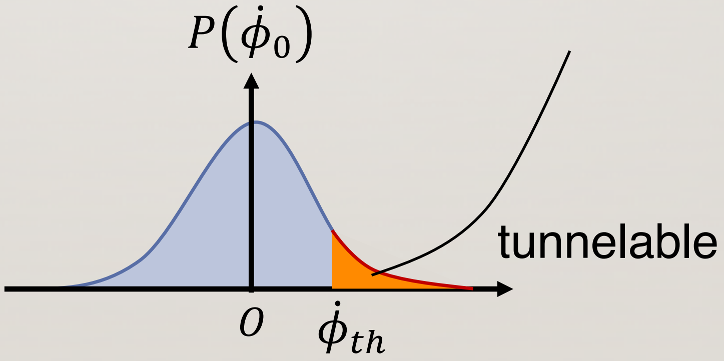
FLYOVER VACUUM DECAY

Concept

Initial velocity $\dot{\phi}_0$ ← distribution function $P(\dot{\phi}_0)$

⇒ Tunneling rate is derived as **a probability of $\boldsymbol{\dot{\phi}_0 > \dot{\phi}_{th}}$**

$$= \int_{\dot{\phi}_{\mathrm{th}}}^{\infty} P(\dot{\phi}_0) d\dot{\phi}_0$$



PROCEDURE

With this formalism, particle production is derived as

1. initial velocity $\dot{\phi}_0$ + EoM of ϕ $\Rightarrow$ **background** $\boldsymbol{\phi(t, \dot{\phi}_0)}$
2. $\phi(t, \dot{\phi}_0)$ + EoM of χ_k $\Rightarrow$ **solution of EoM** $\boldsymbol{\chi_k(t, \dot{\phi}_0)}$
3. $\chi_k(t, \dot{\phi}_0)$ $\Rightarrow$ **produced particle number density** $\boldsymbol{n_k(\dot{\phi}_0)}$
4. $n_k(\dot{\phi}_0)$ + probability distribution of initial velocity $P(\dot{\phi}_0)$

   $\Rightarrow$ **expected value** $\boldsymbol{\langle n_k \rangle}$ $\left(= \int n_k(\dot{\phi}_0) P(\dot{\phi}_0) d\dot{\phi}_0\right)$

How to solve the EoM of χ_k under $\omega_k(t)$?

N. Birrell and P. Davies, *Quantum Fields in Curved Space* (1984).

WKB APPROXIMATION

Exact solution of EoM ⇐ almost impossible…

WKB approximation

$$\chi_k(t) = \frac{c_1}{\sqrt{2W_k}} \exp\left[-i\int^t W_k(t')dt'\right] + \frac{c_2}{\sqrt{2W_k}} \exp\left[+i\int^t W_k(t')dt'\right]$$

$$W_k^{(0)} = \omega_k$$

$$\left(W_k^{(n+1)}\right)^2 \quad \cdots \quad {}^2 \quad \cdots \quad 1\left(W_k^{(n)''} \quad \cdots \quad \frac{3}{2}\left(W_k^{(n)'}\right)^2 \quad \cdots\right)$$

But this is *divergent* series

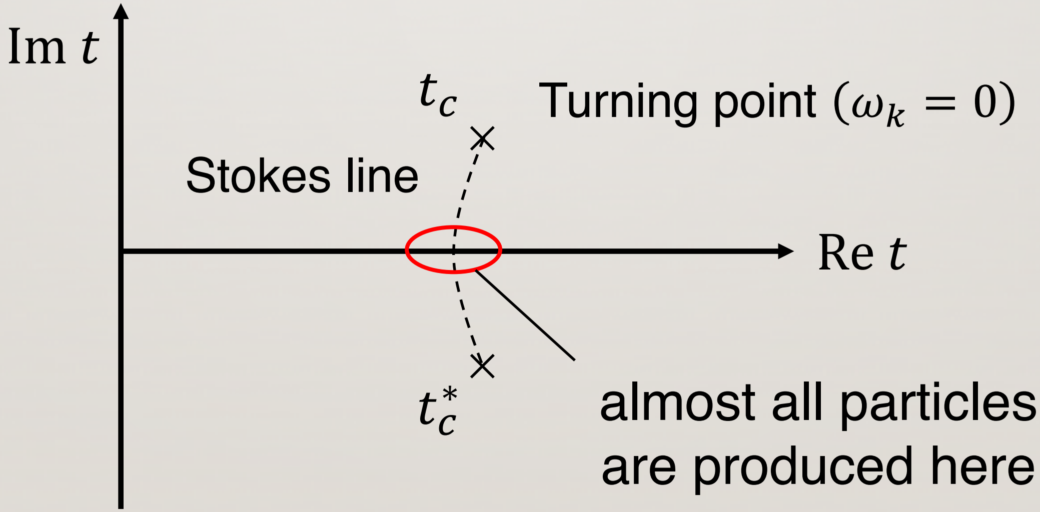
It can't include **non-perturbative** effect

N. Fröman and Ö. Dammert, *Nucl. Phys.* **A147** (1970) 627.

STOKES PHENOMENON METHOD

Particle production occurs mainly around **Stokes lines** connecting pairs of **turning points** t_c where $\omega_k(t_c) = 0$ in *complex* time

$=$ **Stokes phenomenon** in math.



