

# Perturbations of Hairy Black Holes in Shift-symmetric Scalar-Tensor theories

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# Introduction

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## **General Relativity (GR) 100th anniversary**

most successful gravitational theory **"now"**

## **Modified Gravity (MG) explains accelerating Universe**

additional field, higher dimension, Lorentz violation, etc...

## **How to compare** MG with GR?

## **Astrophysical tests**

"gravitational waves

"not studied in MG

## **using Black Holes**

# Motivation

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## BH Hair

Many ST theory: No-hair theorem

(Brans Dicke, K-essence, Galileon, etc...)

mass, charge, angular momentum

Bekenstein (1995); Hui, Nicolis(2014)...

Shift-symmetric ST theory **with time-dependent scalar field**

 BH solution found with non-trivial **scalar hair**

(this scalar is regular at the horizon)

Bavichev, Charmousis(**2014**)

## Stealth Schwarzschild

Schwarzschild metric with nontrivial configuration of the scalar field

## Self-tuned Schwarzschild-de-sitter

screening of the bare huge cosmological constant

# Motivation

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***However...***

**GR Schwarzschild**

**Stealth Schwarzschild**

**Distinguishable?**



**Perturbation**

**Stability?**

**Can exist?**

**Gravitational waves?**

Dressing BH in  
**Shift-symmetric**  
**Scalar-Tensor Theory**

# Dressing BH in Shift-symmetric ST theory

Shift & reflection symmetry:  $\phi \rightarrow \phi + \text{const.}$ ,  $\phi \rightarrow -\phi$

$$\mathcal{L} = [\zeta R - \eta(\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2\Lambda] \quad \zeta > 0, \eta, \beta : \text{const}$$

$\Lambda : \text{cosmological constant}$

Shift symmetry

EOM for scalar

$$\phi \rightarrow \phi + \text{const.} \longrightarrow \nabla_\mu J^\mu = 0$$

$$J^\mu = (\eta g^{\mu\nu} - \beta G^{\mu\nu}) \partial_\nu \phi$$

## Assumptions

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$J^r = 0 \longrightarrow \text{Current } J^2 = J_\mu J^\mu \text{ regular at the horizon}$$

$$\phi(t, r) = qt + \psi(r) \longrightarrow \text{Space-time is static in Shift-symmetric theory}$$

# Dressing BH in Shift-symmetric ST theory

$$\mathcal{L} = [\zeta R - \eta(\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu\phi\partial_\nu\phi - 2\Lambda] \quad \phi(t, r) = qt + \psi(r)$$

## Stealth Schwarzschild

$$f(r) = h(r) = 1 - \frac{\mu}{r} \quad \mu : \text{const.}$$

$$\phi_\pm = qt \pm q\mu \left[ 2\sqrt{\frac{r}{\mu}} + \log \frac{\sqrt{r} - \sqrt{\mu}}{\sqrt{r} + \sqrt{\mu}} \right] + \phi_0$$

## Self-tuned Schwarzschild-de-sitter

$$f(r) = h(r) = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2 \quad \longrightarrow \quad \Lambda_{\text{eff}} = -\frac{\zeta\eta}{\beta}$$

this metric represent Schwarzschild BH  
in the presence of cosmological constant

**present model  $\Lambda$  does not appear!**

we do not conceive huge bare  $\Lambda$  through the metric



# Dressing BH in Shift-symmetric ST theory

$$\mathcal{L} = [\zeta R - \eta(\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu\phi\partial_\nu\phi - 2\Lambda]$$

## Stealth Schwarzschild

$$f(r) = h(r) = 1 - \frac{\mu}{r} \quad \mu : \text{const.}$$

$$\phi_\pm = qt \pm q\mu \left[ 2\sqrt{\frac{r}{\mu}} + \log \frac{\sqrt{r} - \sqrt{\mu}}{\sqrt{r} + \sqrt{\mu}} \right] + \phi_0$$

## Self-tuned Schwarzschild-de-sitter

$$f(r) = h(r) = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2 \quad \longrightarrow \quad \Lambda_{\text{eff}} = -\frac{\zeta\eta}{\beta}$$

$$\psi(r)' = \pm \frac{q}{h(r)} \sqrt{1 - h(r)} \quad \Lambda > \Lambda_{\text{eff}}$$