C. Dumlu and G. Dunne, *Phys. Rev. Lett.* **104** (2010) 250402.

STOKES PHENOMENON METHOD

Stokes phenomenon tells us produced particle number density as

$$n_k \approx \left| \sum_p \underbrace{\exp\left(i \int_{t_{c_p}}^{t_{c_p}^*} \omega_k(t)dt \right)} \underbrace{\exp\left(2i \int_{s_1}^{s_p} \omega_k(t)dt \right)} \right|^2$$

particle production around p-th Stokes line

interference between Stokes lines

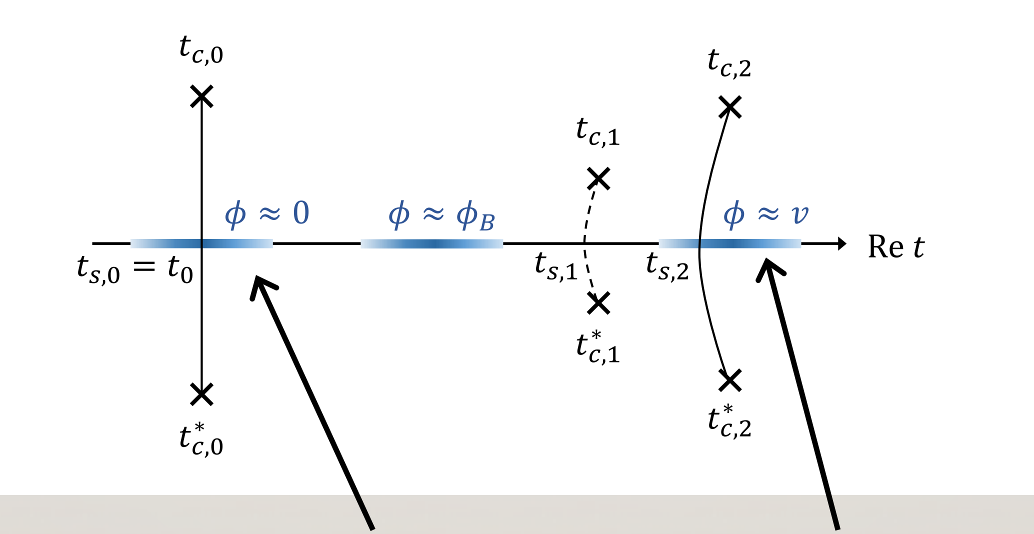
Non-perturbative effect **included**!

(Dabrowski & Dunne 2014)

arXiv:2006.10986 [hep-th]

STOKES PHENOMENON METHOD

- Where are Stokes lines in our case?
  = When are χ particles produced?



When ϕ is around each vacuum (potential minimum) arXiv:2006.10986 [hep-th]

RESULT

- How many χ particles are produced?

$$\langle n_k \rangle \approx \Gamma\left(\exp\left[-2\Delta \ln\left(\frac{512}{e^2}\Delta^2\right)\sqrt{\frac{g}{\lambda}}\right] + \exp\left[-D\Delta^{1+\delta}\sqrt{\frac{g}{\lambda}}\right]\right)$$

Γ : decay rate

$D(\approx 10.5), \delta(\approx 0.13)$: numerical factor

$\Delta^2 = \omega_k^2/\Delta M^2$

In reality, particle production is suppressed exponentially (or even stronger)

SH, Y. Yamada and J. Yokoyama in prep.

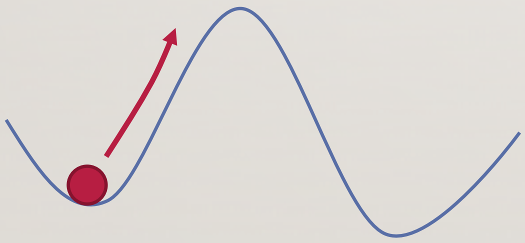
RESULT

- Backreaction on vacuum decay (preliminary)

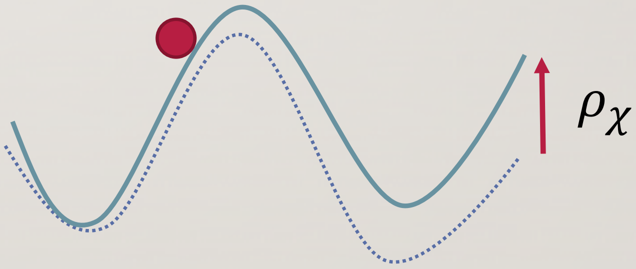
$$\Gamma \;\to\; \left(1 - 40\pi^3 \frac{\rho_\chi}{m_F}\right)\Gamma$$

Is this consistent with imaginary time formalism?

⇒ validity of real time formalism



χ particles are produced



$V(\phi)$ is lifted up by ρ_χ

SUMMARY

- Particle production induced by vacuum decay
  by using
  - real time formalism (flyover vacuum decay)
  - Stokes phenomenon method
- Backreaction on vacuum decay (preliminary)

Future work

- Backreaction (more detailed)
- Application to Higgs instability
- Play with Stoke phenomenon method!