**Bavichev, Charmousis(2014) can be generalized**

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

# **BH perturbations with time-dependent scalar**

**HO, Kobayashi, Suyama  
in preparation**

# BH perturbations with **time-dependent scalar**

**The most general 2nd-order theory with shift & reflection symmetries**

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

$$\phi(t, r) = qt + \psi(r)$$

**Perturbations can be written as following eq (odd-parity)**

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

$$h_{tt} = 0, \quad h_{tr} = 0, \quad h_{rr} = 0$$

$$h_{ta} = \sum_{l,m} h_{0,lm}(t, r) E_{ab} \partial^b Y_{lm}(\theta, \varphi)$$

$$h_{ra} = \sum_{l,m} h_{1,lm}(t, r) E_{ab} \partial^b Y_{lm}(\theta, \varphi)$$

$$h_{ab} = \sum_{l,m} h_{2,lm}(t, r) [E_a^c \nabla_c \nabla_b Y_{lm}(\theta, \varphi) + E_b^c \nabla_c \nabla_a Y_{lm}(\theta, \varphi)]$$

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2 d\Omega^2$$

$$E_{ab} = \sqrt{\det\gamma} \epsilon_{ab}$$

$\gamma_{ab}$  two-dim metric on the sphere

$\epsilon_{ab}$  Levi-Civita symbol

**Detail: Suyama san's poster "BH perturbations in Horndeski theory"**

# BH perturbations with **time-dependent scalar**

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \quad ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2d\Omega^2$$

$$\phi(t, r) = qt + \psi(r)$$

**action 2nd-order in perturbations**

**field redefinition**

**master eq**

**Determination of perturbations behavior**

# BH perturbations with **time-dependent scalar**

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \quad ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2d\Omega^2$$
$$\phi(t, r) = qt + \psi(r)$$

**action 2nd-order in perturbations**

**time-dependent scalar(my result)**

**mixing term!**

$$\frac{2l+1}{2\pi}\mathcal{L}^{(2)} = A_1h_0^2 + A_2h_1^2 + A_4h_0h_1$$
$$\dot{h}_i := \frac{\partial h_i}{\partial t}, \quad h'_i := \frac{\partial h_i}{\partial r}$$
$$+ A_3 \left( \dot{h}_1^2 - 2h'_0\dot{h}_1 + h_0'^2 + \frac{4}{r}h_0\dot{h}_1 \right)$$

**static scalar(Kobayashi, Motohashi, Suyama(2012))**

$$\frac{2l+1}{2\pi}\mathcal{L}^{(2)} = a_1h_0^2 + a_2h_1^2$$
$$+ a_3 \left( \dot{h}_1^2 - 2\dot{h}_1h'_0 + h_0'^2 + \frac{4}{r}\dot{h}_1h_0 \right)$$

# BH perturbations with **time-dependent scalar**

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \quad ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2d\Omega^2$$

$$\phi(t, r) = qt + \psi(r)$$

**time-dependent**

$$A_3 = G_4 - \left( \frac{q^2}{A} - B\Psi'^2 \right) G_{4X}$$

$$\begin{aligned} \frac{2l+1}{2\pi} \mathcal{L}^{(2)} = & A_1 h_0^2 + A_2 h_1^2 + A_4 h_0 h_1 \\ & + A_3 \left( \dot{h}_1^2 - 2h_0' \dot{h}_1 + h_0'^2 + \frac{4}{r} h_0 \dot{h}_1 \right) \end{aligned}$$

$$\begin{aligned} \frac{2l+1}{2\pi} \mathcal{L}^{(2)} = & a_1 h_0^2 + a_2 h_1^2 \\ & + a_3 \left( \dot{h}_1^2 - 2\dot{h}_1 h_0' + h_0'^2 + \frac{4}{r} \dot{h}_1 h_0 \right) \end{aligned}$$

**static**

$$a_3 = G_4 - B\Psi'^2 G_{4X}$$

other coefficients are modified similarly but not in exactly the same way

 **Implications (difference time-dependent between static)**

propagation of speed changes?

stability conditions changes?

# Summary and Outlook

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## **BH with time-dependent scalar**

found some interesting Black Hole solutions

## **BH stability in Shift-symmetric ST theory**

*obtained action up to 2nd-order in perturbations*

**BH stability conditions may change**

## **Outlook**

derive master equation

obtain BH stability conditions